Centre for Logic and Philosophy of Science
Centrum voor Logica en Wetenschapsfilosofie

UNIVERSITEIT GENT

FACULTY OF ARTS AND PHILOSOPHY

A formal explication of the search for explanations. The adaptive logics approach to abductive reasoning.

Hans Lycke

Centre for Logic and Philosophy of Science
Ghent University
Hans.Lycke@Ugent.be
http://logica.ugent.be/hans

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Outline

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Abduction?

Searching for Explanations

Backwards reasoning: from the phenomena to be explained to possible explanations.

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Example

- A physician in search of the right diagnosis for a patient's symptoms,
- a technician trying to find out why a machine broke down,
- a scientist trying to find an explanation for an empirical phenomenon contradicting some predictions derived from an accepted theory,...

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Example

Given the background theory Γ, A is an explanation for B iff

- $\Gamma \cup \{A\} \vdash B$
- $\Gamma \nvdash \neg A$
- $\Gamma \nvdash B$; $A \nvdash B$
- ...

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- FOR Focus is on abductive consequence, not on abductive reasoning
  - $\Rightarrow$ Search procedures instead of a proof theory
    - e.g. Tableau methods (Aliseda–Llera 2006, Mayer&Pirri 1993)

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- (Some of) the conditions of **BD** can be incorporated.
- A nice proof theory for abductive reasoning is provided.

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Advantages

- (Some of) the conditions of **BD** can be incorporated.
- A nice proof theory for abductive reasoning is provided.

⇒ The adaptive logics approach provides a more realistic explication of the application of abductive inferences in human reasoning!

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To present a general approach towards the explication of abductive reasoning based on the Adaptive Logics Programme.

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  = To characterize the abductive inference rule in general.

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Three Steps

- The deductive frame
  = To spell out the relation between abduction and deduction.
- On defeasible inference
  = To characterize the abductive inference rule in general.
- Enter adaptive logics
  = To characterize some adaptive logics for abductive reasoning.

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Abduction vs Deduction

Intertwinement

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  $\Rightarrow$ Abductive inference steps are applied against a deductive background!

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Language Schema of **RBK**

language	letters	logical symbols	set of formulas
$\mathcal{L}$	$\mathcal{S}$	$\neg, \wedge, \vee, \supset$	$\mathcal{W}$
$\mathcal{L}^{\mathcal{M}}$	$\mathcal{S}$	$\neg, \wedge, \vee, \supset, \Box_n, \Box_e, \Diamond_n, \Diamond_e$	$\mathcal{W}^{\mathcal{M}}$

- $\Box_n$ expresses *nomological necessity*.
- $\Box_e$ expresses *empirical necessity*.

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Proof Theory of **RBK**

= the axiom system of **CL**, extended by

AM1n	$\Box_n(A \supset B) \supset (\Box_n A \supset \Box_n B)$	AM1e	$\Box_e(A \supset B) \supset (\Box_e A \supset \Box_e B)$
AM2n	$\Box_n A \supset A$	AM2e	$\Box_e A \supset A$
NECn	From $\vdash A$ to $\vdash \Box_n A$	NECe	From $\vdash A$ to $\vdash \Box_e A$
AM3	$\Box_n A \supset \Box_n \Box_n A$		
AM4	$\Box_n A \supset \Box_e A$		

$\Diamond_n A =_{df} \neg \Box_n \neg A$
$\Diamond_e A =_{df} \neg \Box_e \neg A$

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  - $\mathcal{W}^{\mathcal{N}} = \{\Box_n A \mid A \in \mathcal{W}\}$ Nomological Facts
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On Defeasible Inference

AC in a Modal Environment

The applications of **AC** that qualify for conditional acceptance are limited to those satisfying the following schema:

AC^m $\Box_n(A \supset B), B, \Delta \vdash A$

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  OTHERWISE It wouldn't be in need of an explanation.

