Error Handling

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Intro

- Psychology: very large literature on practical errors, see e.g. [Reason, 1990], [Woods, 2010], [Dekker, 2011];
- Epistemology&Philosophy of Science: error detection and resolution in paradigm definition and change (Popper, Lakatos, Kuhn, Bayesian epistemology); see e.g. [Mayo, 1996], [Allchin, 2001], [Mayo, 2010];
- Applications: fault tolerance ([Goodenough, 1975], [Cristian, 1982]); specification design; technological malfunctioning; see e.g. [Turner, 2011].

Logic

- Philosophical Logic: defeasible conditions and bounded resources for knowledge as approximations to errors; see e.g. [Williamson, 1992]; [Williamson, 2002]; [Woods, 2004]; [Sundholm, 2012]; [Bonnay and Egre', 2011];
- What about formal logic?: PDEL; proof-theoretical approach.

Tasks

- formulate a full, empirically-informed characterization of error states for informational systems; presented in [Primiero, 2013];
- 2 TODAY: a formal model of logical processes with error states.

Outline



2 Making and Eliminating Errors in 3 Steps



Image: A matrix

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Types

• Types are semantic categories of objects:

- n = 0 and S(n) are terms of the type Nat;
- true; false are terms of type Bool;
- values on a continuous line are terms of type *Real*;
- proofs are terms of the type Prop;
- programs are terms of the type Spec;

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- Complex expressions are construed by functional operations:
 - introduction and elimination for connectives (proof-theoretical style)
 - abstraction and applications (λ-style)
 - predications over sets of functions for quantifiers (Σ, Π)

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 - predications over sets of functions for quantifiers (Σ, Π)
- validity is by type-checking (truth for *Prop*; termination for *Spec*).

A Basic Language for the Type of Propositions

Definition (Syntax)

The basic alphabet is composed as follows:

Terms := x | a, variables and constants;

 $\texttt{Types} := \alpha \mid \bot \mid \alpha + \beta \mid \alpha \times \beta \mid \alpha \to \beta$

Connectives

 $\frac{\mathbf{a}:\alpha \quad \mathbf{b}:\beta}{(\mathbf{a},\mathbf{b}):\alpha\times\beta} I \times \qquad \frac{(\mathbf{a},\mathbf{b}):\alpha\times\beta}{\mathbf{a}:\alpha} \mathbf{E}\times(I)$ $\frac{a:\alpha}{a:\alpha+\beta}I+(1) \qquad \frac{b:\beta}{b:\alpha+\beta}I+(2)$ $\frac{\alpha + \beta}{c;\gamma} = \frac{a: \alpha \vdash c: \gamma}{c;\gamma} = \frac{b: \beta \vdash c: \gamma}{c+\gamma} E + C$ $\frac{a:\alpha \quad \alpha \vdash b:\beta}{(x)b:\alpha \to \beta} I \to \frac{(x)b:\alpha \to \beta}{\alpha \vdash b:\beta} E \to$

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Standard \perp

Standard rule for falsehood (a similar approach in PDEL):

 $\frac{\Delta, \Gamma \vdash m: \bot}{\Delta; \Gamma \vdash ABORT(m): \alpha}$

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No use of the distinction between local Γ and global resources Δ;

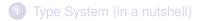
- No internal structure for the error term m;
- No available recovery from ABORT.

Wishlist

Control on local/global validity;

Define minimal (in-)correctness conditions;

- Analyse errors in view of the different conditions breaches;
- Obesign formal strategies to identify and correct errors.







A Premise: Errors as Limits of Validity

- Errors as emergence of uncertainty;
- To express uncertainty, we need to limit validity;
- Our calculus will admit *local* validity.

Step 1: Add Local Execution Functions

- *run_i*: execution of a term bounded to a location; call by value;
- exec: global unbounded value, valid for every other term to call; call by name;
- \rightarrow : to express implication from *exec*;
- \supset : to express implication from *run*;
- *synchro*: to pass a \supset construction to an *exec* value.

