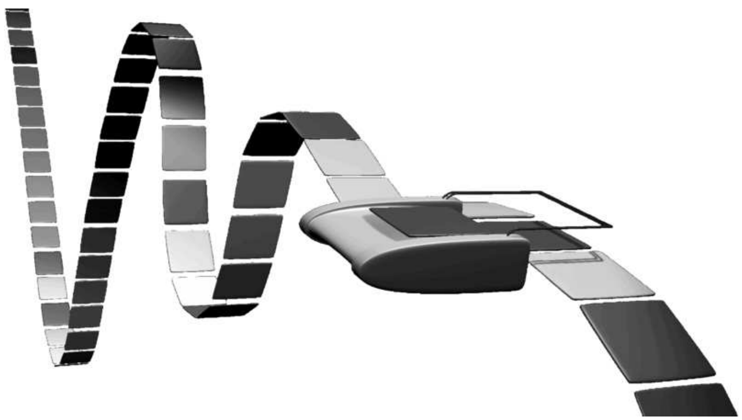
From Practices of Mathematical Logic to a natural Law? The Case of A. Church and E.L. Post



Liesbeth De Mol
Centre for Logic and Philosophy of Science
Universiteit Gent, Belgium
elizabeth.demol@ugent.be

Introduction

Specific Question: How did Church, Post (and Turing) arrive at their respective theses, i.e., the formalization of certain intuitive notions (1936)? What was the meaning of what is now known as the Church-Turing thesis *originally*?

General Question: What is the role of "practices" of symbolic logic for the "discovery" of a result like the Church-Turing thesis? Focus on the role of formalisms!

Introduction (Continued)

- Historical context of the Church-Turing thesis
- Post's practice
- Church's practice
- "you can't hide behind a definition": the Church-Turing thesis as a natural law (Post) or a definition (Church)?
- Discussion

1. Church-Turing thesis: Historical context

Church-Turing thesis: Historical context

- **Mathematical Logic** "professional philosophers have taken very little interest in it, presumably because they found it too mathematical. On the other hand, most mathematicians, have taken very little interest in it, because they found it too philosophical" (Skolem, 1928)
- **Formalizing mathematics and study foundations** *Principia Mathematica*, decision problems
- **Significance** "[T]he contemporary practice of mathematics, using as it does heuristic methods, only makes sense because of this undecidability. When the undecidability fails then mathematics, as we now understand it, will cease to exist; in its place there will be a mechanical prescription for deciding whether a given sentence is provable or not" (Von Neumann, 1927)

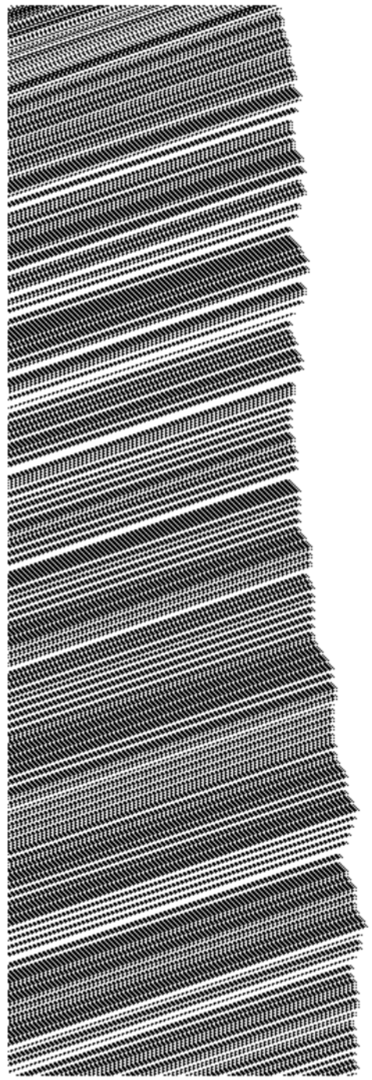
Church's thesis "We now define the notion [...] of an **effectively calculable** function of positive integers by identifying it with the notion of a **recursive function** of positive integers (or of a **λ-definable** function of positive integers.)"