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- Certain additional conditions have to be fulfilled before **AC^m** may be applied.

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$\Rightarrow$ Abduction processes are layered processes!

  $\Rightarrow$ The adaptive logics needed are *prioritized adaptive logics*.

Outline

Multiple Abduction Processes

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Practical Abduction

In case of multiple possible explanations, only the disjunction of all possible explanations is derivable.

$\Rightarrow$ The logic $\mathbf{AbL^p}$

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Prioritized Abduction

In case of multiple possible explanations, only the most plausible explanations are derivable.

$\Rightarrow$ The logic $\mathbf{AbL}^{\mathbf{pt}}$

Multiple Abduction Processes

Earlier Attempts

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- J. Meheus et al. Ampliative Adaptive logics and the foundation of logic-based approaches to abduction. In: L. Magnani, N. Nersessian and C. Pizzi. *Logical and Computational Aspects of Model-Based Reasoning*, Kluwer, Dordrecht, 2002, pp. 39–71.

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- J. Meheus and D. Batens. A formal logic for abductive reasoning. *Logic Journal of the IGPL*, vol. 14, 2006, pp. 221–236.

  BUT Only abductive inferences at the predicate level.
  BUT Only practical abduction could be characterized.

  $\Rightarrow$ Abductive reasoning is captured in a limited way.

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General Characterization

Prioritized Adaptive Logics

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Prioritized Adaptive Logics

1. A Lower Limit Logic (**LLL**)
   - The **LLL** determines the inference rules that can be applied unrestrictedly.
2. A Set of Abnormalities ($\Omega = \Omega_0 > \Omega_1 > \ldots > \Omega_n$)
   - Elements of Ω are interpreted as false as much as possible
   - The result: some conditionally derived consequences
     - $\frac{A \vee B^{\in \Omega}}{A}$, unless B cannot be interpreted as false.
   - Prioritized: Ω is a structurally ordered set of sets.
     - Consequences obtained by falsifying abnormalities of a certain priority may necessitate the withdrawal of consequences obtained by falsifying abnormalities of a lower priority.
3. An Adaptive Strategy
   - The adaptive strategy determines which of the conditionally derived formulas have to be withdrawn.

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General Characterization: Practical Abduction

The Adaptive Logic **AbL^p**

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2. Set of Abnormalities $\Omega = \Omega_{bk} > \Omega_p$

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     - ▷ $B \in \mathcal{S} \cup \mathcal{S}^{\neg}$,
     - ▷ A in *Conjunctive Normal Form*, and
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3. Adaptive Strategy = Reliability

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     $\wedge\neg(A_1 \wedge \ldots \wedge A_n) \mid$
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     - ▷ $A_1, \ldots, A_n, B \in \mathcal{S} \cup \mathcal{S}^{\neg}$,
     - ▷ B is not a subformula of $A_1 \wedge \ldots \wedge A_n$, and
     - ▷ $\neg[A_1^n \supset B] =_{df} \neg\Box_n((A_2 \wedge \ldots \wedge A_n) \supset B) \wedge \neg\Box_n((A_1 \wedge A_3 \wedge \ldots \wedge A_n) \supset B) \wedge \ldots \wedge \neg\Box_n((A_1 \wedge \ldots \wedge A_{n-1}) \supset B)$

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By integrating the knowledge of priorities in the background knowledge.

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General Characterization: Prioritized Abduction

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How to Make Use of Priorities?

There are multiple possibilities!

HERE in a straightforward way.

