Centre for Logic and Philosophy of Science
Centrum voor Logica en Wetenschapsfilosofie

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FACULTY OF ARTS AND PHILOSOPHY

The Use of Defaults in Commonsense Reasoning. How Abnormality Markers Point Out the Bad Guys.

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Outline

1. Defaults, Commonsense Reasoning, and AI
2. The Adaptive Logics Approach
   - Characterizing Default Rules
   - Characterizing Default Inference
   - Example
3. Conclusion

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Defaults, Commonsense Reasoning, and AI

Default Rules

Rules of the form “Typically A’s are B’s"

EXAMPLE: Typically birds fly.

Defaults, Commonsense Reasoning, and AI

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    $\Rightarrow$ Obtained consequences may be withdrawn later on.

Defaults, Commonsense Reasoning, and AI

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  - In a defeasible way!

    $\Rightarrow$ Obtained consequences may be withdrawn later on.
  - Nature’s way to lessen the cognitive load on people’s minds.

    $\Rightarrow$ Plausible consequences are drawn more swiftly, disregarding all possible exceptions.

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Defaults, Commonsense Reasoning, and AI

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Defaults, Commonsense Reasoning, and AI

Resolving Conflicts Between Defaults

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Resolving Conflicts Between Defaults

In general, conflicts are resolved by relying on two requirements:

- specificity
  = more specific defaults are preferred over less specific ones.
- inheritance
  = members belonging to a particular class should inherit as much characteristics as possible from the overarching classes.

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Defaults, Commonsense Reasoning, and AI

Resolving Conflicts in AI

Specificity and inheritance are formally implemented by

- rewriting the default base,
- generating priorities among defaults, or
- imposing restrictions on the domain of applications.

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AIM: To propose an approach to default reasoning that is grounded syntactically, and as such, comes closer to actual human reasoning.

- by relying on the *Adaptive Logics Programme*.

The Adaptive Logics Approach

The Adaptive Logics $\mathbf{D^r}$ and $\mathbf{D^{ram}}$

As all adaptive logics (**AL**), $\mathbf{D^r}$ and $\mathbf{D^{ram}}$ are characterized by means of

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  - Lays down the stable, non–defeasible inference rules

    $\Rightarrow \quad Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma)$

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- a set of abnormalities Ω
  - Defines the defeasible inference steps

    $\Rightarrow \quad \dfrac{A \vee B^{\in\Omega}}{A}$, unless B cannot be interpreted as false.

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- an adaptive strategy
  - Regulates the withdrawal of defeasibly derived consequences

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Characterizing Default Rules: The Logic $\mathbf{CL}_{\overline{\forall}}$

Language Schema

language	letters	logical symbols	set of wffs
$\mathcal{L}$	$\mathcal{S}$	$\neg, \wedge, \vee, \supset, \equiv, \exists, \forall, =$	$\mathcal{W}$
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The Universal Quantifier $\overline{\forall}$

Both gluts and gaps are allowed with respect to $\overline{\forall}$.

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Default Rules

Default rules are defined as formulas of the form $(\overline{\forall}_\alpha)(A_\alpha \supset B_\alpha)$, with $A_\alpha, B_\alpha \in \mathcal{L}$.

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Characterizing Default Inference: the Logic $\mathbf{D^r}$

Defining the $\mathbf{D^r}$–Consequence Relation (1)

$\Gamma \vdash_{\mathbf{D^r}} A$ iff $\Gamma \vdash_{\mathbf{CL}_{\triangledown}} A \vee Dab(\Delta)$ and ...

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BUT: Is F_t a final consequence of Γ?

Characterizing Default Inference: the Logic $\mathbf{D^r}$

Definition

Dab(Δ) is a minimal *Dab*–consequence of Γ iff $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dab(\Delta)$ and there is no $\Delta' \subset \Delta$ such that also $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dab(\Delta')$.

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$\Gamma \vdash_{\mathbf{D^r}} A$ iff $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} A \vee Dab(\Delta)$ and there is no finite Θ such that $Dab(\Theta)$ is a minimal Dab–consequence of Γ and $\Theta \cap \Delta \neq \emptyset$.

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- The condition on defeasible derivation laid down by the reliability strategy!

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EXAMPLE: $\Gamma = \{ (\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha), B_t, \neg F_t \}$

$\Rightarrow$
- $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} F_t \vee ((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t)$, and
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$\Rightarrow$ F_t is a defeasible, but not a final consequence of Γ.

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  $\vee((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t)$
- $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} \neg F_t \vee ((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$

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- $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} \neg F_t \vee ((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$

$\Rightarrow$ B_t, F_t, and $\neg F_t$ are defeasible consequences of Γ.

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Characterizing Default Inference: the Logic $\mathbf{D}^r$

Too many default consequences are withdrawn!