Turing's thesis "The expression 'there is a general process for determining...' has been used throughout this section as equivalent to 'there is a machine which will determine...'. This usage can be justified if and only if we can justify our definition of **'computable'** [...] According to my definition, a number is computable if its decimal expansion can be written down by **a machine**"

Turing's main question: "What are the possible processes which can be carried out in computing a number?" (Turing, 1936)

$\Rightarrow$ If true, then there are problems that cannot be decided in finite time (e.g. the halting problem)

2. Post's practices



Post's practices: Two (hypo)theses

1921 (!): Post's thesis I (P1) Every **generated set of sequences** on a given set of letters $a_1, a_2, ..., a_\mu$ is a subset of the set of assertions of a **system in normal form** with primitive letters $a_1, a_2, ..., a_\mu, a'_1, a'_2, ..., a'_\nu$, i.e., the subset consisting of those assertions of the normal system involving the letters $a_1, a_2, ..., a_\mu$

1936: Post's thesis II (P2) A decision problem is considered **intuitively solvable** iff. the problem is **1-given and one can set-up a finite 1-process which is a 1-solution** to the problem.

$\Rightarrow$ *Where do these two logically equivalent formulations come from?*

The starting point: Post's PhD (1920)

Introduction to a general theory of elementary propositions, 1921

⇒ **Principia Mathematica** $(\sim, \vee)$

– "[...] 'Principia' is but one particular development of the theory [...] and so [one] might construct a general theory of such developments." (Post, 1921)

⇒ **Survey of Symbolic Logic** (Clarence I. Lewis, 1918)

– "Mathematics without Meaning"

– "This meaning of + and - is convenient to bear in mind as a guide to thought, but in the actual development they are to be considered merely as symbols which we manipulate in a certain way"

Post's PHD

$*54{\cdot}42.\ \vdash :: \alpha \in 2 . \supset :. \beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv . \beta \in \iota``\alpha$

Dem.

$\vdash . *54{\cdot}4 . \supset \vdash :: \alpha = \iota`x \cup \iota`y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \equiv : \beta = \Lambda . \vee . \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha : \exists ! \beta :$

$[*24{\cdot}53{\cdot}56 . *51{\cdot}161] \quad \equiv : \beta = \iota`x . \vee . \beta = \iota`y . \vee . \beta = \alpha \qquad (1)$

$\vdash . *54{\cdot}25 . \mathrm{Transp} . *52{\cdot}22 . \supset \vdash : x \neq y . \supset . \iota`x \cup \iota`y \neq \iota`x . \iota`x \cup \iota`y \neq \iota`y :$

$[*13{\cdot}12] \quad \supset \vdash : \alpha = \iota`x \cup \iota`y . x \neq y . \supset . \alpha \neq \iota`x . \alpha \neq \iota`y \qquad (2)$

$\vdash . (1) . (2) . \supset \vdash :: \alpha = \iota`x \cup \iota`y . x \neq y . \supset :.$

$\beta \subset \alpha . \exists ! \beta . \beta \neq \alpha . \equiv : \beta = \iota`x . \vee . \beta = \iota`y :$

$[*51{\cdot}235] \quad \equiv : (\exists z) . z \in \alpha . \beta = \iota`z :$

$[*37{\cdot}6] \quad \equiv : \beta \in \iota``\alpha \qquad (3)$

$\vdash . (3) . *11{\cdot}11{\cdot}35 . *54{\cdot}101 . \supset \vdash . \mathrm{Prop}$

$*54{\cdot}43.\ \vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

$[*51{\cdot}231] \quad \equiv . \iota`x \cap \iota`y = \Lambda .$

$[*13{\cdot}12] \quad \equiv . \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$

$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash . \mathrm{Prop}$

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

$\xrightarrow{?}$ Post's thesis 1

Post's "programme" (1918–1920)

⇒ Two directions of generalization to study systems of symbolic logic

1. Development of a *general* theory of systems of symbolic logic:

   - *Generalization by Postulation, systems in canonical form A* and many-valued logics

