

On the Presumptive Meaning of Logical Connectives The Adaptive Logics Approach to Gricean Pragmatics

Hans Lycke

Centre for Logic and Philosophy of Science Ghent University Hans.Lycke@Ugent.be http://logica.ugent.be/hans

Vereniging voor Analytische Filosofie (VAF) January 22–23 2009, Tilburg

(ロ) (日) (日) (日) (日)

Outline



Introduction

- Conversational Implicatures
- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

- 3 The Adaptive Logics Approach
 - Introduction
 - Taking Utterances Seriously
 - The Adaptive Logic CL^{gci}

4 Conclusion



Outline



Conversational Impli

- Conversational Implicatures
- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

- 3 The Adaptive Logics Approach
 - Introduction
 - Taking Utterances Seriously
 - The Adaptive Logic CL^{gci}

4 Conclusion



Conversational Implicatures

Conversational Implicatures

The pragmatic rules allowing the hearer in a conversation to derive the intended informational content of the speaker's message.



Conversational Implicatures

Conversational Implicatures

The pragmatic rules allowing the hearer in a conversation to derive the intended informational content of the speaker's message.

= to derive what is **meant** (by the speaker) from what is **said** (by the speaker)!



Conversational Implicatures

Conversational Implicatures

The pragmatic rules allowing the hearer in a conversation to derive the intended informational content of the speaker's message.

= to derive what is **meant** (by the speaker) from what is **said** (by the speaker)!

The Cooperative Principle

Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1989, p. 26)



Image: A marked and A mar A marked and A

Conversational Implicatures

Conversational Implicatures

The pragmatic rules allowing the hearer in a conversation to derive the intended informational content of the speaker's message.

= to derive what is **meant** (by the speaker) from what is **said** (by the speaker)!

The Cooperative Principle

Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1989, p. 26)

- \Rightarrow The Gricean Maxims
 - Specific instantiations of the Cooperative Principle
 - Presumptions about utterances a hearer relies on to get at the intended meaning of an utterance, and a speaker exploits to get a message transferred successfully.



Outline



Introduction

• Conversational Implicatures

Generalized Conversational Implicatures

Aim of this talk

2 Some Earlier Attempts

3 The Adaptive Logics Approach

- Introduction
- Taking Utterances Seriously
- The Adaptive Logic CL^{gci}

4 Conclusion



Generalized Conversational Implicatures

Generalized Conversational Implicatures (GCI)

Conversational implicatures that only depend on **what** is said and not on the extra–linguistic context.



Generalized Conversational Implicatures

Generalized Conversational Implicatures (GCI)

Conversational implicatures that only depend on **what** is said and not on the extra–linguistic context.

Distinctive Properties of GCI

- Calculability
 - GCI are calculable from the utterance of a sentence in a particular context.



Generalized Conversational Implicatures

Generalized Conversational Implicatures (GCI)

Conversational implicatures that only depend on **what** is said and not on the extra–linguistic context.

Distinctive Properties of GCI

- Calculability
 - GCI are calculable from the utterance of a sentence in a particular context.
- Nondetachability
 - the GCI related to some utterance would also have been triggered in case the literal content of the utterance would have been expressed differently (in the same context).



Generalized Conversational Implicatures

Generalized Conversational Implicatures (GCI)

Conversational implicatures that only depend on **what** is said and not on the extra–linguistic context.

Distinctive Properties of GCI

- Calculability
 - GCI are calculable from the utterance of a sentence in a particular context.
- Nondetachability
 - the GCI related to some utterance would also have been triggered in case the literal content of the utterance would have been expressed differently (in the same context).
- Cancellability



 GCI might be refuted because they are in conflict with other utterances (of the speaker) or with background knowledge (of the hearer) about the context.

Generalized Conversational Implicatures

Hence

GCI are **defeasible steps** in the uncovering of the intended meaning of an utterance.



- E - N

Generalized Conversational Implicatures

Hence

GCI are **defeasible steps** in the uncovering of the intended meaning of an utterance.

⇒ What is pragmatically derived by the hearer doesn't follow logically from what is said by the speaker.



Generalized Conversational Implicatures

Hence

GCI are **defeasible steps** in the uncovering of the intended meaning of an utterance.

