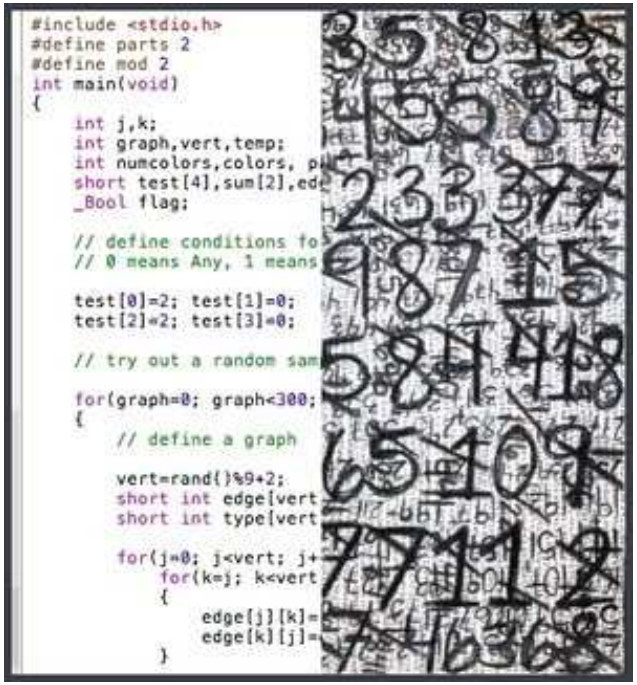
What is the impact of the computer on mathematics? A quantitative approach



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Intro.

Introduction

⇒ **Motivation I:** Debates on the impact of the computer on mathematics: "computers [are] *changing* [m.i.] the way we do mathematics"

⇒ **Motivation II:** "[The first school of thought is concerned with] the improvement of highways between the well-established parts of mathematics and the outposts of the realm [favoring] the extension of existing methods of proof to more general situations" [The second school is concerned with] "the establishment of new outposts [...] This school favors *explorations* as a means of discovery" (Lehmer,1966)

⇒ ***But how important is the computer, really?***

⇒ **A possible approach 1:** Study of particular cases of computer-assisted practices

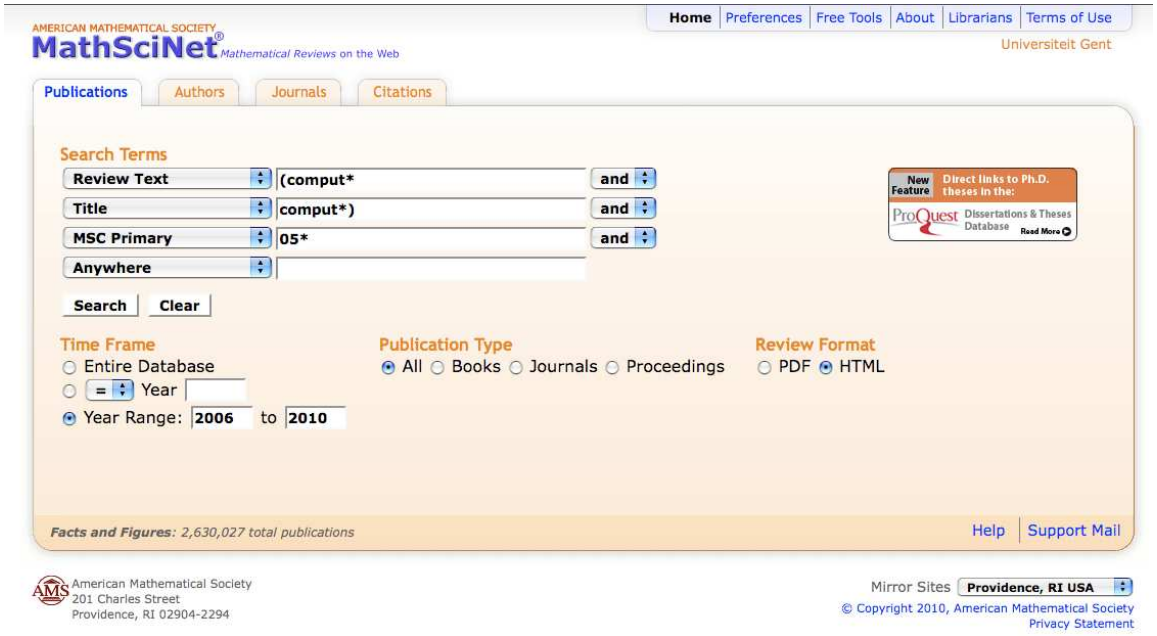
⇒ **Problem:** How to select relevant case studies and how to relate these "micro" studies with macro developments?

> "[L]a focalisation sur quelques articles ou quelques personnes ne fait dans de nombreux cas qu'éviter, et non permettre, une véritable réflexion sur l'innovation." (Goldstein, 1999)

⇒ **A possible approach 2:** A quantitative study of the impact of the computer on math – zooms out on the everyday "practice" of the use of the computer in mathematics

Note: *Research in Progress!* Preliminary results and evaluation

Details of the approach



Details of the approach: Scope

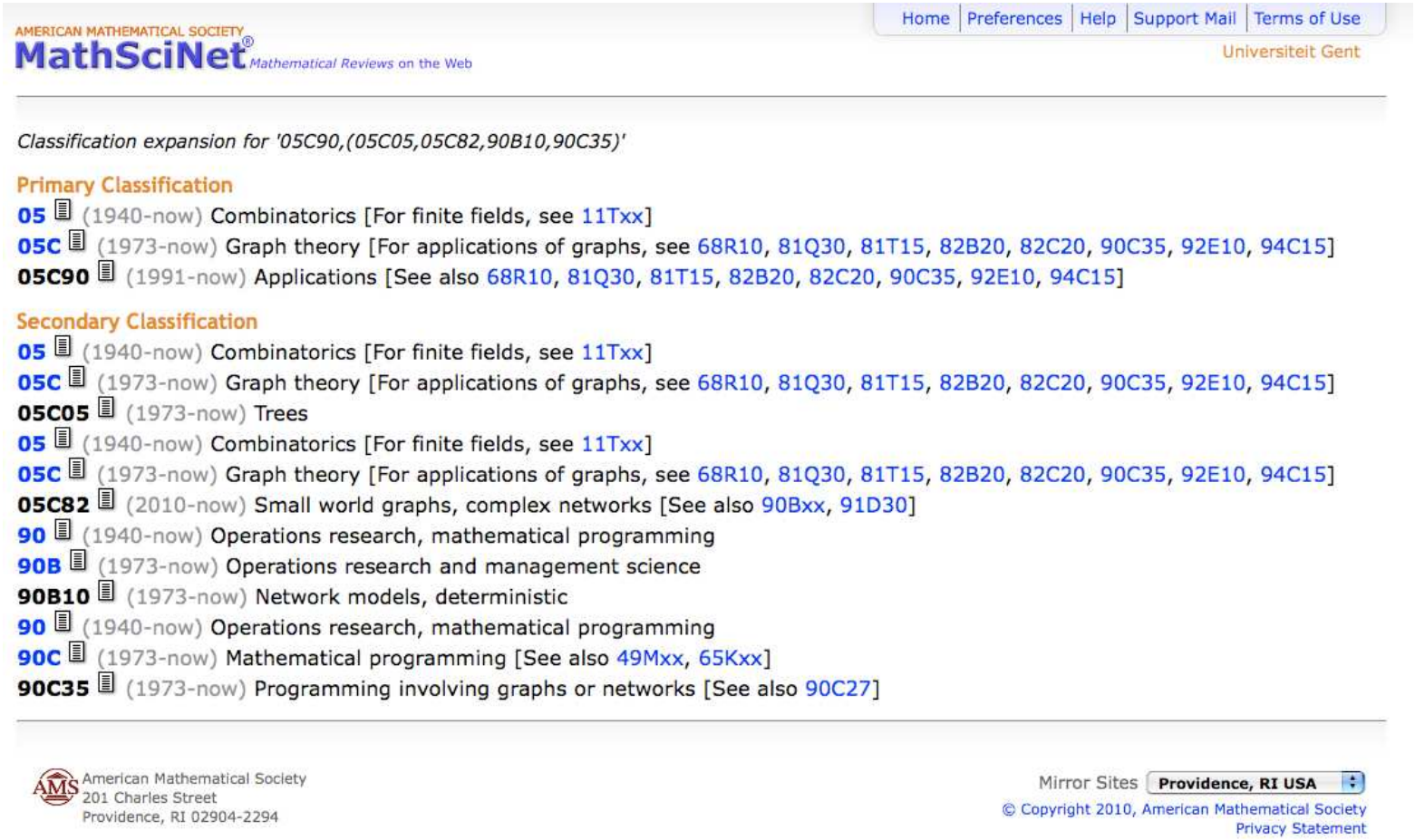
⇒ Study of the *evolution* in the use of certain groups of words (e.g. computation) in MathSciNet (mainly)

1. Evaluate the quantitative impact of the computer *as such* in published research: Group I = compute*, computi*, computa*, calcula*, machine*
2. Evaluate Lehmer's words, i.e., the significance of "experimental computer-assisted mathematics" in published research: Group II = experiment*, heurist*, conject*, explor*, inspect*
3. Evaluate the significance of computer-related terminology: Group III = algorit*, program*, digital*, simulat*, automat*

⇒ Study of the evolution of these "groups" in general but also within disciplines

Details of the approach: Disciplines

⇒ Use of MSC2010 (MathSubjectClassification) – primary ⇒ An example:



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Classification expansion for '05C90,(05C05,05C82,90B10,90C35)'

Primary Classification

05 (1940-now) Combinatorics [For finite fields, see 11Txx]
05C (1973-now) Graph theory [For applications of graphs, see 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15]
05C90 (1991-now) Applications [See also 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15]

Secondary Classification

05 (1940-now) Combinatorics [For finite fields, see 11Txx]
05C (1973-now) Graph theory [For applications of graphs, see 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15]
05C05 (1973-now) Trees
05 (1940-now) Combinatorics [For finite fields, see 11Txx]
05C (1973-now) Graph theory [For applications of graphs, see 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15]
05C82 (2010-now) Small world graphs, complex networks [See also 90Bxx, 91D30]
90 (1940-now) Operations research, mathematical programming
90B (1973-now) Operations research and management science
90B10 (1973-now) Network models, deterministic
90 (1940-now) Operations research, mathematical programming
90C (1973-now) Mathematical programming [See also 49Mxx, 65Kxx]
90C35 (1973-now) Programming involving graphs or networks [See also 90C27]

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⇒ Total of 57 main disciplines (history, general and pedagogy excluded)

Details of the approach: Grouping the Disciplines

1. Mathematical Logic and foundations: 03*
2. Combinatorics: 05*
3. Number theory: 11*
4. Algebra: 06* or 08* or 12* or 13* or 14* or 15* or 16* or 17* or 18* or 19* or 20* or 22* (e.g., associative rings and algebras or group theory)
5. Analysis: 26* or 28* or 30* or 31* or 32* or 33* or 34* or 35* or 37* or 39* or 40* or 41* or 42* or 43* or 44* or 45* or 46* or 47* or 48* (e.g., "Difference and functional equations" or "Ordinary differential equations")
6. Geometry: 51* or 52* or 53* or 54* or 55* or 57* or 58* (e.g., "General Topology" or "Differential Geometry")
7. Statistics and probability theory: 60* or 62*
8. Applied mathematics: 70* or 74* or 76* or 80* or 81* or 82* or 83* or 85* or 86* or 91* or 92* (e.g., "Mechanics of particles and Systems" or "Biology and other mathematical sciences")
9. Mathematics, computing and computer science: 65* or 68* or 90* or 93* or 94* (e.g., "mathematical programming" or "computer science")