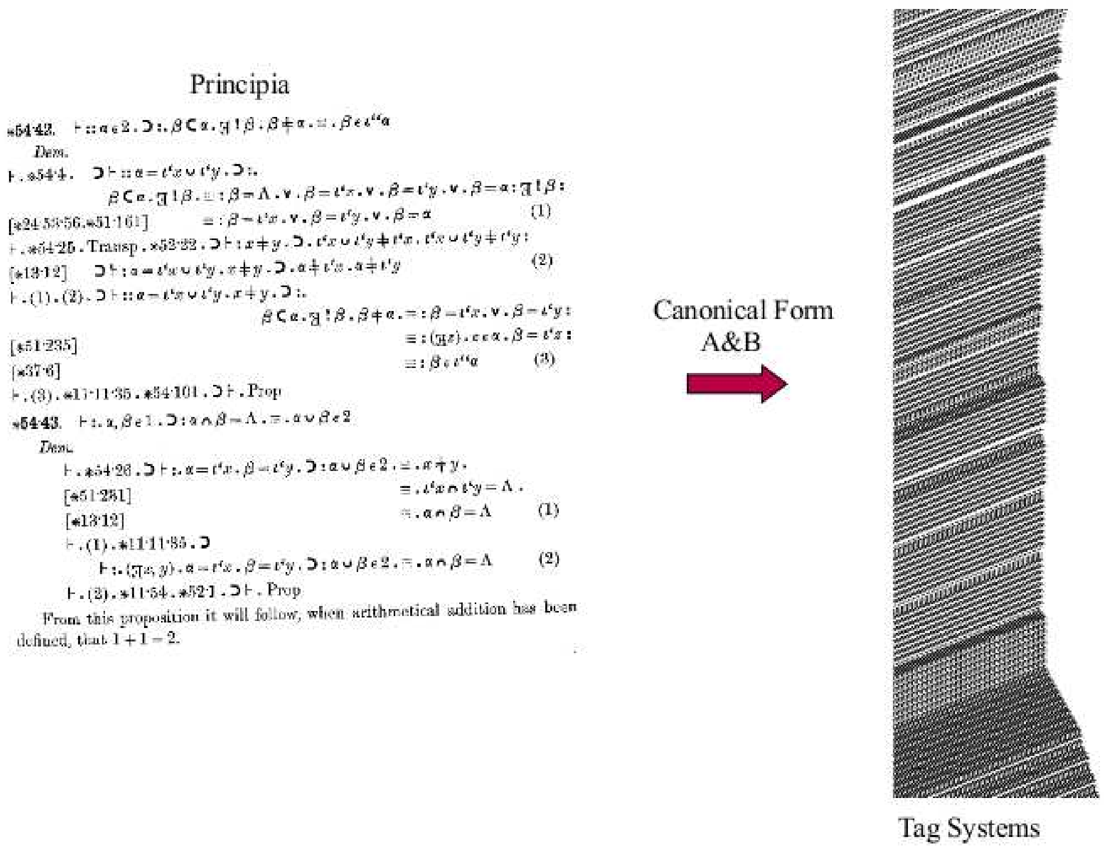
2. Generalization of the main results to other parts of *Principia* and ultimately mathematics. "Since *Principia* was intended to formalize all of existing mathematics, Post was proposing no less than to find a single algorithm for all of mathematics." (Davis, 1994)

⇒ Post's method ∼ Lewis' mathematics without meaning

Simplification through generalization: "Perhaps the chief difference in method between the present development and its more complete successors is its preoccupation with the outward forms of symbolic expressions, and possible operations thereon. [This] allows greater freedom of method and technique."

Account of an anticipation: towards the reversal of Post's programme

Method (influence Lewis): Simplification through generalization:



Definition of tag systems. A (relatively) famous Example

Let T_{Post} be defined by $\Sigma = \{0, 1\}, v = 3, 1 \rightarrow 1101, 0 \rightarrow 00$

$A_0 = 10111011101000000 \Rightarrow$ Primitive assertion

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$\cancel{001}\underbrace{10111011101000000}_{A_0} \Rightarrow$ Periodicity!

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$\cancel{010}0000011011101110100$

$\cancel{000}001101110111010000$

$\cancel{001}\underbrace{10111011101000000}_{A_0} \Rightarrow$ Periodicity!