General Characterization: Prioritized Abduction

The Adaptive Logic $\mathbf{AbL}^{pt}$

General Characterization: Prioritized Abduction

The Adaptive Logic $\mathbf{AbL}^{\mathbf{pt}}$

1. Lower Limit Logic (**LLL**) = the logic **RBK**
2. Set of Abnormalities $\Omega = \Omega_{bk} > \Omega_{pt_1} > \Omega_{pp_1} > \Omega_{pt_2} > \Omega_{pp_2} > \ldots > \Omega_t$
   - Ω_{bk} and Ω_t as for theoretical abduction.
   - Ω_{pt_i} =
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     $\wedge \neg\Box_e B \wedge \neg(A_1 \wedge \ldots \wedge A_n) \wedge \neg[A_1^n \supset B] \mid$
     - ▷ For the most part as for theoretical abduction, except for

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     - For the most part as for theoretical abduction, except for
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   - $\Omega_{pp_i} = \{\Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge \Box_n\Diamond^i_e((C_1 \wedge \ldots \wedge C_m) \wedge B) \wedge B$
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General Characterization: Prioritized Abduction

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     - ▷ For the most part as for theoretical abduction, except for
     - ▷ $\Box_n \underbrace{\Diamond_e \ldots \Diamond_e}_{i \text{ times}}((A_1 \wedge \ldots \wedge A_n) \wedge B)$

     $\}$
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     $\wedge \neg \Box_e B \wedge \neg(A_1 \wedge \ldots \wedge A_n) \mid$
     - ▷ For the most part as for theoretical abduction, except
     - ▷ that $\neg[A^n_1 \supset B]$ is absent, and
     - ▷ that $C_1, \ldots, C_m \in \{A_1, \ldots, A_n\}$.

     $\}$
3. Adaptive Strategy = Reliability

Outline

Proof Theory (1)

General Features

- An **AbL$^{\mathbf{x}}$**–proof is a succession of stages, each consisting of a sequence of lines.
  - Adding a line to a proof is to move on to a next stage.

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  - As all **AbL**^{x} are based on the same **LLL**, the deduction rules are the same for all of them.
- Marking Criterium
  - As all **AbL**^{x} are based on the same adaptive strategy, the marking criterium is the same for all of them.
  - Dynamic proofs

Proof Theory (2)

Dab–Formulas

$Dab^x(\Delta) = \bigvee(\Delta)$, with $\Delta \subset \Omega_x$

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Deduction Rules

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{RBK}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{RBK}} B \vee Dab^x(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Proof Theory (3)

Minimal Dab^x–consequences

$Dab^x(\Delta)$ is a minimal Dab^x–consequence of Γ at stage s of a proof, iff (1) it occurs on an unmarked line at stage s, (2) all members of its adaptive condition belong to a $\Omega_{x'}$ such that $\Omega_{x'} > \Omega_x$, and (3) there is no $\Delta' \subset \Delta$ for which the same applies.

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The Set of Unreliable Formulas of a Certain Priority

$U^x_s(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$ for $Dab^x(\Delta_1)$, $Dab^x(\Delta_2)$,... the minimal Dab^x–consequences of Γ at stage s of the proof.

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Marking Definition

Line i is marked at stage s of the proof iff, where Δ is its condition, $\Delta \cap U^x_s(\Gamma) \neq \emptyset$.

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Line i is marked at stage s of the proof iff, where Δ is its condition, $\Delta \cap U^x_s(\Gamma) \neq \emptyset$.

Marking Proceeds Stepwise

First for the highest priority level, and afterwards for the lower ones.

Proof Theory (4)

Derivability

A is derived from Γ at stage s of a proof iff A is the second element of an unmarked line at stage s.

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Final Derivability

- A is finally derived from Γ on a line i of a proof at stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) every extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.
- $\Gamma \vdash_{\mathbf{AbL^x}} A$ iff A is finally derived on a line of a proof from Γ.