Details of the approach: More details

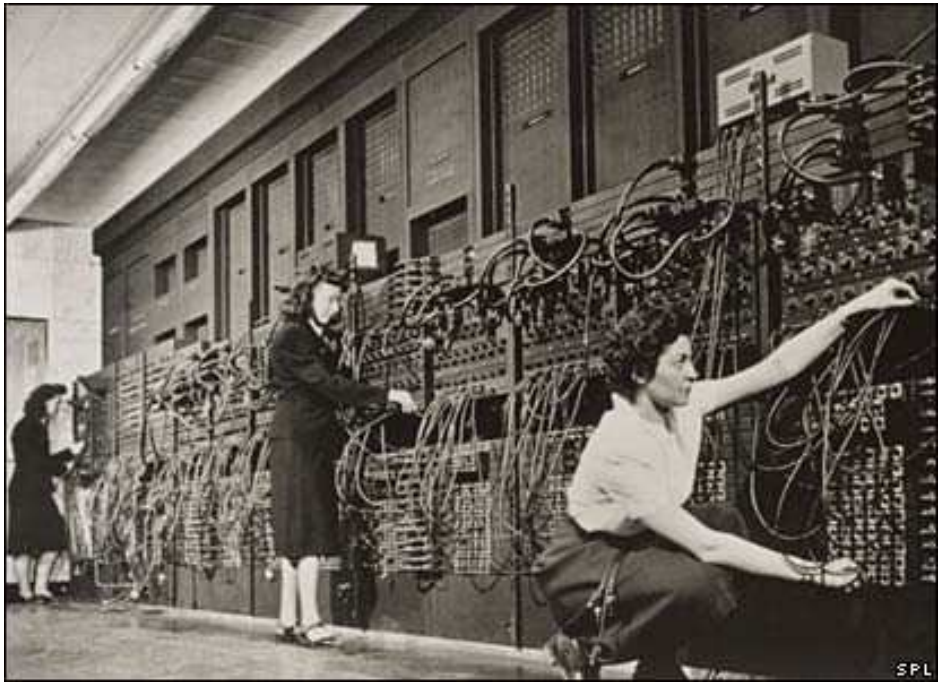
- From 1936 till 2010, blocks of five years
- Given a (group of) terms, search the total number of publications that contain term in **review or title** for every block of five years, e.g., (Review Text=((calcula*) OR Title=(calcula*)) AND MSC Primary=(11*)) AND pubyear in [1936 1940]
- Use of **relative numbers**, i.e., the percentage of publications that contains a given (group of) word(s) with respective to the total number of publications.
- (Note: Research itself is heavily computer-assisted (GnuPlot, MathSciNet, Internet, etc)

Method: some possibilities

- Does the computer have a significant impact on published mathematical research, and, if yes, in what way?
- Evolution of computing within the disciplines? How fast?
- Correlations between Group I and II/ Group I and III over the years?

Results

Results: Group I = compute*, computa*, computi*, calcula*, machine*



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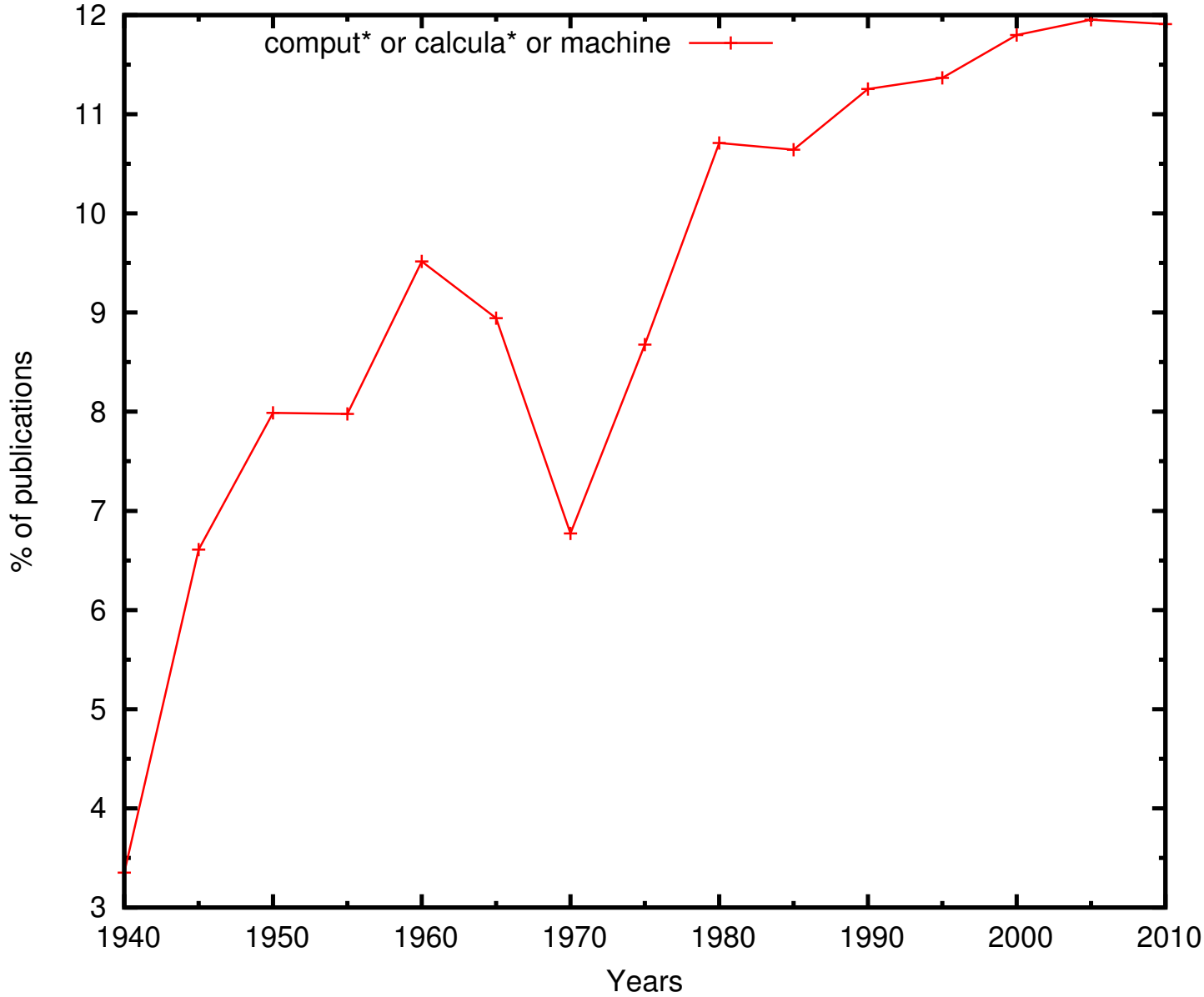

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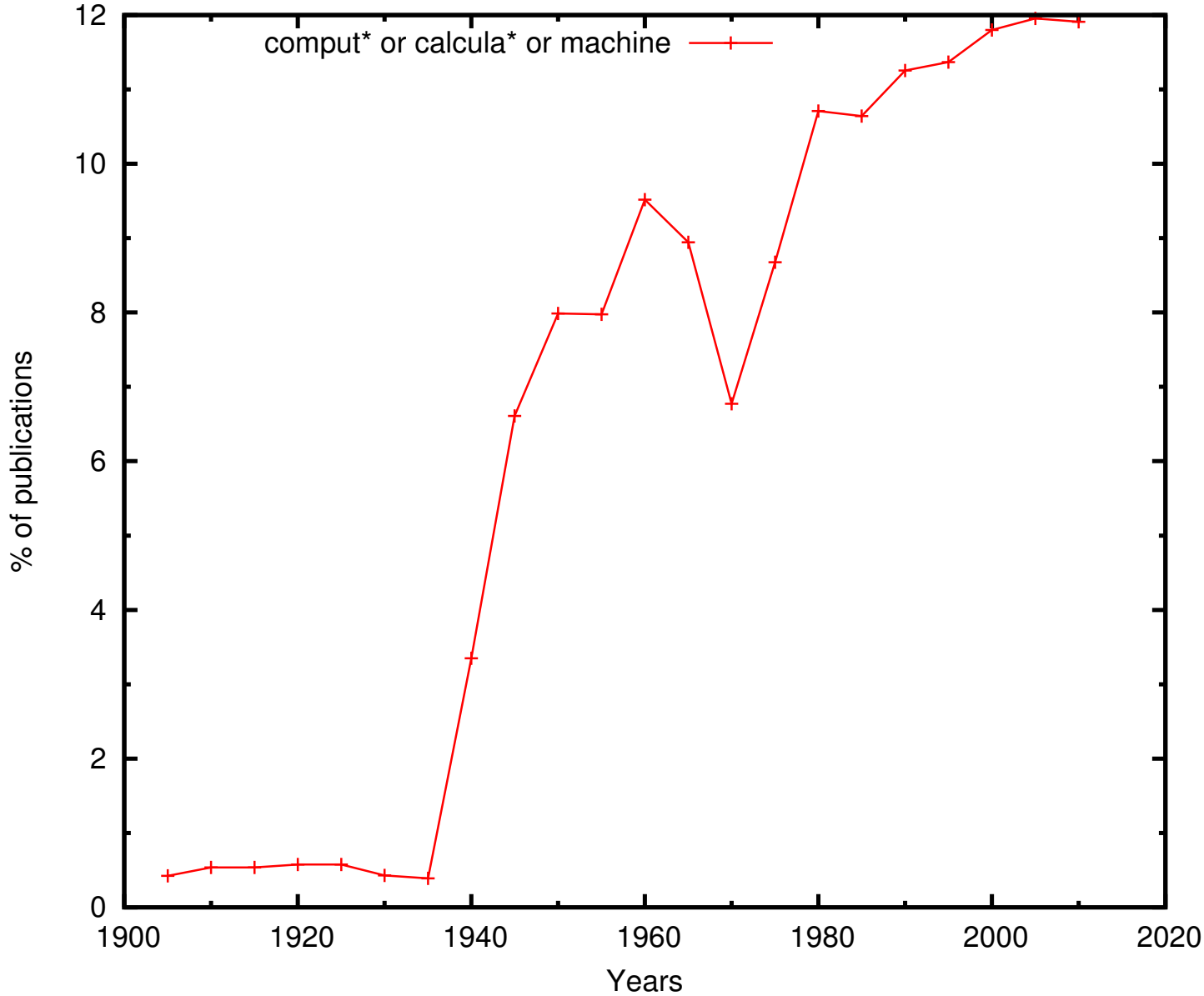
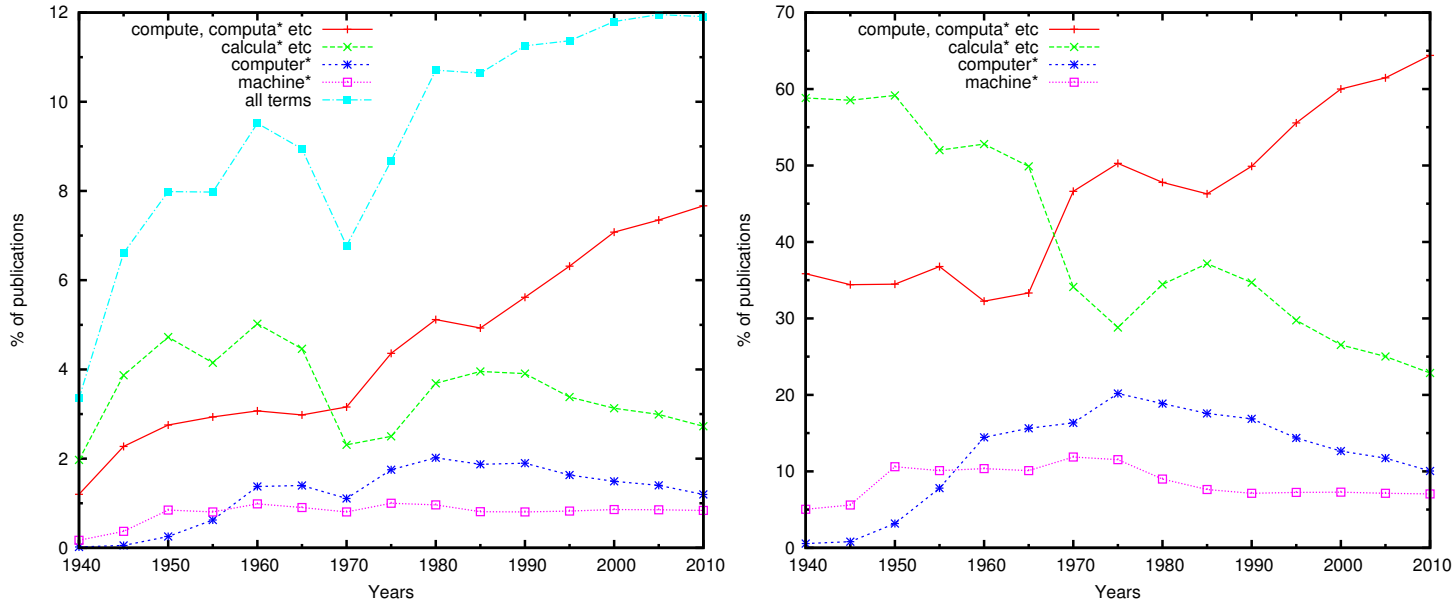