Modified Syntax

Definition (Syntax)

Functions and their values are given as:

 $\begin{array}{l} \text{Terms} := \textbf{x}_i \mid \textbf{a}_i, \text{ for } i \in \text{I} \\ \text{Types} := \alpha \mid \perp \mid \alpha \times \beta \mid \alpha + \beta \mid \alpha \to \beta \mid \alpha \supset \beta \\ \text{Locations} := \textbf{1} < \cdots < \textbf{n} \\ \text{Operations} := \textbf{exec}(\alpha) \mid \textbf{run}_i(\alpha) \mid \textbf{run}_{i \cup j}(\alpha \cdot \beta) \mid \textbf{run}_{i \cap j}(\alpha \cdot \beta) \mid \\ \textbf{synchro}_j(\beta(\textbf{exec}(\alpha))), \text{ where } \cdot \in \{+, \times\} \end{array}$

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Connectives I

$$\frac{\overline{\Delta_{i}, a_{i}: \alpha \vdash exec(\alpha)} \quad \text{Global}}{\underline{a_{i}: \alpha \vdash exec(\alpha)} \quad \underline{b_{j}: \beta \vdash exec(\beta)} \quad I \times \\
\frac{a_{i}: \alpha \vdash exec(\alpha)}{\vdash run_{i\cap j}(\alpha \times \beta)} \quad E \times (I)$$

$$\frac{a_{i}: \alpha \vdash exec(\alpha)}{\vdash run_{i}(\alpha + \beta)} \quad I + (I) \quad \frac{b_{j}: \beta \vdash exec(\beta)}{run_{j}(\alpha + \beta)} \quad I + (r)$$

$$\frac{\vdash run_{i\cup j}(\alpha + \beta) \quad run_{i}(\alpha) \vdash c_{k}: \gamma \quad run_{j}(\beta) \vdash c_{k}: \gamma}{\vdash run_{i\cap j\cap k}(\gamma)} \quad E + run_{i\cap j\cap k}(\gamma)$$

$$\frac{\vdash run_{i}(\alpha) \qquad x_{i}: \alpha \vdash run_{j}(\beta)}{run_{i\cap j}(\alpha \supset \beta)} I \supset \frac{\vdash run_{i\cap j}(\alpha \supset \beta) \qquad x_{i}: \alpha}{x_{i}: \alpha \vdash run_{j}(\beta)} E \supset x_{i}: \alpha \vdash run_{j}(\beta)$$

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Connectives II

$$\frac{\vdash exec(\alpha) \qquad a_{i}: \alpha \vdash exec(\beta)}{\vdash run_{i\cup j}(\alpha \to \beta)} I \to \frac{\vdash run_{i\cup j}(\alpha \to \beta)}{exec(\alpha) \vdash exec(\beta)} E \to \frac{\vdash run_{i\cap j}(\alpha \supset \beta) \qquad a_{i}: \alpha}{\vdash synchro_{j}(\beta(exec(\alpha)))}$$
Synchro

A Premise: Local means Mobile

- Locally valid terms can be made mobile;
- Errors occur for terms that can be fetched, accessed, broadcasted;
- Our calculus will admit *mobility*.

Step 2: Add Modal Functions for Mobility

- GLOB: is a □-Intro rule; it says that an exec value is everywhere valid;
- BROAD: is a ⇔-Intro rule; it says that a *run* term can be transmitted to a given location;
- RET: is a □-Elim rule; it says that a GLOB value can be returned to a specific address;
- SEND: is a ◇-Elim rule; it moves a run term from a place to another.

Modified Syntax

Definition (Syntax)

Mobility is given as:

Remote Operations := $GLOB(\Box_{i\cup j}\Gamma, \alpha) \mid BROAD(\diamondsuit_{i\cap j}\Gamma, \alpha)$ Portable Code := $RET(\Gamma_{i\cup j}, \alpha) \mid SEND(\Gamma_{i\cap j}, \alpha)$

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Rules for Modality (Intro & Elimination)

$$\frac{\Gamma_{i}, x_{j}: \alpha \vdash run_{j}(\alpha) \qquad \Box_{i}\Gamma, x_{j}(a_{j}): \alpha \vdash exec(\alpha)}{GLOB(\Box_{i\cup j}\Gamma, \alpha)} \text{ RPC1}$$

$$\frac{\Gamma_{i}, x_{j}: \alpha \vdash run_{j}(\alpha) \qquad \diamond_{i}\Gamma \vdash run_{j}(\alpha)}{BROAD(\diamond_{i\cap j}\Gamma, \alpha)} \text{ RPC2}$$

$$\frac{\Box_{i}\Gamma, a_{j}: \alpha \vdash exec(\alpha) \qquad GLOB(\Box_{i\cup j}\Gamma, \alpha)}{RET(\Gamma_{i\cup j}, \alpha)} \text{ PORT1}$$

$$\frac{\Box_{i}\Gamma, x_{j}: \alpha \vdash run_{i\cap j}(\alpha) \qquad BROAD(\diamond_{i\cap j}\Gamma, \alpha)}{SEND(\Gamma_{i\cap j}, \alpha)} \text{ PORT2}$$