⇒ What is pragmatically derived by the hearer doesn't follow logically from what is said by the speaker.

Definition

to follow logically = derivable by means of classical logic (CL)



Generalized Conversational Implicatures

Hence

GCI are **defeasible steps** in the uncovering of the intended meaning of an utterance.

⇒ What is pragmatically derived by the hearer doesn't follow logically from what is said by the speaker.

Definition

to follow logically = derivable by means of classical logic (CL)

Levinson's Claim

GCI should be modeled formally as non-monotonic inference rules!



Outline



Introduction

- Conversational Implicatures
- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

- 3 The Adaptive Logics Approach
 - Introduction
 - Taking Utterances Seriously
 - The Adaptive Logic CL^{gci}

4 Conclusion



< 3 ×

Aim of this talk

A Threefold Aim

• I will consider some of the earlier attempts to model **GCI** as non-monotonic inference rules.



Aim of this talk

A Threefold Aim

- I will consider some of the earlier attempts to model **GCI** as non-monotonic inference rules.
- I will contend that GCI can be captured satisfactorily by relying on the adaptive logics approach.



Aim of this talk

A Threefold Aim

- I will consider some of the earlier attempts to model **GCI** as non-monotonic inference rules.
- I will contend that **GCI** can be captured satisfactorily by relying on the adaptive logics approach.
- [I will argue that cooperative communication is a very dynamic and context-dependent problem-solving activity.]



1) Verhoeven & Horsten (Studia Logica, 2005)



A B > A B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A

1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).



1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Verhoeven & Horsten's Proposal



1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Verhoeven & Horsten's Proposal

- BUT: This approach doesn't capture the informational strength of the or-implicature to a full extent.
 - \Rightarrow The premises $A \lor B$ and A do not lead to $\neg B$.



1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Verhoeven & Horsten's Proposal

- BUT: This approach doesn't capture the informational strength of the or-implicature to a full extent.
 - \Rightarrow The premises $A \lor B$ and A do not lead to $\neg B$.
 - The approach confuses the viewpoint of the speaker with the viewpoint of the hearer.



1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Verhoeven & Horsten's Proposal

- BUT: This approach doesn't capture the informational strength of the or-implicature to a full extent.
 - $\Rightarrow \quad \text{The premises } A \lor B \text{ and } A \text{ do not lead to } \neg B.$
 - The approach confuses the viewpoint of the speaker with the viewpoint of the hearer.
 - \Rightarrow Cooperative communication is a problem–solving activity.



1) Verhoeven & Horsten (Studia Logica, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Verhoeven & Horsten's Proposal

- BUT: This approach doesn't capture the informational strength of the or-implicature to a full extent.
 - $\Rightarrow \quad \text{The premises } A \lor B \text{ and } A \text{ do not lead to } \neg B.$
 - The approach confuses the viewpoint of the speaker with the viewpoint of the hearer.
 - \Rightarrow Cooperative communication is a problem–solving activity.
 - ⇒ Meaning is dependent on the context (= problem–solving situation).



2) Horsten (Synthese, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).



2) Horsten (Synthese, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Horsten's Proposal

Substitute in a sentence every formula of the form $A \lor B$ by the formula $(A \lor B) \land \neg (A \land B)$ (a substitutional approach).



2) Horsten (Synthese, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Horsten's Proposal

Substitute in a sentence every formula of the form $A \lor B$ by the formula $(A \lor B) \land \neg (A \land B)$ (a substitutional approach).

- BUT: Horsten does not provide a mechanism to reject implicatures in case this is necessary.
 - ⇒ Implicatures are not modeled as non-monotonic inference rules!



2) Horsten (Synthese, 2005)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Horsten's Proposal

Substitute in a sentence every formula of the form $A \lor B$ by the formula $(A \lor B) \land \neg (A \land B)$ (a substitutional approach).

- BUT: Horsten does not provide a mechanism to reject implicatures in case this is necessary.
 - ⇒ Implicatures are not modeled as non-monotonic inference rules!



• This is a formal approach, but not a logic.

3) Wainer (Journal of Logic, Language and Information, 2007)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).



3) Wainer (Journal of Logic, Language and Information, 2007)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Wainer's Proposal

Also a substitutional approach!