Outline

Example: Practical Abduction

Definition

$\langle A, B \rangle =_{df} \Box_n (A \supset B) \wedge B \wedge \neg \Box_e B \wedge \neg A$

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Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
3	q	–;PREM	$\emptyset$

Set of Unreliable Formulas

$U_3^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_3^{p}(\Gamma) = \emptyset$

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1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
3	q	–;PREM	$\emptyset$
4	$\neg\Box_e q$	–;RC	$\{\Box_e q\}$
5	p	$1, 3, 4$;RC	$\{\Box_e q, \langle p, q\rangle\}$
6	r	$2, 3, 4$;RC	$\{\Box_e q, \langle r, q\rangle\}$

Set of Unreliable Formulas

$U_6^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

$U_6^{p}(\Gamma) = \emptyset$

Example: Practical Abduction

Definition

$\langle A, B\rangle =_{df} \Box_n(A \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
3	q	–;PREM	$\emptyset$
4	$\neg\Box_e q$	–;RC	$\{\Box_e q\}$
5	p	1, 3, 4;RC	$\{\Box_e q, \langle p, q\rangle\}$
6	r	2, 3, 4;RC	$\{\Box_e q, \langle r, q\rangle\}$
7	$\langle p, q\rangle \vee \langle r \wedge \neg p, q\rangle$	1,2,3,4;RU	$\{\Box_e q\}$
8	$\langle r, q\rangle \vee \langle p \wedge \neg r, q\rangle$	1,2,3,4;RU	$\{\Box_e q\}$

Set of Unreliable Formulas

$U_8^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_8^{p}(\Gamma) = \{\langle p, q\rangle, \langle r \wedge \neg p, q\rangle, \langle r, q\rangle, \langle p \wedge \neg r, q\rangle\}$

Example: Practical Abduction

Definition

$\langle A, B\rangle =_{df} \Box_n(A \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$	
3	q	–;PREM	$\emptyset$	
4	$\neg\Box_e q$	–;RC	$\{\Box_e q\}$	
5	p	1, 3, 4;RC	$\{\Box_e q, \langle p, q\rangle\}$	✓
6	r	2, 3, 4;RC	$\{\Box_e q, \langle r, q\rangle\}$	✓
7	$\langle p, q\rangle \vee \langle r \wedge \neg p, q\rangle$	1,2,3,4;RU	$\{\Box_e q\}$	
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$U_8^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
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$\langle A, B \rangle =_{df} \Box_n(A \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A$

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1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$	
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4	$\neg\Box_e q$	–;RC	$\{\Box_e q\}$	
5	p	1, 3, 4;RC	$\{\Box_e q, \langle p, q\rangle\}$	✓
6	r	2, 3, 4;RC	$\{\Box_e q, \langle r, q\rangle\}$	✓
7	$\langle p, q\rangle \vee \langle r \wedge \neg p, q\rangle$	1,2,3,4;RU	$\{\Box_e q\}$	
8	$\langle r, q\rangle \vee \langle p \wedge \neg r, q\rangle$	1,2,3,4;RU	$\{\Box_e q\}$	
9	$p \vee r$	1,2,3,4;RC	$\{\Box_e q, \langle p \vee r, q\rangle\}$	

Set of Unreliable Formulas

$U_9^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_9^{p}(\Gamma) = \{\langle p, q\rangle, \langle r \wedge \neg p, q\rangle, \langle r, q\rangle, \langle p \wedge \neg r, q\rangle\}$

Example: Theoretical Abduction

Definition

$\langle A_1 \wedge \ldots \wedge A_n, B\rangle =_{df} \Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \wedge \neg[A_1^n \supset B]$

Example: Theoretical Abduction

Definition

$\langle A_1 \wedge \ldots \wedge A_n, B\rangle =_{df} \Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \wedge \neg[A_1^n \supset B]$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
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Set of Unreliable Formulas

$U_3^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_3^{t}(\Gamma) = \emptyset$

Example: Theoretical Abduction

Definition

$\langle A_1 \wedge \ldots \wedge A_n, B\rangle =_{df} \Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \wedge \neg[A_1^n \supset B]$

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1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
3	q	–;PREM	$\emptyset$
4	p	1, 3;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle\}$
5	r	2, 3;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle\}$