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A Premise: Which Errors

- We consider two main kinds of errors:
 - Mistakes; semantic or syntactic errors, generated by non-valid terms at locations;
 - Failures: procedural errors, generated by failing mobility instructions.

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Step 3: Add Error Functions

Two kinds of errors:



1 fail $\mathbb{Q}_i(\tau)$: the state of the system where a term of type τ induces a failure when accessed at index i;

2 mistake(τ): the state of the system where reference to type τ induces an error.

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Two kinds of errors:



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Two concurrency functions:

• access $\mathbb{Q}_i(t: \tau \setminus \bot)$: a command to access a term at a given address:



2 $\phi(\tau)WITH(\Pi \cup I \cup \Gamma)$: a command to execute a resource $\phi := \{run, exec\}$ with a local term;

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Modified Syntax

Definition (Syntax)

Errors are defined/use:

 $H(\text{Error Functions}) := fail@_i(t) | mistake(\tau)$

 $C(Concurrency Functions) := access@_i(t) | \\ \phi(\tau)WITH(\Pi \cup I \cup \Gamma)$

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Rules for Concurrency

$$\frac{\Delta_i, a_i : \alpha \vdash exec(\alpha)}{\Delta_i \vdash access@_i(a:\alpha)} @I \qquad \frac{\Delta_i; \Gamma_i \vdash access@_i(a:\alpha)}{\Delta_i; \Gamma_i, x_i : \alpha \vdash run_i(\alpha)} @E$$

$$\frac{\Delta_i; \Gamma_i \vdash \phi(\alpha)}{\Delta_i \vdash \phi(\alpha) WITH(t, \Gamma)}$$
 WITH-I

 $\frac{\Delta_i \vdash \phi(\alpha) WITH(t, \Gamma) \qquad \Delta_i; \Gamma \vdash t: \alpha}{\Delta_i; \Gamma, t: \alpha \vdash \phi(\alpha)} WITH-E$

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Mistakes: semantic errors

Mistake1: a dependency from a locally invalid value

$$\frac{\Delta_i; \Gamma_i \vdash run_i(\tau \to \bot) \qquad \Delta_i; x_i : \tau \vdash run_i(\upsilon)}{\Gamma_i; \Delta_i \vdash mistake(exec(\upsilon)WITH(\tau))}$$
Mistake1

Image: A matrix

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Mistakes: semantic errors

Mistake1: a dependency from a locally invalid value

$$\frac{\Delta_i; \Gamma_i \vdash run_i(\tau \to \bot) \qquad \Delta_i; x_i : \tau \vdash run_i(\upsilon)}{\Gamma_i; \Delta_i \vdash mistake(exec(\upsilon)WITH(\tau))}$$
Mistake1

HandleMistake1: re-define dependency from local resource:

$$\frac{\Gamma_{i} \vdash \textit{mistake}(\textit{exec}(\upsilon)\textit{WITH}(\tau)) \qquad \Gamma_{i}, t_{j} : \tau' \vdash \textit{exec}(\tau')}{\Gamma_{i} \vdash \textit{synchro}_{j}(\upsilon(\textit{exec}([\tau/\tau'])))} \text{ HandleMistake1}$$

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Mistakes: syntax error

Mistake2: dependency from a locally invalid term

 $\frac{\Delta_i; \Gamma_i \vdash run_i(t \supset \bot) \qquad \Gamma_i, x_i: \tau \vdash run_i(v)}{\Gamma_i; \Delta_i \vdash mistake(run_i(v)WITH(t_i))}$ Mistake2

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Mistakes: syntax error

Mistake2: dependency from a locally invalid term

$$\frac{\Delta_i; \Gamma_i \vdash run_i(t \supset \bot) \qquad \Gamma_i, x_i: \tau \vdash run_i(\upsilon)}{\Gamma_i; \Delta_i \vdash mistake(run_i(\upsilon) WITH(t_i))}$$
Mistake2