3) Wainer (Journal of Logic, Language and Information, 2007)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Wainer's Proposal

Also a substitutional approach!

MOREOVER: Wainer does provide a mechanism to reject implicatures in case this is necessary (by means of circumscription).



3) Wainer (Journal of Logic, Language and Information, 2007)

The quantitative scalar or-implicature

If a disjunction is asserted in a conversation, it should be interpreted as an exclusive disjunction.

Formally: If a speaker says A or B, it is implicated that not (A and B).

Wainer's Proposal

Also a substitutional approach!

- MOREOVER: Wainer does provide a mechanism to reject implicatures in case this is necessary (by means of circumscription).
- BUT: There is no proof theoretic characterization, only a semantic one.
 - ⇒ Implicatures are not modeled as non-monotonic inference rules!



A (1) > A (2) > A

Outline



Conversational Implicatures

- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

3 The Adaptive Logics Approach

Introduction

- Taking Utterances Seriously
- The Adaptive Logic CL^{gci}

Conclusion


Introduction

Adaptive Logics?

Adaptive Logics are formal logics that were developed to explicate dynamic (reasoning) processes (both monotonic and non–monotonic ones).

e.g. Induction, abduction, default reasoning,...



Introduction

Adaptive Logics?

Adaptive Logics are formal logics that were developed to explicate dynamic (reasoning) processes (both monotonic and non–monotonic ones).

e.g. Induction, abduction, default reasoning,...

Example

I will show how two of the best-known **GCI** can be captured by means of an adaptive logic, viz.

- The or-implicature: A or B implicates not (A and B).
- The existential-implicature: (some α)A(α) implicates not (all α)A(α).

A (1) > A (2) > A

Outline



Conversational Implicatures

- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

3 The Adaptive Logics Approach

- Introduction
- Taking Utterances Seriously
- The Adaptive Logic CL^{gci}

Conclusion



Taking Utterances Seriously

Main Idea

GCI are triggered by the utterances made by the speaker in a conversation.



Taking Utterances Seriously

Main Idea

GCI are triggered by the utterances made by the speaker in a conversation.

+ There is a difference between the utterances made by the speaker and the consequences derived from those utterances by the hearer.



Taking Utterances Seriously

Main Idea

GCI are triggered by the utterances made by the speaker in a conversation.

- + There is a difference between the utterances made by the speaker and the consequences derived from those utterances by the hearer.
 - ⇒ This difference should be taken into account when formalizing GCI.

 \Rightarrow the logic **CL^d**



Taking Utterances Seriously

Main Idea

GCI are triggered by the utterances made by the speaker in a conversation.

- + There is a difference between the utterances made by the speaker and the consequences derived from those utterances by the hearer.
 - ⇒ This difference should be taken into account when formalizing GCI.

 \Rightarrow the logic **CL^d**

Preview

The logic **CL^d** will be used to capture **GCI**

by means of the adaptive logic CL^{gci}, based on CL^d

Taking Utterances Seriously

Language Schema of CL^d

language	letters	connectives	set of formulas
\mathcal{L}	S	$\neg, \land, \lor, \supset, \equiv, \exists, \forall, =$	\mathcal{W}
\mathcal{L}^+	S	$\neg, \land, \lor, \supset, \equiv, \exists, \forall, =$	\mathcal{W}^+
		$ \dot{\neg},\dot{\wedge},\dot{\lor},\dot{\supset},\dot{\equiv},\dot{\exists},\dot{\forall},\dot{=}$	



A (1) > A (2) > A

Taking Utterances Seriously

Language Schema of CL^d

language	letters	connectives	set of formulas
\mathcal{L}	S	$\neg, \land, \lor, \supset, \equiv, \exists, \forall, =$	\mathcal{W}
\mathcal{L}^+	S	$\neg, \land, \lor, \supset, \equiv, \exists, \forall, =$	\mathcal{W}^+
		$\dot{\neg}, \dot{\wedge}, \dot{\lor}, \dot{\supset}, \dot{\equiv}, \dot{\exists}, \dot{\forall}, \dot{=}$	

Representing Utterances

In order to express that a formula is an utterance, it will be formalized using *dotted connectives* only.