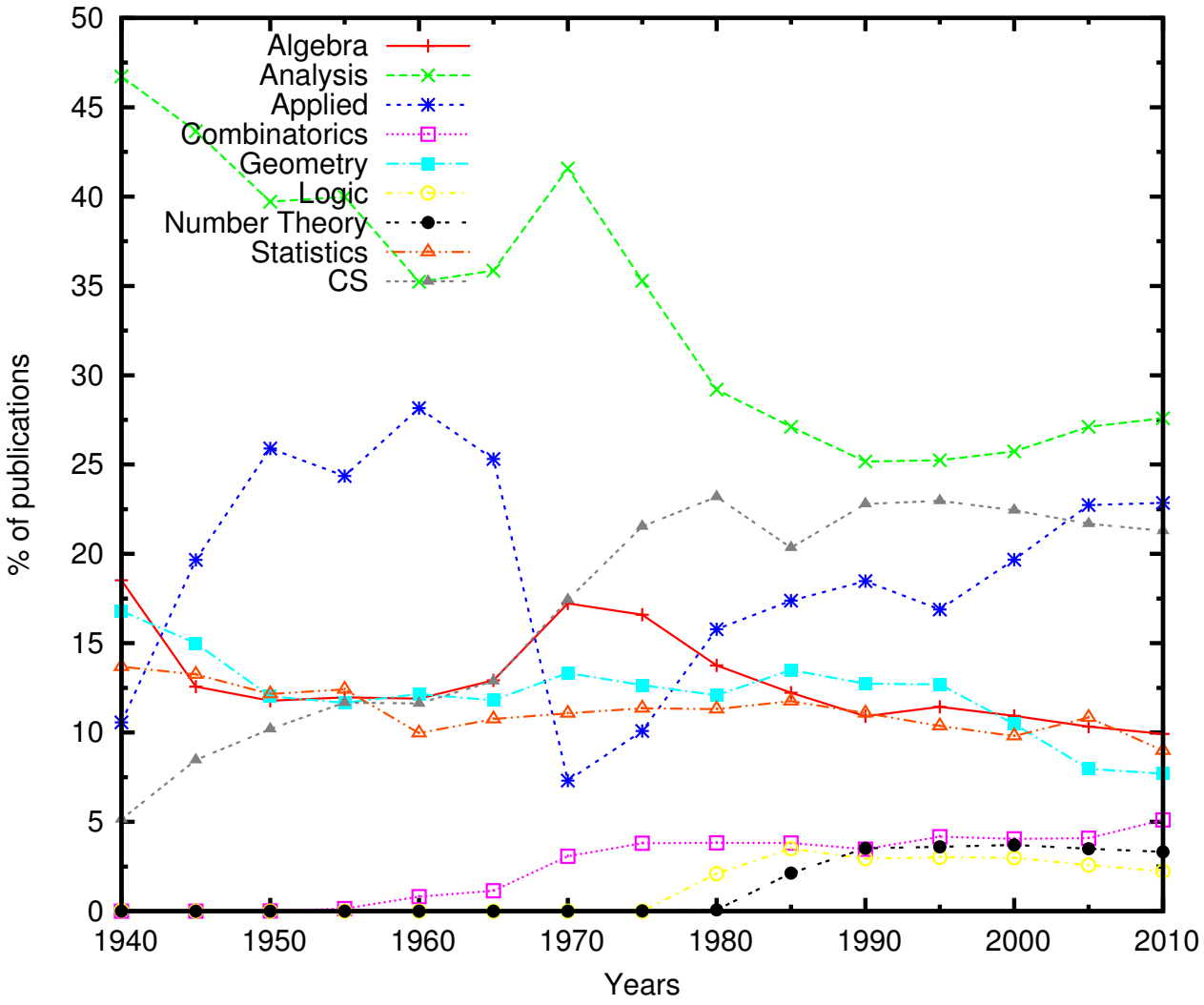
Results: Group I = compute*, computa*, computi*, calcula*, machine*