⇒ Definition of a *class* of symbolic logics according to a form

⇒ Two decision problems (finiteness problems) for tag systems: the halting and reachability problem

The frustrating problem of "Tag" and the reversal of Post's programme

⇒ **Exploring tag systems: pencil-and-paper computations and "observations" – "experimentation as exploration"**

- "Observation" of three classes of behavior: periodicity, halt, unbounded growth.
- Three decidable classes ($v = 1;\mu = 1;\ \mu = v = 2$) (Wang, 1963; De Mol, 2010)
- Infinite class with $\mu = 2, v = 3$: "intractable" (Minsky, 1967; De Mol, 2009)
- Infinite class with $\mu > 2, v = 2$: "bewildering complexity"

⇒ *Principia* vs. Lewis-like Abstract form ("mathematics without meaning") → shift to an analysis of the behavior → limitations of Lewis' ideal mathematics

⇒ **The reversal** "[T]he general problem of "tag" appeared hopeless, and with it our entire program of the solution of finiteness problems. This *frustration* [my emphasis], however, was largely based on the assumption that "tag" was but a minor, if essential, stepping stone in this wider program." (Post,1965)

From canonical form C to systems in normal form....

Defined by Σ, one initial word $\in \Sigma^*$ and a finite set of production rules of the form:

$$\begin{array}{ccc} g_iP_i & 1101P_i & \cancel{1101}11011101000000 \\ & \textit{produces} & \\ P_ig_{i'} & P_i001 & 11011101000000001 \end{array}$$

with each $g_i, g_{i'}, P_i \in \Sigma^*$.

...to "the most beautiful theorem in mathematics" (Minsky, 1961)

"May I suggest that the tricks employed in my paper [...] were forced on me by ~~b~~ the ever more restricted formal means left me by the required ever simpler forms of basis. " (Post in a letter to Church, dated July 29, 1943)

The anticipation....

- **"Mathematics without meaning" $\rightarrow$ tag systems $\rightarrow$ normal form $\rightarrow$ Normal form theorem:**
  "In view of the generality of the system of *Principia Mathematica*, and its seeming inability to lead to any other generated set of sequences on a given set of letters than those given by our normal systems, we are led to the following generalization", i.e., Post's thesis I (Davis,1982):

  > **Post's Thesis I.** *Every* ***generated*** *set of sequences on a given set of letters* $a_1, a_2, ..., a_\mu$ *is a subset of the set of assertions of a system in* ***normal form*** *with primitive letters* $a_1, a_2, ..., a_\mu, a'_1, a'_2, ..., a'_\mu$*, i.e., the subset consisting of those assertions of the normal system involving the letters* $a_1, a_2, ..., a_\mu$.

- **Given thesis I + idea reversal programme:**

  > "[...] the finiteness problem for the class of all normal systems is unsolvable"
  >
  > "A complete logic is impossible"

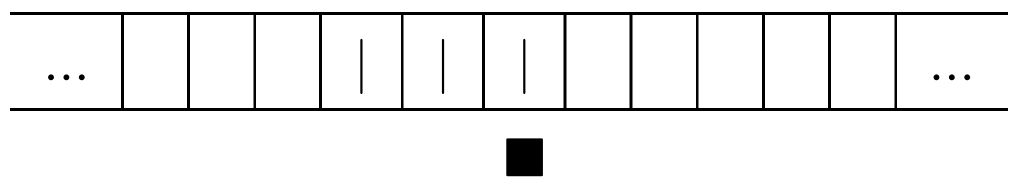
From normal form (1921) to Post's machines (1936)

$Post's\ Thesis\ I \quad \overset{?}{\Rightarrow} \quad Post's\ thesis\ II$

~~1101~~11011101000000

produces $\overset{?}{\Rightarrow}$

11011101000000001



$Generated\ sets \quad \overset{?}{\Rightarrow} \quad Solvability$

Post's thesis II

⇒ Motivation

"[for the thesis to obtain its full generality] an analysis should be made of all the possible ways the human mind can set up finite processes to generate sequences."

"Establishing this universality is not a matter for mathematical proof, but of *psychological analysis of the mental processes involved in combinatory mathematical processes* [m.i.].