Set of Unreliable Formulas

$U_5^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_5^{t}(\Gamma) = \emptyset$

Example: Theoretical Abduction

Definition

$\langle A_1 \wedge \ldots \wedge A_n, B\rangle =_{df} \Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \wedge \neg[A_1^n \supset B]$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$	
3	q	–;PREM	$\emptyset$	
4	p	1, 3;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle\}$	✓
5	r	2, 3;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle\}$	✓
6	$\langle p, q\rangle \vee \langle r \wedge \neg p, q\rangle$	1,2,3;RC	$\{\Box_e q, \Box_n q, \Box_n(r \supset q), \Box_n(\neg p \supset q)\}$	
7	$\langle r, q\rangle \vee \langle p \wedge \neg r, q\rangle$	1,2,3;RC	$\{\Box_e q, \Box_n q, \Box_n(p \supset q), \Box_n(\neg r \supset q)\}$	

Set of Unreliable Formulas

$U_7^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

$U_7^{t}(\Gamma) = \{\langle p, q\rangle, \langle r \wedge \neg p, q\rangle, \langle r, q\rangle, \langle p \wedge \neg r, q\rangle\}$?

Example: Theoretical Abduction

Definition

$\langle A_1 \wedge \ldots \wedge A_n, B\rangle =_{df} \Box_n((A_1 \wedge \ldots \wedge A_n) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \wedge \neg[A_1^n \supset B]$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
3	q	–;PREM	$\emptyset$
4	p	1, 3;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle\}$
5	r	2, 3;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle\}$
6	$\langle p, q\rangle \vee \langle r \wedge \neg p, q\rangle$	1,2,3;RC	$\{\Box_e q, \Box_n q, \Box_n(r \supset q), \Box_n(\neg p \supset q)\}$ ✓
7	$\langle r, q\rangle \vee \langle p \wedge \neg r, q\rangle$	1,2,3;RC	$\{\Box_e q, \Box_n q, \Box_n(p \supset q), \Box_n(\neg r \supset q)\}$ ✓

Set of Unreliable Formulas

$U_7^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$
$U_7^{t}(\Gamma) = \emptyset$

Example: Prioritized Abduction

Definition

$\langle A, B \rangle_t \in \Omega_t$, $\langle A, B \rangle_{pt_i} \in \Omega_{pt_i}$ and $\langle A, B \rangle_{pp_i} \in \Omega_{pp_i}$

Example: Prioritized Abduction

Definition

$\langle A, B\rangle_t \in \Omega_t$, $\langle A, B\rangle_{pt_i} \in \Omega_{pt_i}$ and $\langle A, B\rangle_{pp_i} \in \Omega_{pp_i}$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n\Diamond_e(p \wedge q)$	–;PREM	$\emptyset$
3	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
4	$\Box_n\Diamond_e\Diamond_e(r \wedge q)$	–;PREM	$\emptyset$
5	q	–;PREM	$\emptyset$

Set of Unreliable Formulas

$U_5^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

Example: Prioritized Abduction

Definition

$\langle A, B\rangle_t \in \Omega_t$, $\langle A, B\rangle_{pt_i} \in \Omega_{pt_i}$ and $\langle A, B\rangle_{pp_i} \in \Omega_{pp_i}$

Example

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6	p	1, 2, 5;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$
7	r	3, 4, 5;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle_{pt_2}\}$

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Example

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2	$\Box_n\Diamond_e(p \wedge q)$	–;PREM	$\emptyset$
3	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
4	$\Box_n\Diamond_e\Diamond_e(r \wedge q)$	–;PREM	$\emptyset$
5	q	–;PREM	$\emptyset$
6	p	1, 2, 5;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$
7	r	3, 4, 5;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle_{pt_2}\}$
8	$\langle p, q\rangle_{pt_1} \vee \langle r \wedge \neg p, q\rangle_{pp_2}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$
9	$\langle r, q\rangle_{pt_2} \vee \langle p \wedge \neg r, q\rangle_{pp_1}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$