Comparison of the use of the terms

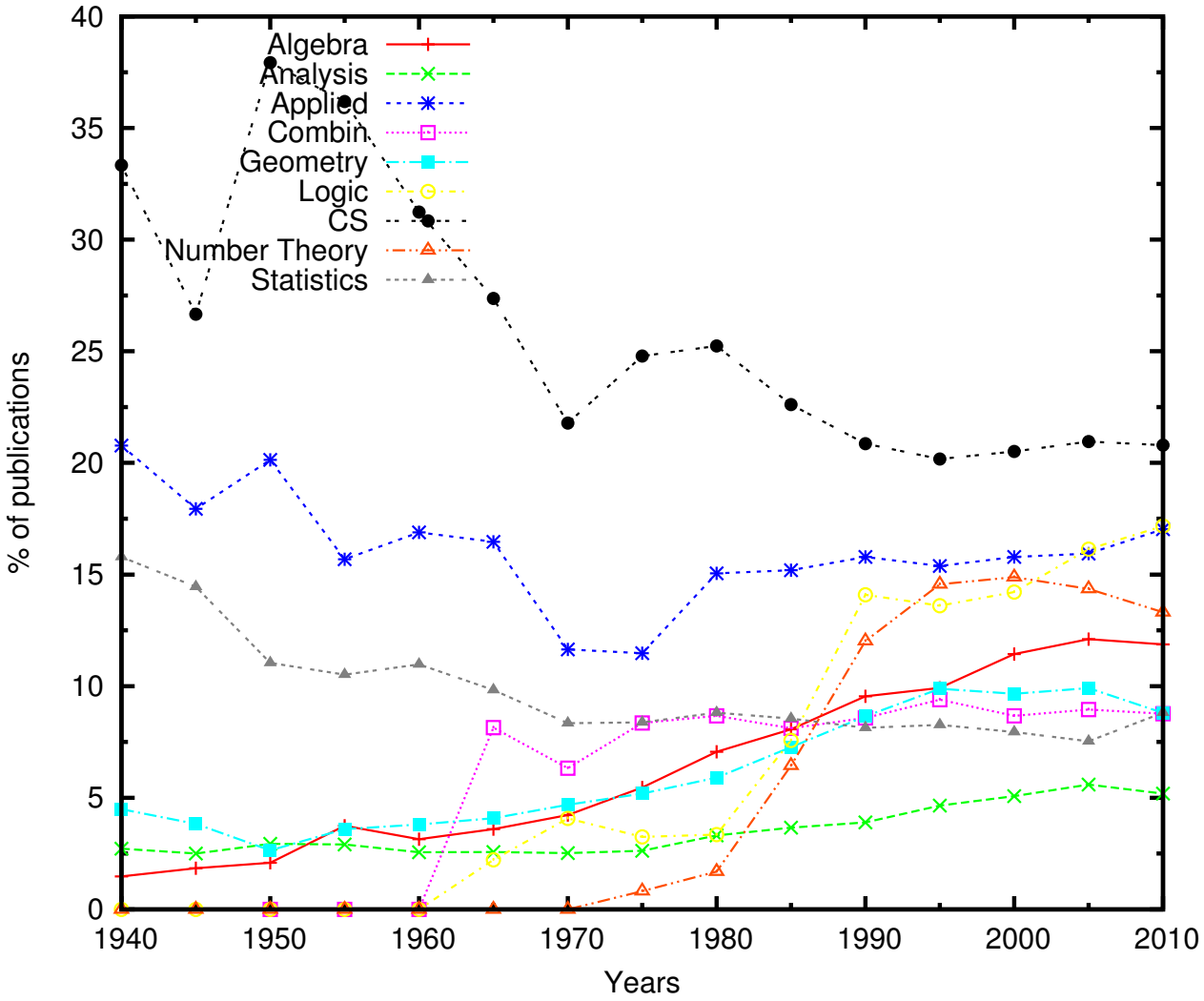


Group I – Distribution in the disciplines

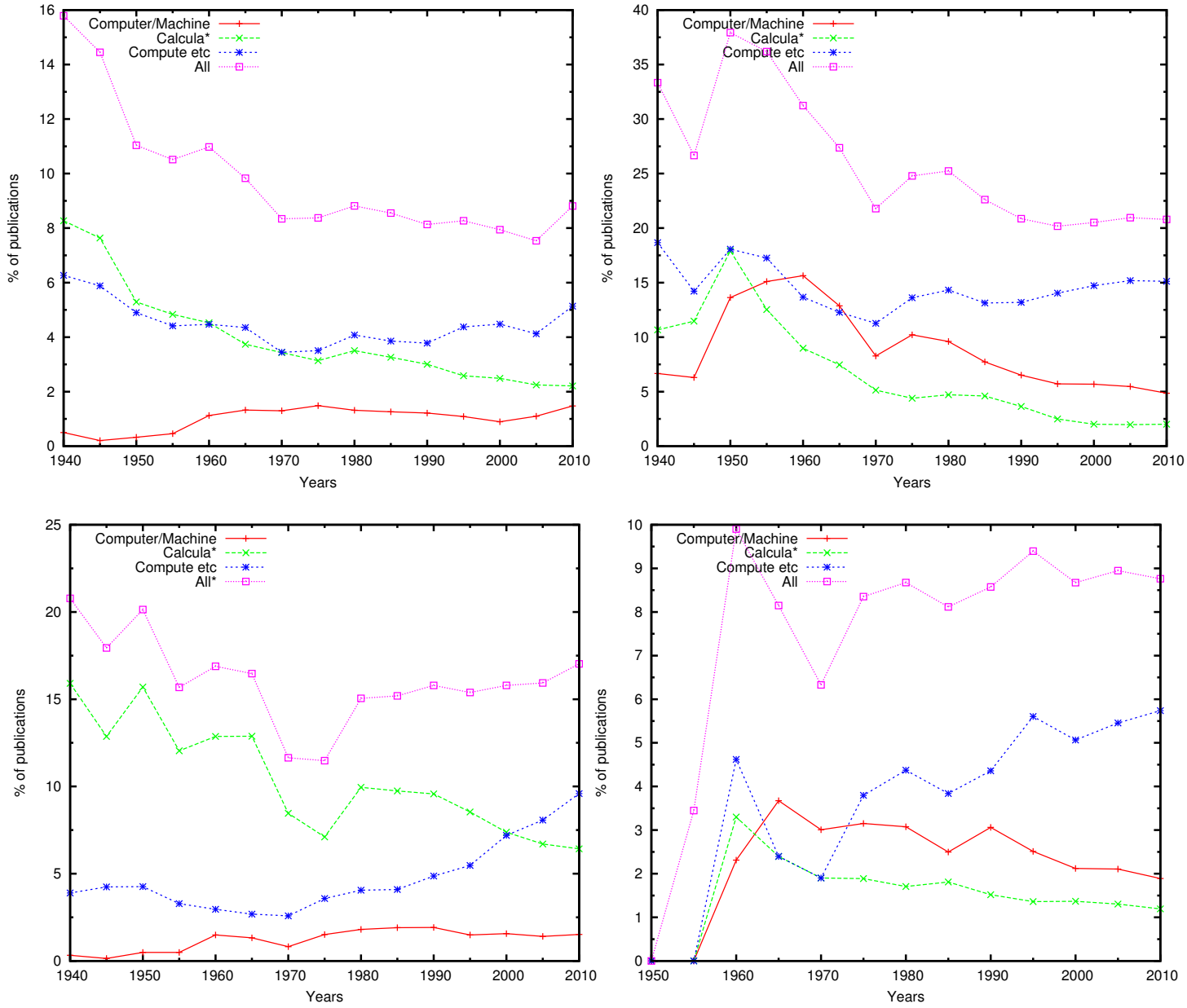
The disciplines – General evolution



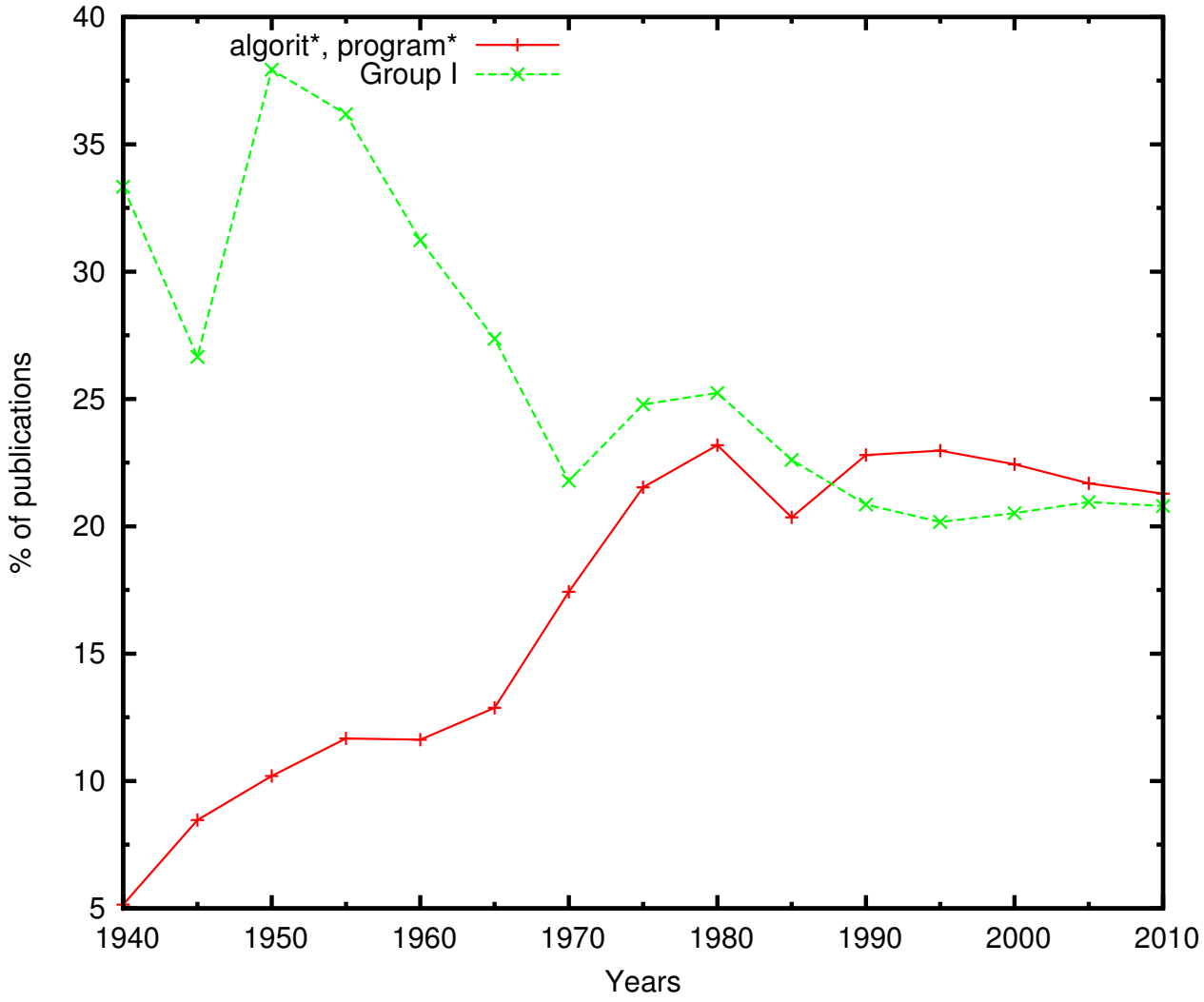
Group I – disciplines: General evolution



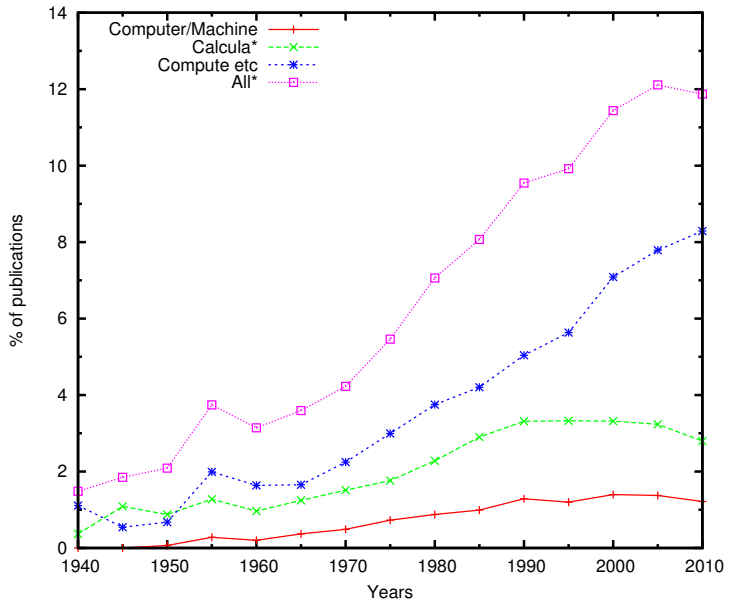
Group I – disciplines: Statistics (Top left),Computer-related math (Top right), Applied (Bottom left), Combinatorics (Bottom right)

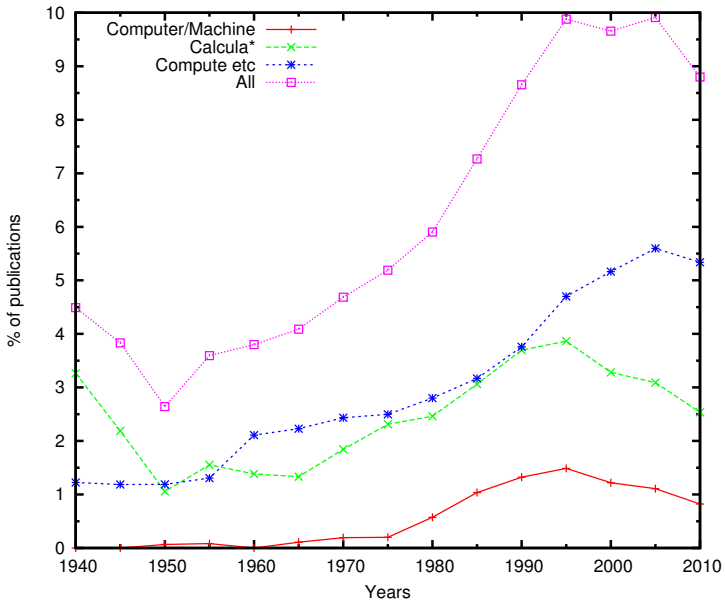


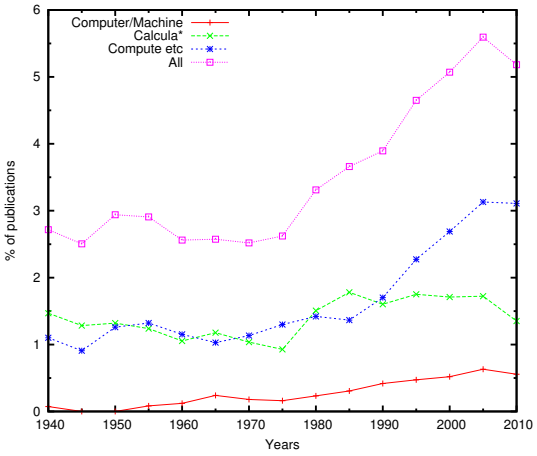
Group I – disciplines: Computer-related math (algorit*, program*)