HandleMistake2: re-define dependency from required resource

$$\frac{\Gamma_i \vdash \textit{mistake}(\textit{run}_i(\upsilon)\textit{WITH}(t_i)) \qquad \Gamma_i, [t_i/x_j]: \tau \vdash \textit{run}_i(\upsilon)}{\Gamma_i \vdash \textit{run}_{j \cap i}(\tau \supset \upsilon)} \text{ HandleMistake2}$$

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Failures: access wrong resources

Failure1: access wrong resources possibly at right location

$$\frac{\Box_{i}\Gamma, t_{j}: \tau \vdash exec(\upsilon) \qquad GLOB(\Box_{i\cup j}\Gamma, \tau)}{\Box_{i}\Gamma, access@_{j}(t': \tau') \vdash fail@_{i\cup j}(\upsilon) \qquad (t' \neq t; \tau' \neq \tau)}$$
FailPort1

Failures: access wrong resources

Failure1: access wrong resources possibly at right location

$$\frac{\Box_i \Gamma, t_j : \tau \vdash exec(\upsilon) \qquad GLOB(\Box_{i \cup j} \Gamma, \tau)}{\Box_i \Gamma, access@_j(t' : \tau') \vdash fail@_{i \cup j}(\upsilon) \quad (t' \neq t; \tau' \neq \tau)}$$
FailPort1

HandleFailure1: re-execute access to local resource

$$\frac{\Box_i \Gamma, access@_j(t':\tau') \vdash fail@_{i\cup j}(\upsilon) \qquad RET(\Gamma_{i\cup j}, \tau) \qquad \tau \neq \tau'}{\Box_i \Gamma, [t'_j:\tau'/t_j:\tau] \vdash exec(\upsilon)} \text{ HFP1}$$

Failures: access wrong locations

Failure2: access right resources at wrong locations

$$\frac{\Box_i \Gamma; x_j: \tau \vdash run_{i \cap j}(\upsilon)}{\diamond_i \Gamma, access@_{k>j}(t:\tau) \vdash fail@_{i \cap j}(\upsilon)}$$
FailPort2

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Failures: access wrong locations

Failure2: access right resources at wrong locations

$$\frac{\Box_i \Gamma; x_j: \tau \vdash run_{i \cap j}(\upsilon)}{\diamond_i \Gamma, access@_{k>j}(t:\tau) \vdash fail@_{i \cap j}(\upsilon)}$$
FailPort2

HandleFailure2: re-execute access to local resource

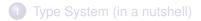
$$\frac{\diamond_i \Gamma, \operatorname{access} \mathbb{Q}_{k>j}(t:\tau) \vdash \operatorname{fail} \mathbb{Q}_{i\cap j}(\upsilon)}{\diamond_i \Gamma, [t_k/x_j]: \tau \vdash \operatorname{run}_{i\cap j}(\upsilon)} \operatorname{SEND}(\Gamma_{i\cap j}, \tau)} \operatorname{HFP2}$$

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- Structural: Weakening, Contraction and Exchange for Mistake and Failure expressions are provable;
- Reductions: Equivalent or Inducible Errors can be substituted preserving Error states; β-reductions and η-expansions hold;
- Termination: either an error state or its handling is at some point a non-further reducible expression (based on non-iteration of errors).







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Philosophical Remarks

- Validity is here intended as a local notion;
- Errors originate from such a restricted understanding of validity and from the additional use of the notion of accessibility;
- Handling is always a procedural request to reformulate the correct version of a given process.

- Data Quality as defined by Errors elimination;
- Definition of Distrust and Mistrust Propagation by Error Production in information channels; with application to Expertise;
- Future Work: Risk, Doubt, Consensus reaching strategies by error resolution.

Thanks

- Enough to think about? Then, Thanks!
- Want to see more? Semantics is waiting for you after this slide...

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A Semantics of Transitions

- Satisfaction: define the operations that allow to transit to the state of the system where that expression holds; for every satisfiable expression of the system there is such a transition;
- Validity: for every transition, there is a final state that is reached; at such state, every valid expression holds;
- Correctness: one needs to show that a satisfiable transition does not invalidate an expression it moves from.