"The real question at issue is: What are the possible processes that can be carried out in computing a number?" (Turing, 1936)

⇒ Robustness? From a hypothesis to a law

0 Post's beliefs and experiences

I Demand for an appeal to intuition argument (for the thesis to obtain its full generality)

II Demand Argument by confluence: contemplating other "models" and prove them reducible to formulation I: "The success of the above program would, for us change this working hypothesis [...] to a *natural law*"

3. Church's practices

$$(\lambda[m]\cdot(\lambda[n]\cdot(\lambda[f]\cdot(\lambda[x]\cdot m[f][n[f][x]]))))[5][2] =$$
$$\lambda[f]\cdot(\lambda[x]\cdot f[f[f[f[f[f[f[x]]]]]]])$$

Church's practices: One definition

1936: Church's thesis "We now define the notion [...] of an *effectively calculable* function of positive integers by identifying it with the notion of a recursive function of positive integers (or of a λ-definable function of positive integers.)"

$\Rightarrow$ *Where does this formulation which is logically equivalent to Post's theses come from?*

"To deny what seems intuitively natural"....

Topic?

The axiom of choice: For any set A, all of whose member are non-empty sets, there exists a set B which contains exactly one element from each of the sets belonging to A

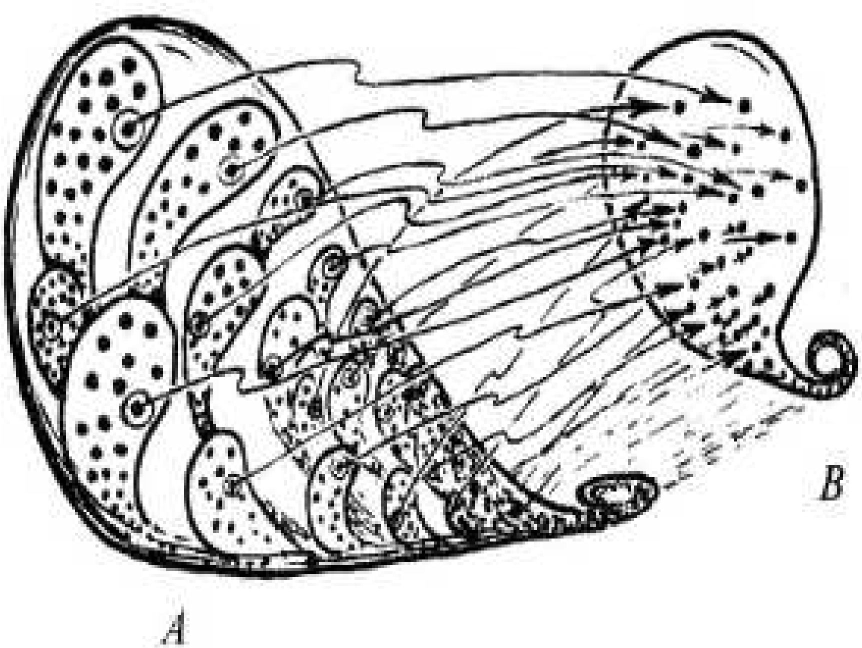
Logical independence C of ZF? "The object of this paper is to consider the possibility of setting up a logic in which the axiom of choice is false."

Church's "experimental" approach?:

"[I]f a considerable body of theory can be developed on the basis of one of these postulates without obtaining inconsistent results, then this body of theory, when developed, could be used as presumptive evidence that no contradiction exists." (Church, 1927)

"We shall examine briefly the consequences of each of the postulates just stated [...] taking the same experimental attitude as that which we [already used]" (Church, 1927)

Church's PhD



Church's Thesis?

"A set of postulates for the Foundation of Logic
⇒ Motivations

Yet another formalization of mathematics *after* (!) Gödel "In this paper we present a set if postulates for the foundation of formal logic" (Church, 1932)

Going beyond Gödel "I was seeking to do the very thing that Gödel proved impossible" (Church in a letter to Dawson, July 25, 1983)
"[...] This is conceivable on account of the entirely formal character of the system which makes it possible to abstract from the meaning of the symbols and to regard the proving of theorems (of formal logic) as a game played with marks on paper according to a certain arbitrary set of rules" (Church 1933) $\sim$ Post

Introduction of the λ-operator to denote functions: **Ex.** "$x^4 + x$ is smaller than 1000" vs. "x^4+x is a primitive recursive function" $\rightarrow \lambda x.x^4+x$

⇒ Criterion of consistency

"We do not attach any character of uniqueness or absolute truth to any particular system of logic." (Church, 1932)

The "experimental" approach (revisited)?