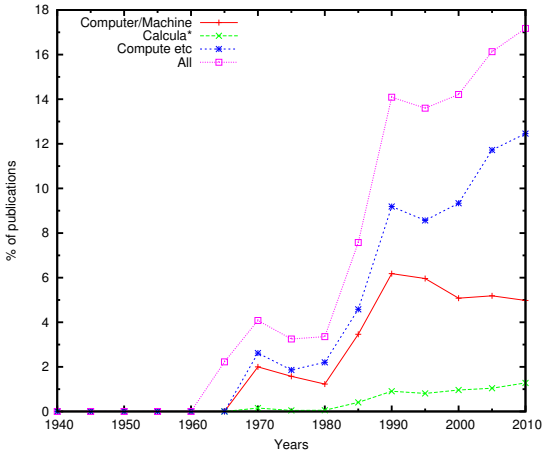


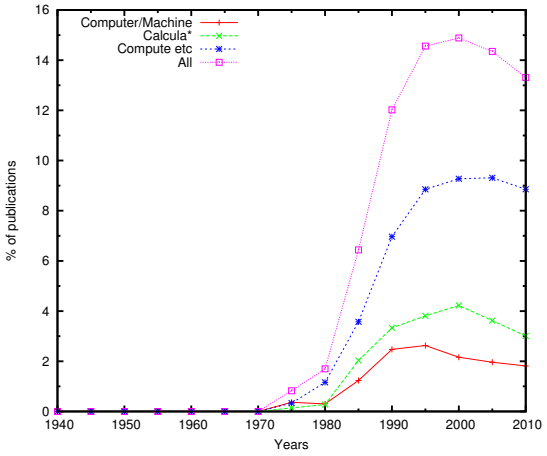
Group I – disciplines: Algebra (Top left), Geometry (Top right), Analysis (Bottom left), Logic (Bottom middle), Number theory (Bottom right)











Results: Group II = experiment*, heuristic*, explor*, inspect*, conjectur*

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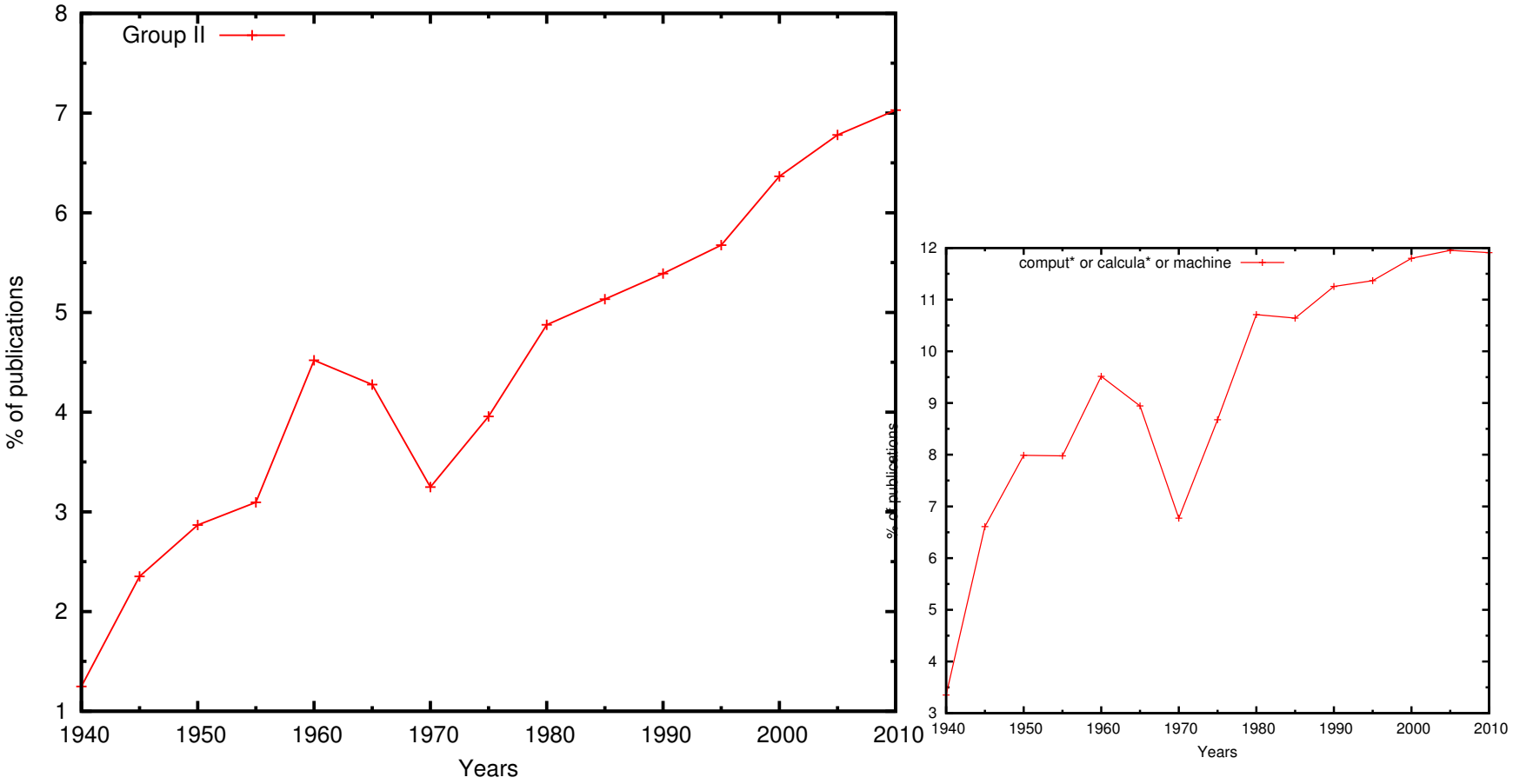
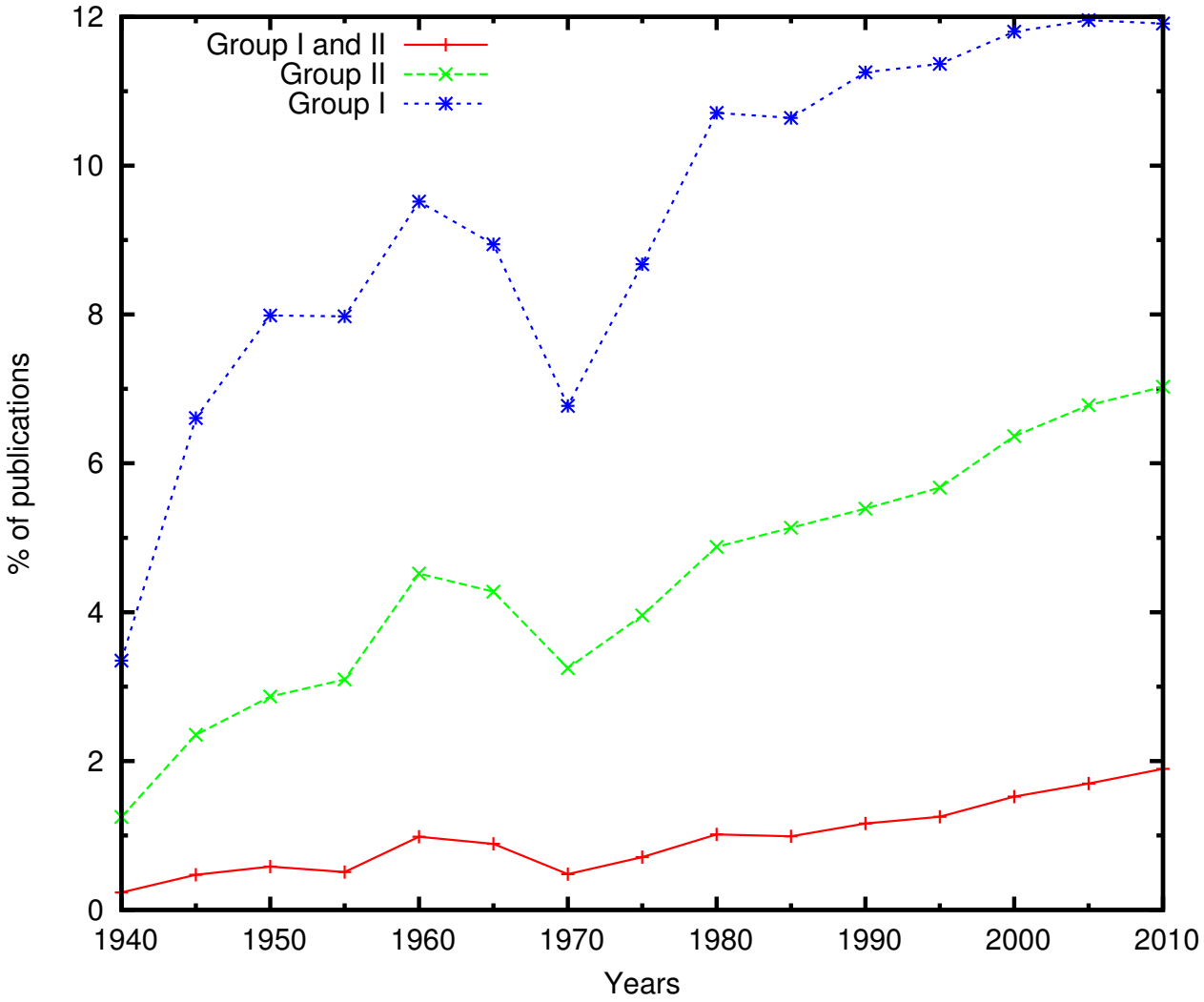
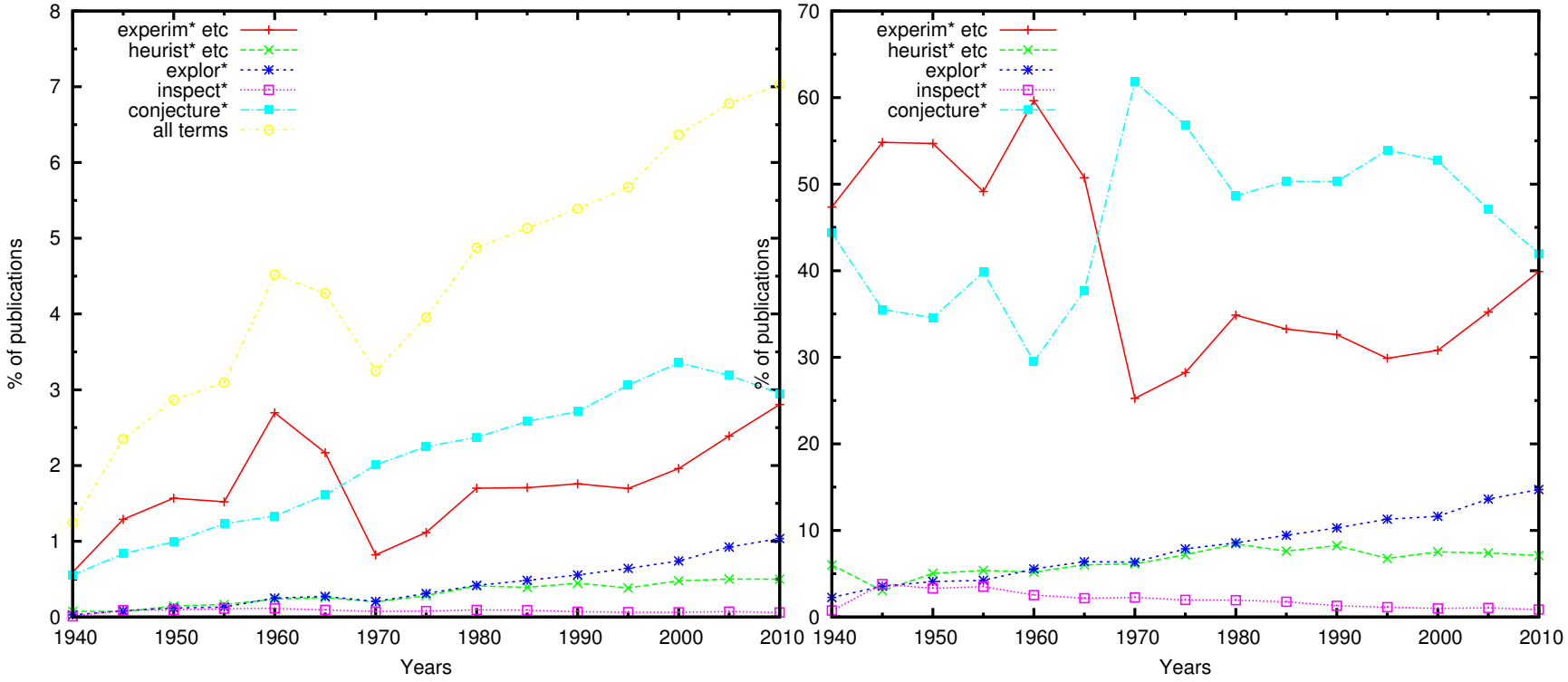