 \Rightarrow *A^d* will be used to express that the formula *A* is an utterance (only contains dotted connectives).



Taking Utterances Seriously

Proof Theory of CL^d

= the axiom system of **CL**, extended by the following axiom schemas,

NDD NDF

$$\begin{array}{ll} \mathsf{DN} & \neg A \supset \neg A \ (A \in \mathcal{S}) \\ \mathsf{DC} & (A \land B) \supset (A \land B) \\ \mathsf{DD} & (A \lor B) \supset (A \lor B) \\ \mathsf{DF} & (\forall \alpha) A(\alpha) \supset (\forall \alpha) A(\alpha) \\ \mathsf{DId} & (\alpha \doteq \beta) \supset (\alpha = \beta) \end{array}$$

DDN
$$\neg \neg A \supset A$$
 (*A* an utterance)
NDC $\neg (A \land B) \supset (\neg A \lor \neg B)$
NDD $\neg (A \lor B) \supset (\neg A \land \neg B)$
NDF $\neg (\forall \alpha) A(\alpha) \supset (\exists \alpha) \neg A(\alpha)$



Taking Utterances Seriously

Proof Theory of CL^d

= the axiom system of CL, extended by the following axiom schemas,

DDN NDC NDD NDF

DN
$$\neg A \supset \neg A \ (A \in S)$$

DC $(A \land B) \supset (A \land B)$
DD $(A \lor B) \supset (A \lor B)$
DF $(\forall \alpha) A(\alpha) \supset (\forall \alpha) A(\alpha)$
DId $(\alpha \doteq \beta) \supset (\alpha = \beta)$

and the following definitions.

$$\begin{array}{l} A \dot{\supset} B =_{df} \dot{\neg} A \dot{\lor} B \\ A \dot{\equiv} B =_{df} (A \dot{\supset} B) \dot{\land} (B \dot{\supset} A) \\ (\dot{\exists} \alpha) A(\alpha) =_{df} \dot{\neg} (\dot{\forall} \alpha) \dot{\neg} A(\alpha) \end{array}$$

VERSITED

Taking Utterances Seriously

Definition

 $\mathcal{W}^d \subset \mathcal{W}^+$ is the set of all formulas A such that all connectives that occur in A are dotted connectives.



Taking Utterances Seriously

Definition

 $\mathcal{W}^d \subset \mathcal{W}^+$ is the set of all formulas *A* such that all connectives that occur in *A* are dotted connectives.

Representing Communicative Situations

Premise sets are restricted to subsets of the set $W \cup W^d$.



Taking Utterances Seriously

Definition

 $\mathcal{W}^d \subset \mathcal{W}^+$ is the set of all formulas A such that all connectives that occur in A are dotted connectives.

Representing Communicative Situations

Premise sets are restricted to subsets of the set $W \cup W^d$.

- \Rightarrow Premise sets only contain
 - utterances (elements of \mathcal{W}^d), and
 - background knowledge about the communicative context that is taken (by the hearer) to be shared by both speaker and hearer (elements of W).



Taking Utterances Seriously

[Appendix 1]

Inferential Strength of the logic CL^d

Some CL-inference rules are not valid for dotted connectives, e.g.

- From a formula A, it is impossible to derive $A \dot{\lor} B$.
- From a formula $A(\beta)$, it is impossible to derive $(\dot{\exists}\alpha)A(\alpha)$.



Taking Utterances Seriously

[Appendix 2]

Definition $\Gamma^d = \{ A^d \mid A \in \Gamma \subset \mathcal{W} \}.$



(I)

Taking Utterances Seriously

[Appendix 2]

Definition $\Gamma^{d} = \{ A^{d} \mid A \in \Gamma \subset \mathcal{W} \}.$

The Classical Consequence Set

For $\Gamma \cup \{A\} \subset \mathcal{W}$, $\Gamma \vdash_{\mathsf{CL}} A$ iff $\Gamma^d \vdash_{\mathsf{CL}^d} A$.



A (10) × (10) × (10)

Taking Utterances Seriously

[Appendix 2]

Definition $\Gamma^{d} = \{ A^{d} \mid A \in \Gamma \subset \mathcal{W} \}.$

The Classical Consequence Set

For $\Gamma \cup \{A\} \subset \mathcal{W}$, $\Gamma \vdash_{\mathsf{CL}} A$ iff $\Gamma^d \vdash_{\mathsf{CL}^d} A$.