"Whether the system of logic which results from our postulates is adequate for the development of mathematics, and whether it is wholly free from contradiction, are questions **which we cannot answer except by conjecture. Our proposal is to seek at least an empirical answer to these questions by carrying out in some detail a derivation of the consequences of our postulates**" (Church, 1932)

"Our present project is to develop the consequences of the foregoing set of postulates, until a contradiction is obtained from them, or until the development has been carried so far consistently as **to make it empirically probable that no contradiction can be obtained from them.**" (Church, 1933)

⇒ **Confronted with the problems of this approach?:** Church's set of postulates proven inconsistent by his PhD students (Kleene and Rosser, 1935)

λ – The ultimate operator?

- Symbols: $\lambda, (,), x, y, z, \ldots$
- λ-formulas:
  - the variables
  - If P is a λ-formula containing x as a free variable then $\lambda x[P]$ $(\lambda x.P)$ is also a λ-formula.
  - If M and N are λ-formulas then so is $\{\mathrm{M}\}(\mathrm{N})$
- Rules of conversion:
  1. *Reduction.* To replace any part $((\lambda x\ \mathrm{M})\ \mathrm{N})$ of a formula by $\mathrm{S}^{x}_{\mathrm{N}}\mathrm{M}|$ provided that the bound variables of M are distinct both from x and from the free variables of N. For example to change $\{\lambda x[x^2]\}(2)$ reduces to 2^2
  2. *Expansion* To replace any part $\mathrm{S}^{x}_{\mathrm{N}}\mathrm{M}|$ of a formula by $((\lambda x\ \mathrm{M})\ \mathrm{N})$ provided that $((\lambda x\ \mathrm{M})\ \mathrm{N})$ is well-formed and the bound variables of M are distinct both from x and from the free variables in N. For example, 2^2 can be expanded to $\{\lambda x[x^2]\}(2)$
  3. *Change of bound variable* To replace any part M of a formula by $\mathrm{S}^{x}_{\mathrm{y}}\mathrm{M}|$ provided that x is not a free variable of M and y does not occur in M. For example changing $\{\lambda x[x^2]\}$ to $\{\lambda y[y^2]\}$

λ – The ultimate operator: an example

- Defining the natural numbers:

$$1 \rightarrow \lambda yx.yx,$$
$$2 \rightarrow \lambda yx.y(yx),$$
$$3 \rightarrow \lambda yx.y(y(yx)),$$
$$\ldots$$

- The successor function S:

$$S \rightarrow \lambda abc.b(abc)$$

$$\begin{aligned}
& \Big(\lambda abc.b(abc)\Big)\Big(\lambda yx.y(yx)\Big) = S(2) \\
\rightarrow \quad & \lambda bc.b\Big(\Big(\lambda yx.y(yx)\Big)bc\Big) \\
\rightarrow \quad & \lambda bc.b\Big(\Big(\lambda x.b(bx)\Big)c\Big) \\
\rightarrow \quad & \lambda bc.b(b(bc)) = 3
\end{aligned}$$

Surprised by λ....

The heuristic method revisited (again)?

> "We kept thinking of specific such functions, and of specific operations for proceeding from such functions to others. I kept establishing the functions to be λ-definable and the operations to preserve λ-definability." (Kleene, 1981)

> "Our object is to prove empirically (!) that the system is adequate for the theory of positive integers" (Kleene, 1935)

> "The results of Kleene are so general and the possibilities of extending them apparently so unlimited that one is led to the conjecture that a formula can be found to represent any particular constructively defined function of positive integers whatever." (Church in a letter to Bernays, January 231935)

$\Rightarrow$ Every effectively calculable function is λ-definable

Convinced by the formalism "Turing's definition of computability was intrinsically plausible, whereas with the other two, a person became convinced only after he investigated and found, much by surprise, how much could be done with the definition." (Kleene in an interview with Aspray, 1985)

...and being "careful" about λ

- First informal formulation Church's thesis I in February 1934; public announcement: April, 1935!
- In need of more support (robustness?): argument by example, argument by confluence + step-by-recursive step argument

$\Rightarrow$ **1936: Church's thesis** "We now define the notion [...] of an *effectively calculable* function of positive integers by identifying it with the notion of a recursive function of positive integers (or of a λ-definable function of positive integers.)"