Group I and Group II: Comparison

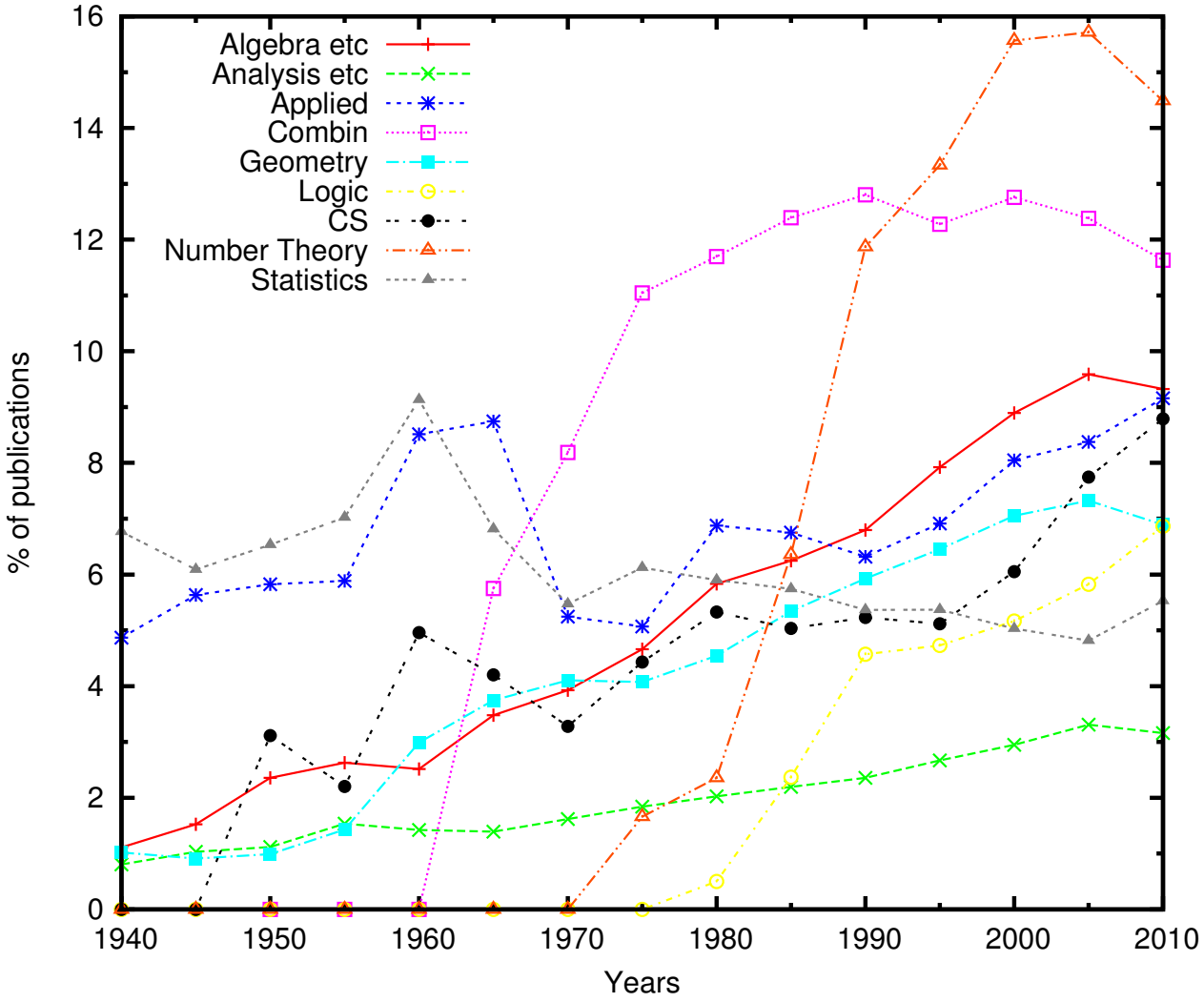


Group II: Comparison of the use of the terms

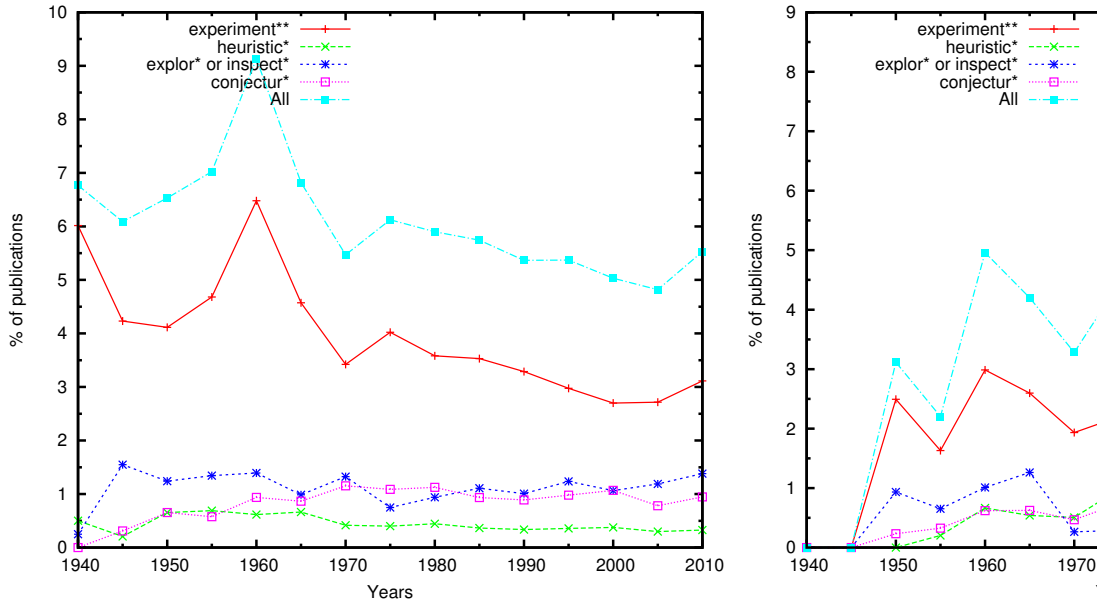


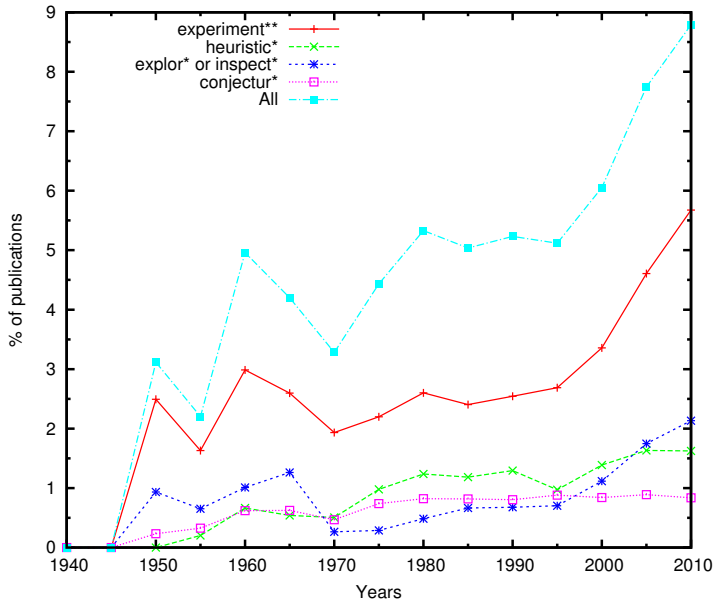
Group II – Distribution in the disciplines

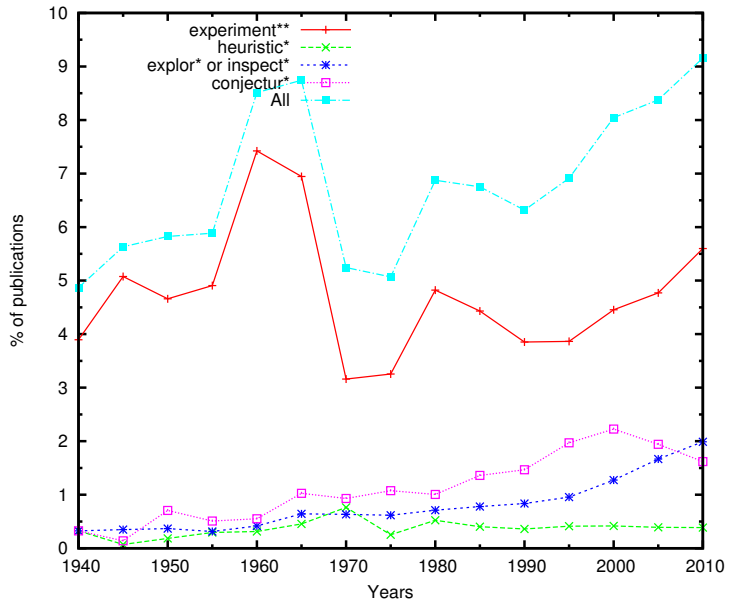
Group II – disciplines: General evolution



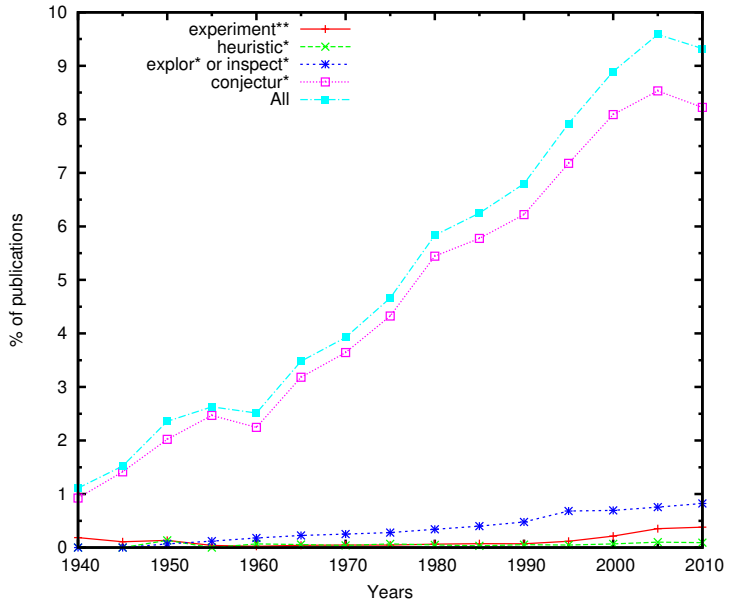
Group II – disciplines: Statistics (Top left),Computer-related (Top right), Applied (Bottom)