 $\Rightarrow~$ The hearer is able to derive all **CL**–consequences from the utter-ances of the speaker.



Outline



Conversational Im

- Conversational Implicatures
- Generalized Conversational Implicatures
- Aim of this talk

2 Some Earlier Attempts

3 The Adaptive Logics Approach

- Introduction
- Taking Utterances Seriously
- The Adaptive Logic CL^{gci}

4 Conclusion



The Adaptive Logic CL^{gci}





The Adaptive Logic CL^{gci}

General Characterization

- 1. Lower Limit Logic (LLL)
- 2. Set of Abnormalities $\Omega = \Omega^{\vee} \cup \Omega^{\exists}$

3. Adaptive Strategy



The Adaptive Logic CL^{gci}

General Characterization

- 1. Lower Limit Logic (LLL): the logic CL^d
- 2. Set of Abnormalities $\Omega = \Omega^{\dot{\vee}} \cup \Omega^{\dot{\exists}}$

3. Adaptive Strategy



The Adaptive Logic CL^{gci}

General Characterization

- 1. Lower Limit Logic (LLL): the logic CL^d
- 2. Set of Abnormalities $\Omega = \Omega^{\dot{\vee}} \cup \Omega^{\dot{\exists}}$

$$\Omega^{\dot{\vee}} = \{ (A^d \dot{\vee} B^d) \land (A \land B) \mid A, B \in \mathcal{W} \}$$

$$\Omega^{\dot{\exists}} = \{ (\dot{\exists}\alpha) A^{d}(\alpha) \land (\forall \alpha) A(\alpha) \mid A \in \mathcal{W} \}$$

3. Adaptive Strategy



The Adaptive Logic CL^{gci}

General Characterization

- 1. Lower Limit Logic (LLL): the logic CL^d
- 2. Set of Abnormalities $\Omega = \Omega^{\dot{\vee}} \cup \Omega^{\dot{\exists}}$

$$\Omega^{\dot{\vee}} = \{ (A^d \dot{\vee} B^d) \land (A \land B) \mid A, B \in \mathcal{W} \}$$

$$\Omega^{\dot{\exists}} = \{ (\dot{\exists}\alpha) A^d(\alpha) \land (\forall \alpha) A(\alpha) \mid A \in \mathcal{W} \}$$

3. Adaptive Strategy: reliability, minimal abnormality, normal selections,...



The Adaptive Logic CL^{gci}

General Characterization

- 1. Lower Limit Logic (LLL): the logic CL^d
- 2. Set of Abnormalities $\Omega = \Omega^{\dot{\vee}} \cup \Omega^{\dot{\exists}}$

$$\Omega^{\dot{\vee}} = \{ (A^d \dot{\vee} B^d) \land (A \land B) \mid A, B \in \mathcal{W} \}$$

$$\Omega^{\dot{\exists}} = \{ (\dot{\exists}\alpha) A^d(\alpha) \land (\forall \alpha) A(\alpha) \mid A \in \mathcal{W} \}$$

3. Adaptive Strategy: reliability, minimal abnormality, normal selections,...



The Adaptive Logic CL^{gci}: Proof Theory (1)



The Adaptive Logic **CL^{gci}**: Proof Theory (1)

- A CL^{gci}-proof is a succession of stages, each consisting of a sequence of lines.
 - Adding a line = to move on to a next stage



The Adaptive Logic **CL^{gci}**: Proof Theory (1)

- A CL^{gci}-proof is a succession of stages, each consisting of a sequence of lines.
 - Adding a line = to move on to a next stage
- Each line consists of 4 elements:
 - Line number
 - Formula
 - Justification
 - Adaptive condition = set of abnormalities



The Adaptive Logic **CL^{gci}**: Proof Theory (1)

- A CL^{gci}-proof is a succession of stages, each consisting of a sequence of lines.
 - Adding a line = to move on to a next stage
- Each line consists of 4 elements:
 - Line number
 - Formula
 - Justification
 - Adaptive condition = set of abnormalities
- Deduction Rules



The Adaptive Logic **CL^{gci}**: Proof Theory (1)