Transitions

Definition (Operational Model)

An operational model of the procedural semantics for the machine is a model where each S is evaluated by transition to some S'. An indexed transition system, called a Network

Network := $(\mathcal{S}, \mapsto, \mathcal{I})$

is a triple with $S \subseteq \texttt{States}, \mathcal{I} \subseteq \texttt{Indices}$ and \mapsto a ternary relation over indexed states $(S \times \mathcal{I} \times S)$. If $S, S' \in S$ and $i, j \in \mathcal{I}$, then $\mapsto (S, i, j, S')$ is written as $S_i \mapsto S'_j$. This means that there is a transition \mapsto from state S valid at index i to state S' valid at index jdefined according to the machine typing rules.

The Domain of Correct Expressions

Definition (Standard Domain)

$$SD := \{S \mid \exists S' : (S, S') \in \texttt{Network \&} (S \mapsto S') : \alpha\}$$

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Moving in SD

Definition (Transitions of the Standard Domain)

The rewriting of a state *S* into a valid state $S' \in SD$ is established by the following transitions:

	$S\mapsto S'$
run	$(\Gamma_i, x_i : \alpha) \mapsto (\diamond_i \Gamma, run_i(\alpha))$
exec	$(\Gamma_i, a_i : \alpha) \mapsto (\Box_i \Gamma, exec(\alpha))$
corun	$(\Gamma_i, \operatorname{run}_i(\alpha) \vdash b_j : \beta) \mapsto (\Box_i \Gamma, \operatorname{run}_{i \cap j}(\alpha(\beta)))$
coexec	$(\Gamma_i, exec(\alpha) \vdash b_j : \beta) \mapsto (\Box_i \Gamma, run_{i \cup j}(\alpha(\beta)))$
synchro	$(\Box_i \Gamma, \operatorname{run}_{i \cup j}(\alpha(\beta)) \mapsto (\Box_i \Gamma, \operatorname{synchro}_j(\beta(\operatorname{exec}(\alpha))))$
product	$(\Gamma_i, exec(\alpha), exec(\beta)) \mapsto (\Box_i \Gamma, run_{i \cap j}(\alpha \times \beta))$
extraction1	$(\Box_i \Gamma, \operatorname{run}_{i \cap j}(\alpha \times \beta)) \mapsto (\Box_i \Gamma, \operatorname{exec}(\alpha))$
extraction2	$(\Box_i \Gamma, \operatorname{run}_{i \cap j}(\alpha \times \beta)) \mapsto (\Box_i \Gamma, \operatorname{exec}(\beta))$
tagunion	$(\Gamma_i, exec(\alpha)) \mapsto (\Box_i \Gamma, run_{i \cup j}(\alpha + \beta))$
patternmatch1	$(\Box_i \Gamma, \operatorname{run}_{i \cup j}(\alpha + \beta) \vdash c_k : \gamma) \mapsto (\Box_i \Gamma, \operatorname{run}_{i \cap k}(\alpha(\gamma)))$
patternmatch2	$(\Box_i \Gamma, \operatorname{run}_{i \cup j}(\alpha + \beta) \vdash c_k : \gamma) \mapsto (\Box_i \Gamma, \operatorname{run}_{j \cap k}(\beta(\gamma)))$
01	$(\Box_i \Gamma, exec(\alpha)) \mapsto (GLOB(\Box_{i \cup j} \Gamma, \alpha))$
□2	$(\Box_{i\cup j}\Gamma,\alpha)\mapsto (RET(\Gamma_{i\cup j},\alpha))$
♦1	$(\diamond_i \overline{\Gamma}, \operatorname{run}_j(\alpha)) \mapsto (\operatorname{BROAD}(\diamond_{i \cap j} \Gamma, \alpha))$
♦2	$(\diamond_{i\cap j}\Gamma, \alpha) \mapsto (SEND(\Gamma_{i\cap j}, \alpha))$

The Semantics of Error Expressions

Definition (Exit Network)

An indexed exit transition system called an ExitNetwork

ExitNetwork := $(\mathcal{S}, \mapsto_{e}, \mathcal{I})$

is a triple with $S \subseteq \text{States}$, $\mathcal{I} \subseteq \text{Indices}$ and \mapsto_e a quaternary relation over indexed states $(S \times \mathcal{I} \times S \times E)$, with E the set of all declared exit points. If $S, S' \in S$ and $i, j \in \mathcal{I}$, then $\mapsto (S, i, j, S', e)$ is written as $S_i \mapsto S_j \mapsto e$. This means that there is a transition \mapsto from state S valid at index i to state S' invalid at index j defined according to the machine typing rules which leads to an exit point e.