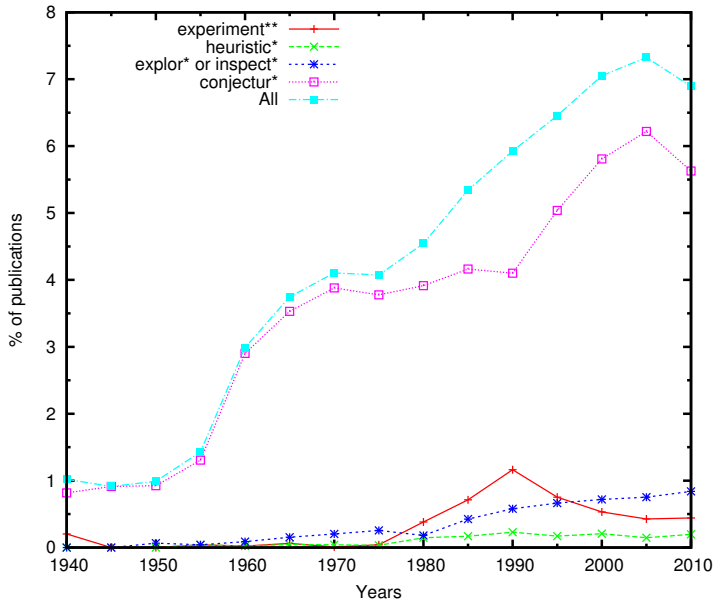


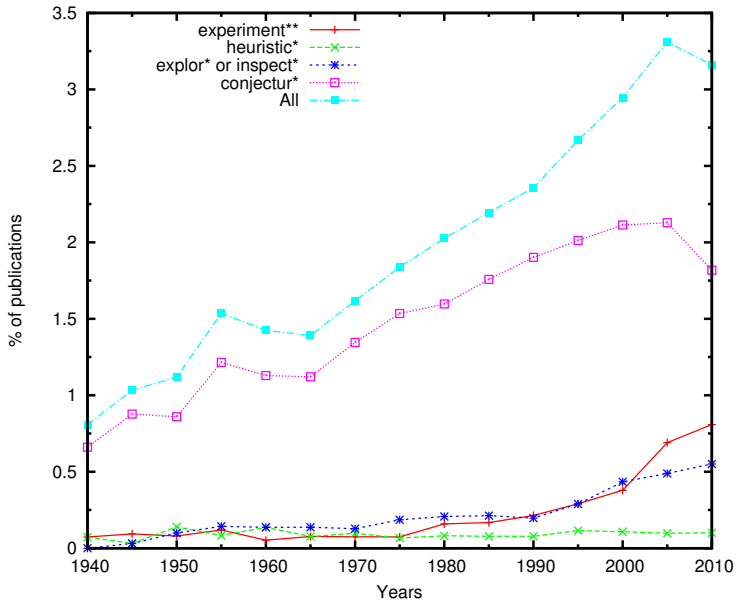




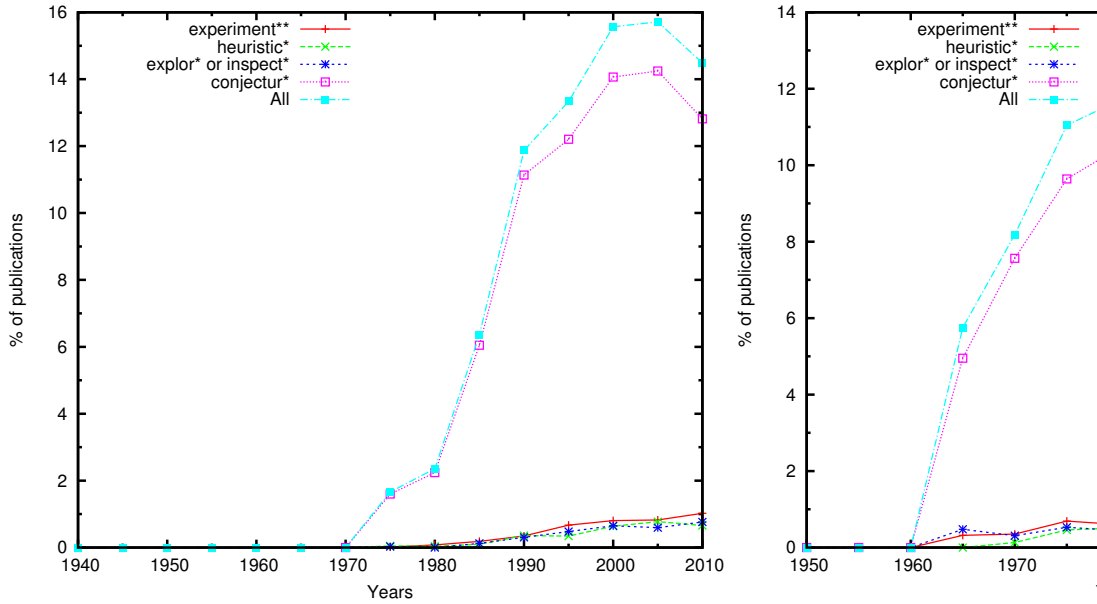
Group II – disciplines: Algebra (Top left),Geometry (Top right), Analysis (Bottom)

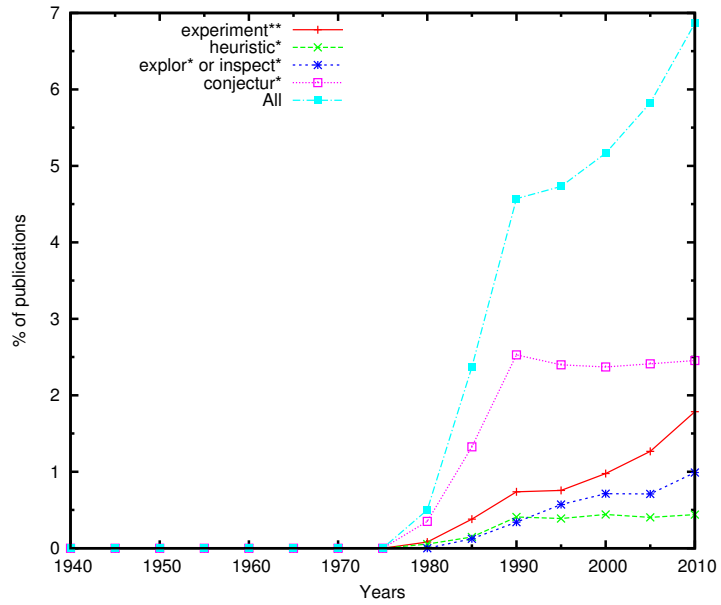






Group II – disciplines: Number Theory (Top left), Combinatorics (Top right), Logic (Bottom)