4. "you can't hide behind a definition"

A definition or a hypothesis?

Post’s position: “Its purpose [...] is not only to present a system of a certain logical potency but also, [...] of psychological fidelity [...] We offer this conclusion at the present moment as a *working hypothesis*. [...] The success of the above program would, for us, change this hypothesis not so much to a definition or to an axiom but to a *natural law*. (Post, 1936)

Church’s position: “[The purpose of this paper is] to propose a definition of effective calculability which is thought to correspond satisfactorily to the somewhat vague intuitive notion in terms of which problems of this class are often stated, and to show that not every problem of this class is solvable [...] This [proposed] definition is thought to be justified by the considerations which follow, so far as positive justification can ever be obtained for the selection of a formal definition to correspond to an intuitive notion”

The disagreement between Church and Post: Post's reaction

- "But to mask this identification under a definition hides the fact that a fundamental discovery in the limitations of the mathematicizing power of *Homo Sapiens* has been made and blinds us to the need of its continual verification." (Post, 1936)
- "For if symbolic logic has failed to give wings to mathematicians this study of symbolic logic opens up a new field concerned with the fundamental limitations of mathematics, more precisely the mathematics of Homo Sapiens." (Post to Church, March 24, 1936)
- "Again you argue that you can always as it were withdraw into your shell and say that that is your definition of effective calculability [...] But then you should be able to bar any ambitious young man from attempting its solution by any means [...] But now you see you can't hide behind merely a definition." (Post to Church, July 10, 1936)
- "But I am not willing to stake my "immortal" soul in it – which I should were I to adopt your original position." (Post to Church, June 9, 1937)

The disagreement between Church and Post: Church's reaction

- "[Y]ou seem to be inclined to put the burden of proof on me, which I do not think justified. The fact is that the intuitive notion is vague and inexact, and that what I proposed was to render it exact by giving a formal definition. [I]f they maintain that there is something in their notion which makes it more general than recursiveness or lambda-definability, they can be legitimately asked to produce an example, and, failing to do so, stand convicted of making an utterly vague and even pointless assertion. I am thus content to let the matter stand as a challenge." (Church to Post, September 18, 1936)
- "I have not been trying to establish an empirical proposition, or a mathematical proposition. Instead, I have been proposing an exact definition of a phrase ("effectively calculable") which has hitherto had only a vague meaning. Under such circumstances proofs are not to be expected, but only considerations of convenience and naturalness and facts of historical usage and generally accepted connotations." (Church to Post, July 10, 1937)

Why this disgreement?

The Case Church: ?? (but, “I have not been trying to establish an empirical proposition, or a mathematical proposition”)

The Case Post:

The human limitations and the frustrating problem of “tag”: “This problem [the problem of “tag”] itself in its entscheidungsproblem form is a special case of my unsolvable problem (which I hope to get to at least before he end of this letter if not of your patience) and should it too prove unsolvable I will be supplied with the perfect alibie [sic] for a year of frustration.” (Post in a latter to Church, May 30, 1936)
“my wife is much worried. So I told her for the first time, the exact history of my mental ups and downs and worse from its first inception in trying to solve the probably unsolvable tag-problem in Princeton and how at 50 experience and lesson of personal importance of failure or success with at best 70-50 $<$ 20 years to go – but I see my 50 years of experience may still not be enough – God help me” (Post in a letter to Church, March 3, 1947)

5. Discussion

Discussion

- Heuristics and "empirical" or "experimental" approaches in Logic lead to and explain results in Logic **vs.** the idea of symbolic logic as a removal of these methods ("[T]he contemporary practice of mathematics, using as it does heuristic methods, only makes sense because of this undecidability.")
- Man-Logic Interactions; exploring formalisms
- $\sim$ the use of the computer (physical "model" of computability) to explore the "universe of mathematics"

  [...] the creativeness of human mathematics has a counterpart inescapable limitation thereof – witness the absolutely unsolvable (combinatory) problems. Indeed, with the bubble of symbolic logic as universal logical machine finally burst, a new future dawns for it as the indispensable means for revealing and developing those limitations. For [...] Symbolic Logic may be said to be Mathematics become self-conscious.

  Emil L. Post, 1920–21.