EXAMPLE: $\Gamma = \{ (\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha)$
$P_t \}$

$\Rightarrow$
- $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} B_t \vee ((\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha) \wedge P_t \wedge \neg B_t)$
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$\Rightarrow$
- $((\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha) \wedge P_t \wedge \neg B_t) \vee$
  $((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee$
  $((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$
  is a minimal *Dab*–consequence of Γ.

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Characterizing Default Inference: the Logic $\mathbf{D}^{\mathbf{r}}$

Too many default consequences are withdrawn!

EXAMPLE: $\Gamma = \{ (\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha)$
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  $((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$
  is a minimal *Dab*–consequence of Γ.

$\Rightarrow$ Neither B_t, F_t, nor $\neg F_t$ is a final consequence of Γ.

Characterizing Default Inference: the Logic $\mathbf{D}^{\mathbf{ram}}$

Abnormality Markers

Formulas that represent the information about specificity and inheritance that is derivable from the premise set.

Characterizing Default Inference: the Logic $\mathbf{D^{ram}}$

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Formulas that represent the information about specificity and inheritance that is derivable from the premise set.

⇒ They point out which of the abnormalities in a *Dab*–consequence are to be blamed for the conflict at hand!

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Preliminary Remark

Default rules are limited to formulas of the form $(\overline{\forall}_\alpha)(A_\alpha \supset B_\alpha)$, with $A_\alpha, B_\alpha \in \mathcal{A}$ (= the set of atomic formulas and negations of atomic formulas).

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Some Shorthand Notations

- $[A^1_\alpha, ..., A^n_\alpha \mid B_\alpha] =_{df} (X_\alpha)(A^1_\alpha \supset A^2_\alpha) \wedge (X_\alpha)(A^2_\alpha \supset A^3_\alpha) \wedge ... \wedge$
  $(X_\alpha)(A^n_\alpha \supset B_\alpha)$, with $X \in \{\forall, \overline{\forall}\}$.

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- $neg(A) =_{df}$ the complement of A.

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Definition

- $\mathcal{M}_i = \{[A^1_\alpha, ..., A^n_\alpha \mid C_\alpha] \wedge [B^1_\alpha, ..., B^m_\alpha \mid neg(C_\alpha)] \wedge A^1_\beta \wedge B^1_\beta \wedge \neg(A^n_\beta \wedge B^m_\beta)$ $\mid A^n_\alpha \notin \{B^1_\alpha, ..., B^m_\alpha\}$, and $B^m_\alpha \notin \{A^1_\alpha, ..., A^n_\alpha\}\}$

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- $\mathcal{M}_s = \{[A^1_\alpha, ..., A^n_\alpha \mid C_\alpha] \wedge [A^1_\alpha, ..., A^n_\alpha, B^1_\alpha, ..., B^m \mid neg(C_\alpha)] \wedge A^1_\beta \wedge \neg(C_\beta \wedge B^m_\beta) \mid \beta \in \mathcal{C}\}$

Characterizing Default Inference: the Logic $\mathbf{D}^{\mathbf{ram}}$

Definition

- $\Theta^{\leqslant 1}$ expresses that Θ is at most a singleton.
- $Dabam(\Delta, \Theta^{\leqslant 1}) = \bigvee(\Delta \cup \Theta^{\leqslant 1})$, with $\Delta \subset \Omega$ and $\Theta^{\leqslant 1} \subset \mathcal{M}_i \cup \mathcal{M}_s$.

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Definition

$Dabam(\Delta, \Theta^{\leqslant 1})$ is a minimal *Dabam*–consequence of Γ iff $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dabam(\Delta, \Theta^{\leqslant 1})$ and there is no $\Delta' \subset \Delta$ and no $\Sigma^{\leqslant 1} \subset \mathcal{M}_i \cup \mathcal{M}_s$ such that also $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dab(\Delta', \Sigma^{\leqslant 1})$.

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Defining the $\mathbf{D^{ram}}$–Consequence Relation

$\Gamma \vdash_{\mathbf{D^{ram}}} A$ iff $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} A \vee Dab(\Delta)$ and there is no finite $\Theta \subset \Omega$ and no $\Sigma^{\leqslant 1} \subset \mathcal{M}_i \cup \mathcal{M}_s$ such that $Dabam(\Theta, \Sigma^{\leqslant 1})$ is a minimal *Dabam*–consequence of Γ and $\Theta \cap \Delta \neq \emptyset$.

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$Dabam(\Delta, \Theta^{\leqslant 1})$ is a minimal *Dabam*–consequence of Γ iff $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dabam(\Delta, \Theta^{\leqslant 1})$ and there is no $\Delta' \subset \Delta$ and no $\Sigma^{\leqslant 1} \subset \mathcal{M}_i \cup \mathcal{M}_s$ such that also $\Gamma \vdash_{\mathbf{CL}_{\overline{\forall}}} Dab(\Delta', \Sigma^{\leqslant 1})$.