- A CL^{gci}-proof is a succession of stages, each consisting of a sequence of lines.
 - Adding a line = to move on to a next stage
- Each line consists of 4 elements:
 - Line number
 - Formula
 - Justification
 - Adaptive condition = set of abnormalities
- Deduction Rules
- Marking Criterium
 - Dynamic proofs

The Adaptive Logic CLgci: Proof Theory (2)





The Adaptive Logic CL^{gci}: Proof Theory (3)

Marking Criterium: Normal Selections Strategy

• Dab-consequences

 $Dab(\Delta)$ is a Dab-consequence of Γ at stage *s* of the proof iff $Dab(\Delta)$ is derived at stage *s* on the condition \emptyset .



The Adaptive Logic CL^{gci}: Proof Theory (3)

Marking Criterium: Normal Selections Strategy

• Dab-consequences

 $Dab(\Delta)$ is a Dab-consequence of Γ at stage *s* of the proof iff $Dab(\Delta)$ is derived at stage *s* on the condition \emptyset .

Marking Definition

Line *i* is marked at stage *s* of the proof iff, where Δ is its condition, $Dab(\Delta)$ is a Dab-consequence at stage *s*.



The Adaptive Logic CL^{gci}: Proof Theory (4)

Derivability

A is derived from Γ at stage s of a proof iff A is the second element of an unmarked line at stage s.



The Adaptive Logic CL^{gci}: Proof Theory (4)

Derivability

A is derived from Γ at stage s of a proof iff A is the second element of an unmarked line at stage s.

Final Derivability

- A is finally derived from Γ on a line *i* of a proof at stage *s* iff (i) A is the second element of line *i*, (ii) line *i* is not marked at stage *s*, and (iii) every extension of the proof in which line *i* is marked may be further extended in such a way that line *i* is unmarked.
- $\Gamma \vdash_{CL^{gci}} A$ iff A is finally derived on a line of a proof from Γ .



The Adaptive Logic CL^{gci}: Example 1

Example



• • • • • • • • • • • • • •
The Adaptive Logic CL^{gci}: Example 1

Example

- 1 $(\exists x)Px$ -;PREM^{*u*} Ø 2 $(\forall x)(Px \land Qx)$ -;PREM^{*bk*} Ø (∃x)Px



・ 同 ト ・ ヨ ト ・ ヨ

The Adaptive Logic CL^{gci}: Example 1

Example		
1 $(\exists x)Px$ 2 $(\forall x)(Px \land Qx)$ 3 $\neg(\forall x)Px$ 4 $(\exists x)\neg Px$	–;PREM [#] –;PREM ^{bk} 1;RC 3;RU	$\emptyset \\ \emptyset \\ \{ (\exists x) Px \land (\forall x) Px \} \\ \{ (\exists x) Px \land (\forall x) Px \} \end{cases}$



Image: A matched and A matc

The Adaptive Logic CL^{gci}: Example 1

Ex	ample		
1 2 3 4 5 6	$(\exists x)Px (\forall x)(Px \land Qx) \neg (\forall x)Px (\exists x)\neg Px (\forall x)Px (\exists x)Px \land (\forall x)Px $	–;PREM ^{<i>u</i>} –;PREM ^{<i>bk</i>} 1;RC 3;RU 2;RU 2;RU 2,5;RU	$\emptyset \\ \{ (\dot{\exists} x) Px \land (\forall x) Px \} \\ \{ (\dot{\exists} x) Px \land (\forall x) Px \} \\ \emptyset \\ \emptyset \end{cases}$



• • • • • • • • • • • • • •

The Adaptive Logic CL^{gci}: Example 1

Ex	ample		
1 2 3 4 5 6	$(\exists x)Px (\forall x)(Px \land Qx) \neg (\forall x)Px (\exists x)\neg Px (\forall x)Px (\exists x)Px \land (\forall x)Px $	-;PREM ^u -;PREM ^{bk} 1;RC 3;RU 2;RU 2;RU 2,5;RU	$\emptyset \\ \{ (\exists x) Px \land (\forall x) Px \} \checkmark \\ \{ (\exists x) Px \land (\forall x) Px \} \checkmark \\ \emptyset \\ \emptyset \\ \emptyset $