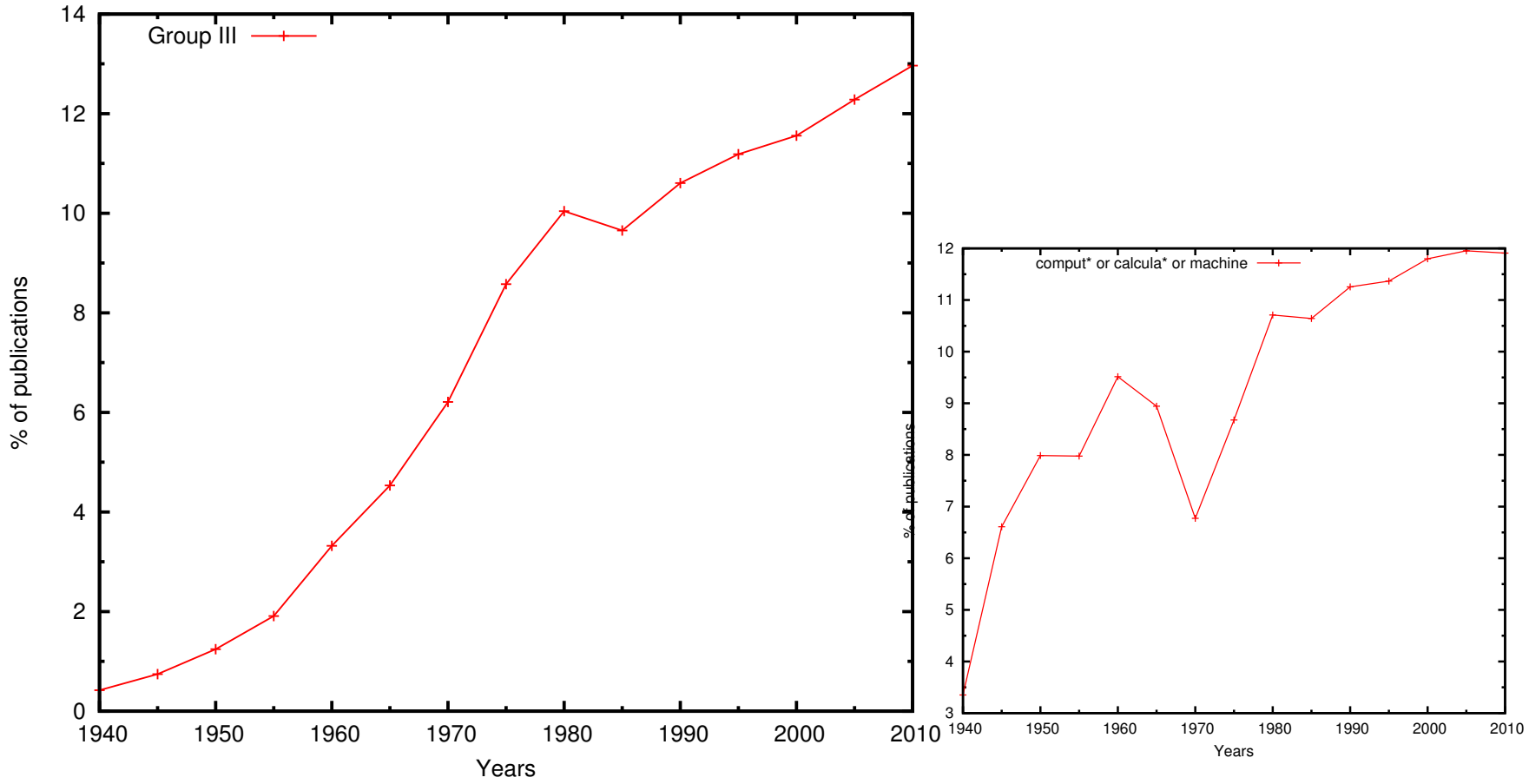




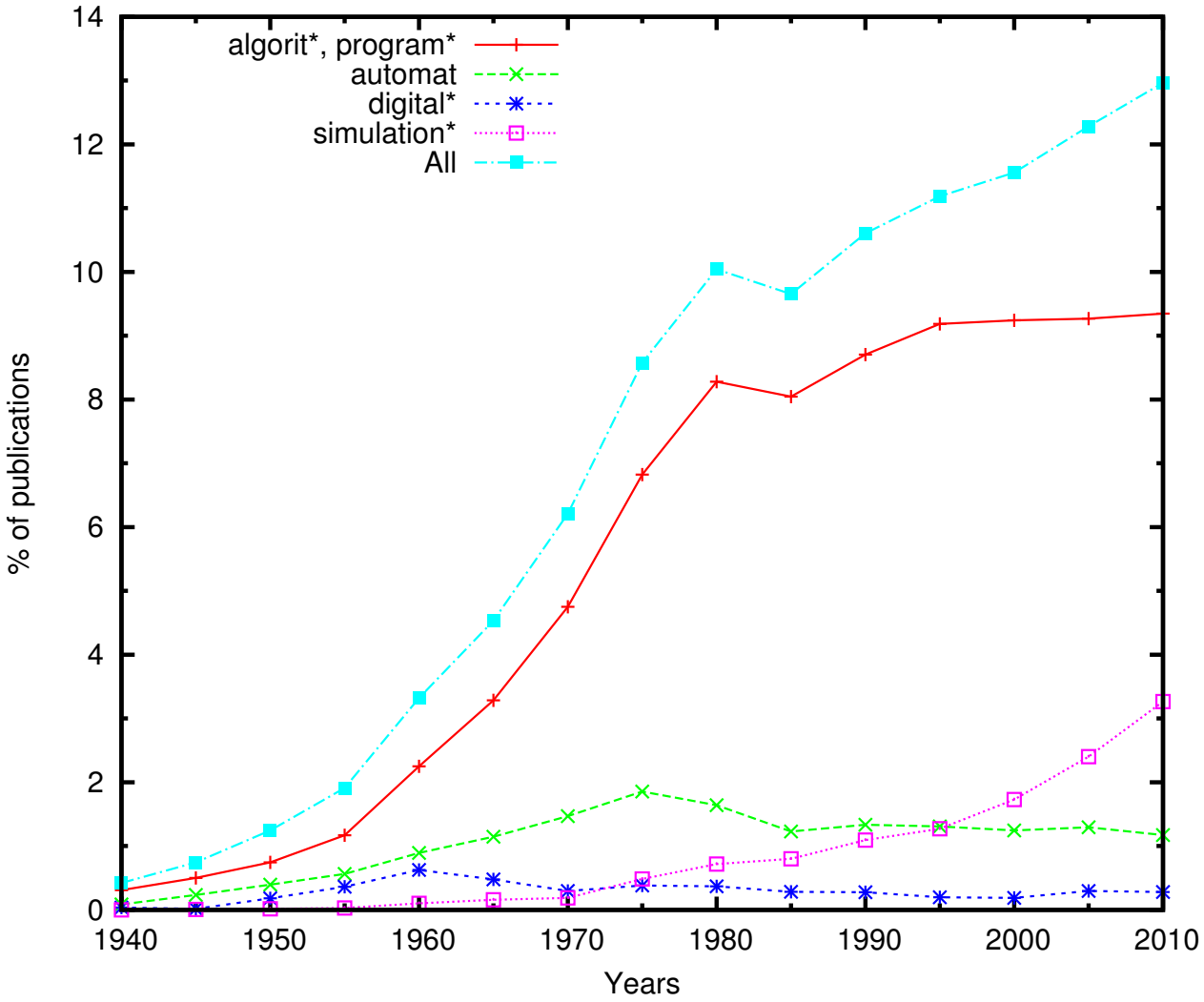
Results: Group III = algorit*, program*, automat*, digital*, simulation*

```
(define (initial-vec seq l n)
  (if (= l 0)
      seq
      (initial-vec (vector-append seq (vector (random n))) (- l 1) n)))
(initial-vec #(0) 300 2)


(define (add-period diff-periods n-period)
  (define (in-list? item seq)
    (cond ((null? seq) #f)
          ((equal? item (car seq)) #t)
          (else (in-list? item (cdr seq)))
          )
    )
  (cond ((= n-period (- 1)) diff-periods)
        ((in-list? n-period diff-periods) diff-periods)
        (else (append (list n-period) diff-periods))))

(add-period '(6) 1)

(define (determine-max r-max n-max?)
  (cond ((>= r-max n-max?) r-max)
        (else n-max?)))


(define (tag-it-vec count words v word ref-word ref-count)
  (define (deletion v word)
    (vector-tail word v))
  (define (produce words v word)
    (let ((appendent (vector-ref words (vector-first word))))
          (if (< (+ (- (vector-length word) v) (vector-length appendent)) 1) #()
              (deletion v (vector-append word appendent)))))
  (define (halt? word)
    (equal? word #()))
  (define (period? w1 w2)
    (equal? w1 w2))
  (define (period count ref-count)
    (- count ref-count))
  (let ((production (produce words v word)))
    (case (modulo count 1000000)
      ((0) (display count) (newline)(display (vector-length word)) (newline)))
    (cond ((period? word ref-word) (list count 0 1 (period count ref-count)))
          ((halt? production) (list count 1 0 (- 1)))
          ;((= count 100000) 1)
          ((= 0 (modulo count 1000)) (tag-it-vec (+ count 1) words v production word count))
          (else (tag-it-vec (+ count 1) words v production ref-word ref-count))
          )
    )
  )
```

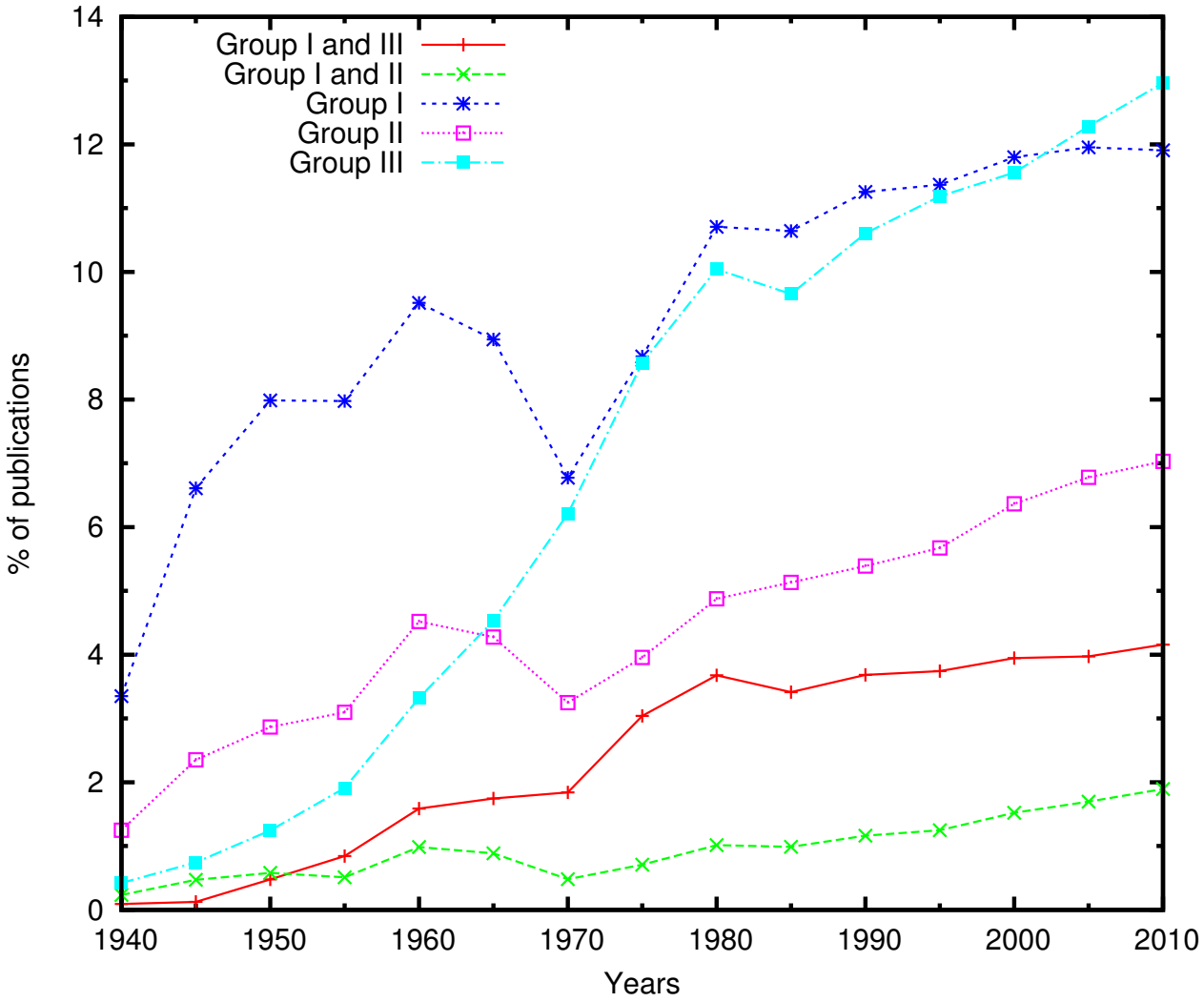
Results: Group III = algorit*, program*, automat*, digital*, simulation*



Group III: Comparison of the Use of the terms



Group I, II and III: comparison



Discussion

Discussion – Summary (1)

- Group I
  - Significant increase of impact computer
  - Unlike expectancies, decrease in applied math, computer-related mah and statistics, increase in the other disciplines. Partial explanation for computer-related math (dominance of algorit* and program*)
  - Drop-down and up in 1970: drop-down statistics, applied and computig sciences; increase for the others
  - Increase in the others around 1970 $\approx$ inverse relation between compute* and calcula* (+machine/computer) $\Rightarrow$ Relation with development time-sharing and high-level programming languages (focus on computing with symbols rather than numbers)

Discussion – Summary (2)

- Group II
  - It is not completely clear in how far the significance of "experimental" math is increasing. Lehmer's explorative interpretation only takes up about 1%.
  - Seeming correlation between increase group I and II → needs to be strengthened! What is "experiment" in applied math, statistics and computer-related math?
  - Statistics, applied and computer-related math: drop-down mainly due to decrease use "experiment*" in 1970 ∼ group I?
  - The remaining 6: Mostly dominance notion conjecture. What are the dominant conjectures in these disciplines and how much are they related to "experimental math" and computer-assisted math"?
- Group III
  - At least as important as group I (!).
  - Dominance of algorit* and program*. Starting around 1970: growing significance simulation*
  - Correlation between I and III more strongly suggested
  - How about distribution of group III within the disciplines?

⇒ What happens in 1970? Good starting point to come to a better under-

standing of question correlation "experimental math" and computer-assisted math?

⇒ Correlations between group I, II and III: about half of the papers from group I are correlated with group II or III

Discussion

- Limitations and problems
  - 1% inspiration, 99% perspiration + necessity to be extremely careful in the interpretation ≈ experimental computer-assisted research
  - Focus on English publications
  - Only published papers → no information on other aspects of computer-assisted mathematics like the use of LaTeX, Google, blogs, etc
  - The micro-macro problem.
- Yes, the computer has a significant impact on mathematics! Notion of computation and computing
- "Content" of the impact? Partial indication – group II and III
- Sensitivity of mathematics and its disciplines to new technological developments within computing
- Quantitative research – a heuristic tools: Indications for further qualitative and quantitative research

⇒ In need of a complementary approach of qualitative combined with quantitative research

- Math is not so "normal (Ferreiros, 2010)
- 20th century math is not so pure