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- The condition on defeasible derivation laid down by the reliability strategy with abnormality markers!

Outline

1 Defaults, Commonsense Reasoning, and AI

2 The Adaptive Logics Approach
- Characterizing Default Rules
- Characterizing Default Inference
- Example

3 Conclusion

Example

$$\Gamma = \{ (\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha), (\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha), P_t \}$$

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$\Rightarrow$ B_t, F_t, and $\neg F_t$ are defeasible consequences of Γ.

- $((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee ([P_\alpha \mid \neg F_\alpha] \wedge [P_\alpha, B_\alpha \mid F_\alpha] \wedge P_t \wedge \neg(\neg F_t \wedge B_t))$ is a minimal *Dabam*–consequence of Γ.

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- $((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee ([P_\alpha \mid \neg F_\alpha] \wedge [P_\alpha, B_\alpha \mid F_\alpha] \wedge P_t \wedge \neg(\neg F_t \wedge B_t))$ is a minimal *Dabam*–consequence of Γ.

$\Rightarrow$ $((\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha) \wedge P_t \wedge \neg B_t) \vee ((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee ((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$ is not a minimal *Dabam*–consequence of Γ.

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$\Rightarrow$ B_t, F_t, and $\neg F_t$ are defeasible consequences of Γ.

- $((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee ([P_\alpha \mid \neg F_\alpha] \wedge [P_\alpha, B_\alpha \mid F_\alpha] \wedge P_t \wedge \neg(\neg F_t \wedge B_t))$ is a minimal *Dabam*–consequence of Γ.

$\Rightarrow$ $((\overline{\forall}_\alpha)(P_\alpha \supset B_\alpha) \wedge P_t \wedge \neg B_t) \vee ((\overline{\forall}_\alpha)(B_\alpha \supset F_\alpha) \wedge B_t \wedge \neg F_t) \vee ((\overline{\forall}_\alpha)(P_\alpha \supset \neg F_\alpha) \wedge P_t \wedge \neg\neg F_t)$ is not a minimal *Dabam*–consequence of Γ.

$\Rightarrow$ F_t isn't a final consequence of Γ, but B_t and $\neg F_t$ are final consequences of Γ!

Conclusion

To Conclude

In comparison to AI–approaches, the approach based on $\mathbf{D}^{\mathbf{ram}}$ comes much closer to actual human reasoning.

More Results

- There is a semantic as well as a proof theoretic characterization of the logic $\mathbf{D}^{\mathbf{ram}}$.
  - + Soundness and completeness have been proven.
- The approach can easily be extended to all default rules of the form $(\overline{\forall}_\alpha)(A_\alpha \supset B_\alpha)$.

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Further Research

- To develop adaptive logics for default inference based on requirements different from specificity and inheritance.
- To compare the $\mathbf{D}^{\mathbf{ram}}$–consequence relation with existing consequence relations for default inference.

References

BATENS, D. Zero logic adding up to classical logic. *Logical Studies 2* (1999), 15pp.

BATENS, D. A general characterization of adaptive logics. *Logique et Analyse 173–175* (2001), 45–68.

BATENS, D. A universal logic approach to adaptive logics. *Logica Universalis 1* (2007), 221–242.

DELGRANDE, J. A preference–based approach to default reasoning: preliminary report. In *AAAI–94*, Seattle (Washington), 1994, pp. 902–908.

DELGRANDE, J., AND SCHAUB, T. Compiling specificity into approaches to nonmonotonic reasoning. *Artificial Intelligence 90* (1997), 301–348.

DELGRANDE, J., AND SCHAUB, T. The role of default logic in knowledge representation. In J. Minker, Ed. *Logic–Based Artificial Intelligence*, Kluwer, Dordrecht, 2000, pp. 107–126.

MCCARTHY, J. Applications of circumscription to formalizing common–sense knowledge. *Artificial Intelligence 13* (1986), 27–39.

REITER, R. On reasoning by default. In *Theoretical Issues in Natural Language Processing*, ACL, Morristown (New Jersey), 1978, pp. 210–218.

REITER, R. A logic for default reasoning. In M. Ginsberg, Ed. *Readings in Nonmonotonic Reasoning*, Kaufmann, Los Altos (Calif.), 1987, pp. 68–93.

REITER, R., AND CRISCUOLO, G. On interacting defaults. In M. Ginsberg, Ed. *Readings in Nonmonotonic Reasoning*, Kaufmann, Los Altos (Calif.), 1987, pp. 94–100.

RINTANEN, J. On specificity in default logic. In *IJCAI*, 1995, pp. 1474–1479.

SEK–WAH, T. AND PEARL, J. Specificity and inheritance in default reasoning. In *IJCAI*, 1995, pp. 1480–1487.