Image: A matched and A matc

The Adaptive Logic CL^{gci}: Example 2





The Adaptive Logic CL^{gci}: Example 2

Example $p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)$ q1 –:PREM ∅ 2 −;PREM Ø



The Adaptive Logic CL^{gci}: Example 2





・ 同 ト ・ ヨ ト ・ ヨ

The Adaptive Logic CL^{gci}: Example 2

Examp	le		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	(¬q∧¬r) ∧¬(¬q∧¬r)) (¬q∧¬r) ¬q	-;PREM -;PREM 1;RC 3;RU 4;RU	



The Adaptive Logic CL^{gci}: Example 2

E	ample		
1	$p \dot{\lor} \dot{\lnot} (\dot{\lnot} q \dot{\land} \dot{\lnot} r)$	-;PREM Ø	
2	q	-;PREM Ø	
3	$\neg (p \land \neg (\neg q \land \neg r))$	1;RC { $(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))$ }	
4	$\neg p \lor (\neg q \land \neg r)$	3;RU { $(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))$ }	
5	$\neg p \lor \neg q$	4;RU { $(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))$ }	
6	$\neg p$	2,5;RU { $(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))$ }	



Image: A matrix

The Adaptive Logic CL^{gci}: Example 2

Ex	ample		
1	$p\dot{ee}\dot{\neg}(\dot{\neg}q\dot{\wedge}\dot{\neg}r)$	–;PREM	Ø
2	q	–;PREM	Ø
6			
	¬p	2,5;RU	$\{(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))\}$



A B > A B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A

The Adaptive Logic CL^{gci}: Example 2

Ex	ample		
1 2	$p \dot{\lor} \neg (\neg q \dot{\land} \neg r) \ q$	–;PREM –;PREM	Ø Ø
 6 7 8	 ¬p ¬(¬q∖¬r) ¬¬q∨̈́¬¬r	 2,5;RU 1,6;RU 7;RU	 $ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)) \land (p \land \neg (\neg q \land \neg r)) \} $ $ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)) \land (p \land \neg (\neg q \land \neg r)) \} $ $ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)) \land (p \land \neg (\neg q \land \neg r)) \} $



• • • • • • • • • • • • • •

The Adaptive Logic CL^{gci}: Example 2

Ex	Example		
1 2	$p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r) \ q$	–;PREM –;PREM	Ø Ø
 6 7 8 9	 ¬ <i>p</i> ¬(<i>¬q</i> ∧ <i>¬r</i>) <i>¬¬q</i> ∧ <i>¬¬r</i>)	 2,5;RU 1,6;RU 7;RU 8;RC	$ \begin{array}{l} & \cdots \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) , \\ (\dot{\neg} \dot{\neg} q \dot{\lor} \neg \gamma) \land (\neg \neg q \land \neg \neg r) \} \end{array} $



Image: A matched black

The Adaptive Logic CL^{gci}: Example 2

Ex	Example			
1 2	$p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r) \ q$	–;PREM –;PREM	Ø Ø	
 6 7 8 9	 ¬ <i>p</i> ¬(¬ <i>q</i> ∧¬ <i>r</i>) ¬¬(¬¬ <i>q</i> ∧¬¬ <i>r</i>)	 2,5;RU 1,6;RU 7;RU 8;RC	$ \begin{array}{l} & \cdots \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)) \} \\ \{ (p \dot{\lor} \dot{\neg} \dot{\neg} r) \land (p \land \neg (\neg q \land \neg r)) \} \end{array} $	
10	$\neg q \lor \neg r$	9;RU	$\{(p \lor \neg (\neg q \land \neg r)) \land (p \land \neg (\neg q \land \neg r)), (\neg q \land \neg r)\}$	
11	¬r	2,10;RU	$ \{ (p \lor \neg (\neg q \land \neg r)) \land (p \land \neg (\neg q \land \neg r)), \\ (\neg \neg q \lor \neg \neg r) \land (\neg \neg q \land \neg -r) \} $	