Set of Unreliable Formulas

$U_9^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

Example: Prioritized Abduction

Definition

$\langle A, B\rangle_t \in \Omega_t$, $\langle A, B\rangle_{pt_i} \in \Omega_{pt_i}$ and $\langle A, B\rangle_{pp_i} \in \Omega_{pp_i}$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$
2	$\Box_n\Diamond_e(p \wedge q)$	–;PREM	$\emptyset$
3	$\Box_n(r \supset q)$	–;PREM	$\emptyset$
4	$\Box_n\Diamond_e\Diamond_e(r \wedge q)$	–;PREM	$\emptyset$
5	q	–;PREM	$\emptyset$
6	p	1, 2, 5;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$
7	r	3, 4, 5;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle_{pt_2}\}$
8	$\langle p, q\rangle_{pt_1} \vee \langle r \wedge \neg p, q\rangle_{pp_2}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$
9	$\langle r, q\rangle_{pt_2} \vee \langle p \wedge \neg r, q\rangle_{pp_1}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$
10	$\langle r \wedge \neg p, q\rangle_{pp_2}$	8;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$
11	$\langle r, q\rangle_{pt_2}$	9;RC	$\{\Box_e q, \Box_n q, \langle p \wedge \neg r, q\rangle_{pp_1}\}$

Set of Unreliable Formulas

$U_{11}^{bk}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

$U_{11}^{pt_2}(\Gamma) = \{\langle r, q\rangle_{pt_2}\}$ $\quad$ $U_{11}^{pp_2}(\Gamma) = \{\langle r \wedge \neg p, q\rangle_{pp_2}\}$

Example: Prioritized Abduction

Definition

$\langle A, B\rangle_t \in \Omega_t$, $\langle A, B\rangle_{pt_i} \in \Omega_{pt_i}$ and $\langle A, B\rangle_{pp_i} \in \Omega_{pp_i}$

Example

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	
2	$\Box_n\Diamond_e(p \wedge q)$	–;PREM	$\emptyset$	
3	$\Box_n(r \supset q)$	–;PREM	$\emptyset$	
4	$\Box_n\Diamond_e\Diamond_e(r \wedge q)$	–;PREM	$\emptyset$	
5	q	–;PREM	$\emptyset$	
6	p	1, 2, 5;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$	
7	r	3, 4, 5;RC	$\{\Box_e q, \Box_n q, \langle r, q\rangle_{pt_2}\}$	✓
8	$\langle p, q\rangle_{pt_1} \vee \langle r \wedge \neg p, q\rangle_{pp_2}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$	
9	$\langle r, q\rangle_{pt_2} \vee \langle p \wedge \neg r, q\rangle_{pp_1}$	1,2,3,4,5;RC	$\{\Box_e q, \Box_n q\}$	
10	$\langle r \wedge \neg p, q\rangle_{pp_2}$	8;RC	$\{\Box_e q, \Box_n q, \langle p, q\rangle_{pt_1}\}$	
11	$\langle r, q\rangle_{pt_2}$	9;RC	$\{\Box_e q, \Box_n q, \langle p \wedge \neg r, q\rangle_{pp_1}\}$	

Set of Unreliable Formulas

$U^{bk}_{11}(\Gamma) = \{\Box_n(p \supset q), \Box_n(r \supset q)\}$

$U^{pt_2}_{11}(\Gamma) = \{\langle r, q\rangle_{pt_2}\}$ $U^{pp_2}_{11}(\Gamma) = \{\langle r \wedge \neg p, q\rangle_{pp_2}\}$

Further Research

Finetuning of

- the adaptive logics for prioritized abduction.

Development of

- adaptive logics for abduction based on inconsistent background knowledge, and of
- adaptive logics that combine abduction with induction.

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