The Semantics of Error Expressions

Definition (Failure Domain)

 $FD := \{S \mid \exists S' : (S, S') \in \texttt{ExitNetwork \& } S \notin SD \cup \texttt{Network}\}$

or in other words $(S \mapsto S' \mapsto e): \neg \alpha$.

Moving in FD

Definition (Transitions of the Failure Domain)

The rewriting of a state $S \in SD$ into an invalid state $S' \in FD$ is established by the following transitions:

	$S\mapsto S'$
mistake1	$(\diamond_i \Gamma, \operatorname{run}_i(\tau(\bot)) \vdash \operatorname{exec}(v)) \mapsto (\Box_i \Gamma, \operatorname{mistake}(\operatorname{exec}(\tau(v))))$
mistake2	$(\diamond_i \Gamma, \operatorname{run}_i(t(\bot)) \vdash \operatorname{run}_i(v)) \mapsto (\diamond_i \Gamma, \operatorname{mistake}(\operatorname{run}_i(\tau(v))))$
failport1	$(GLOB(\Box_{i\cup j}\Gamma, \tau(v))) \mapsto (RET(\Box_{i\cup j}\Gamma, fail(t'(v))))$
failport2	$(BROAD(\diamond_{i\cap j}\Gamma,(\tau(\upsilon))))\mapsto (SEND(\Box_{i\cap k>j}\Gamma,fail(t(\upsilon))))$

Moving to Resolve

Definition (Resolve Transitions)

The rewriting of an invalid state $S \in FD$ into a valid state $S' \in SD$ is established by the following transitions:

	$S \mapsto^{res} S'$
resolveM1	$(\Box_i \Gamma, \textit{mistake}(\textit{exec}(\tau(v)))) \mapsto^{\textit{res}} (\Box_i \Gamma, \textit{run}_{i \cup j}(\tau'(v)))$
resolveM2	$(\diamond_i \Gamma, \textit{mistake}(\textit{run}_i(\tau(\upsilon))) \mapsto^{\textit{res}} (\Box_i \Gamma, \textit{run}_{i \cap j}(\tau'(\upsilon)))$
resolveFP1	$(RET(\Box_{i\cup j}\Gamma, fail(t(\upsilon)))) \mapsto^{res} (\Box_i\Gamma, exec(\tau') \vdash u_j : \upsilon)$
resolveFP2	$(SEND(\Box_{i\cap k>j}\Gamma, fail(t(v)))) \mapsto^{res} (\Box_i\Gamma, run_j(\tau) \vdash u_j:v)$

Moving to Abort

Definition (Abort Transitions)

The final abort state is a valid state $S \in SD$ established by the following transitions:

	$S \mapsto^{abort} S'$
abortM1	$(GLOB(\Box_i\Gamma, mistake(exec(\tau(v))))) \mapsto^{ab} (\Gamma_{i\cup j}, abort(\tau(v)))$
abortM2	$(BROAD(\diamond_i \Gamma, mistake(run_i(\tau(\upsilon)))) \mapsto^{ab} (\Gamma_{i\cap j}, abort(\tau(\upsilon)))$
abortFP1	$(\Box_{i\cup j}\Gamma, fail(t(\upsilon))) \mapsto^{ab} (\Gamma_{i\cup j}, abort(\tau(\upsilon)))$
abortFP2	$(\Box_{i\cap k>j}\Gamma, fail(t(\upsilon))) \mapsto^{ab} (\Gamma_{i\cap k}, abort(\tau(\upsilon)))$

Image: Image:

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Theorem (Progress)

For every state in SD, either there is a transition or its type is the output value. For every state in FD, either there is a resolution transition or an abort transition and the type of the latter is the output value.

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Theorem (Preservation)

For every state in SD, its transition is type-preserving. For every state in FD, every resolution transition is type-preserving w.r.t. another state and every abort transition is of output value.

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Theorem (Preservation)

For every state in SD, its transition is type-preserving. For every state in FD, every resolution transition is type-preserving w.r.t. another state and every abort transition is of output value.

Theorem (Type Safety)

The ExitNetwork semantics is safe: for every state in SD, there is either a type-preserving transition or the state provides the output value; for every state in FD, there is either a type-preserving transition to a resolved state or the \perp -type provides the output value.

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