A B > A B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A

H. Lycke (Ghent University)

The Adaptive Logic CL^{gci}: Example 2

Example			
1	$p \dot{ee} \dot{\neg} (\dot{\neg} q \dot{\wedge} \dot{\neg} r) \ q$	–;PREM	Ø
2		–;PREM	Ø
6	¬p	2,5;RU	$\{(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))\}$
			$ \begin{array}{l} \overset{\cdots}{\{(p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)) \land (p \land \neg (\neg q \land \neg r)), \\ (\dot{\neg} \dot{\neg} q \dot{\lor} \dot{\neg} \dot{\neg} r) \land (\neg \neg q \land \neg \neg r) \end{array} $
11	¬r	2,10;RU	



• • • • • • • • • • • • • •

The Adaptive Logic **CL^{gci}**: Example 2

Ex	Example			
1 2	$p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r)$ q	−;PREM Ø −;PREM Ø		
 6	 ¬p	 2,5;RU { $(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))$ }		
 11	 ¬r	2,10;RU { $(p \lor \neg (\neg q \land \neg r)) \land (p \land \neg (\neg q \land \neg r)),$ $(\neg \neg q \lor \neg r) \land (\neg \neg q \land \neg \neg r)$ }		
12	r	–;PREM ∅		



A B > A B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A
 B > A

The Adaptive Logic CL^{gci}: Example 2

Ex	Example			
1 2	$p\dot{ee}\dot{\neg}(\dot{\neg}q\dot{\wedge}\dot{\neg}r) \ q$	–;PREM –;PREM	Ø Ø	
 6	 ¬p	 2,5;RU	 $\{(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))\}$	
 11	 ¬r	 2,10;RU	$ \begin{array}{l} \cdots \\ \{ (p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \neg r)) \land (p \land \neg (\neg q \land \neg r)), \\ (\dot{\neg} \neg q \dot{\lor} \neg \neg r) \land (\neg \neg q \land \neg \neg r) \} \end{array} $	
12	r	-;PREM	Ø	
13	$\neg \neg q \land \neg \neg r$	2,12;RU	Ø	
14	$((\dot{\neg}\dot{\neg}q\dot{\lor}\dot{\neg}r)\wedge(\neg\neg q)(\dot{\neg}q\dot{\land}\dot{\neg}r))\wedge(p)$	$(\wedge \neg \neg r)) \land \neg (\neg q /$	/ 1,13;RU ∅ \¬r)))	



(4) The h

The Adaptive Logic **CL^{gci}**: Example 2

Example			
1 2	$p \dot{\lor} \dot{\neg} (\dot{\neg} q \dot{\land} \dot{\neg} r) \ q$	–;PREM –;PREM	Ø Ø
 6	 ¬p	 2,5;RU	 $\{(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\dot{\neg}r))\land(p\land\neg(\neg q\land\neg r))\}$
 11	 ¬r	 2,10;RU	$ \begin{array}{l} \dots \\ \{(p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}\neg r))\land(p\land\neg(\neg q\land\neg r)), \checkmark \\ (\dot{\neg}\dot{\neg}q\dot{\lor}\dot{\neg}\neg r)\land(\neg\neg q\land\neg -r)\} \end{array} $
12	r	-;PREM	Ø
13	$\neg \neg q \land \neg \neg r$	2,12;RU	Ø
14	$((\dot{\neg}\dot{\neg}q\dot{\lor}\dot{\neg}r)\wedge(\neg\neg q))$ $((p\dot{\lor}\dot{\neg}(\dot{\neg}q\dot{\land}r))\wedge(p))$	$(\wedge \neg \neg r)) \land \neg (\neg q \land$	/ 1,13;RU ∅ ∖ ¬r)))



(4) The h

Conclusion

Conclusion

It is possible to capture **GCI** as non-monotonic inference rules by relying on the adaptive logics approach.



Conclusion

Conclusion

It is possible to capture **GCI** as non-monotonic inference rules by relying on the adaptive logics approach.

Further Research

- To extend the approach to all scalar implicatures (to n-tuples).
- To extend the approach to non-scalar implicatures.
- To extend the approach to multiple speakers.



Conclusion

Conclusion

It is possible to capture **GCI** as non-monotonic inference rules by relying on the adaptive logics approach.

Further Research

- To extend the approach to all scalar implicatures (to n-tuples).
- To extend the approach to non-scalar implicatures.
- To extend the approach to multiple speakers.

Thank you!

