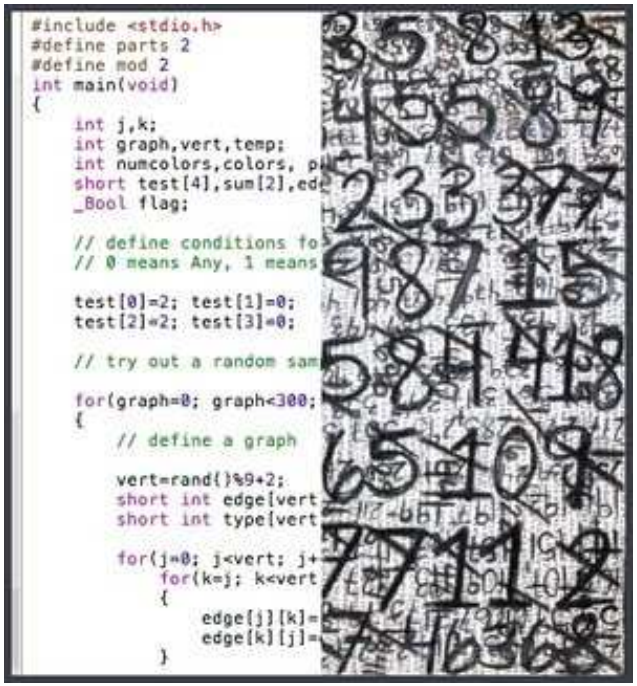
The computer (as a medium) in (the philosophy of) mathematics



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Some publicity first...

www.computing-conference.ugent.be

Intro.

Introduction

⇒ **Motivation:** "computers [are] changing the way we do mathematics" (Borwein, 2008) In the meantime, the machine is largely underestimated in philosophy (of mathematics) and math proper

⇒ **Extent impact??**

- Mathematics proper
- Philosophy of Mathematics

⇒ ... and their interactions

General Approach(es)

General Approach(es): "Traditional" approach

⇒ "Traditional" problems from philosophy of mathematics in the light of computer

- Are aspects of **mathematical knowledge** "quasi-heuristic" (Tymoczko, 1979)
- What is **mathematical understanding** in the context of computer-assisted research? (Avigad, 2008)
- ...

⇒ Computer is not *so* special: "*[N]one of the core issues are specific to the use of the computer per se*"

⇒ "**Ask not what the use of computers in mathematics can do for philosophy***; ask what philosophy can do for computers in mathematics [...]* **What we need now is not a philosophy of computers in mathematics***; what we need is simply a better philosophy of mathematics*" (Avigad, 2008)

⇒ **(Problem 1)** Neglect of technical details and history of CS

⇒ **(Problem 2)** Risk of not detecting problems that *are* inherent to the use of computer *per se* and could affect math and phil of math

General Approach(es): Another approach?

⇒ Bottom-up – and see where one gets

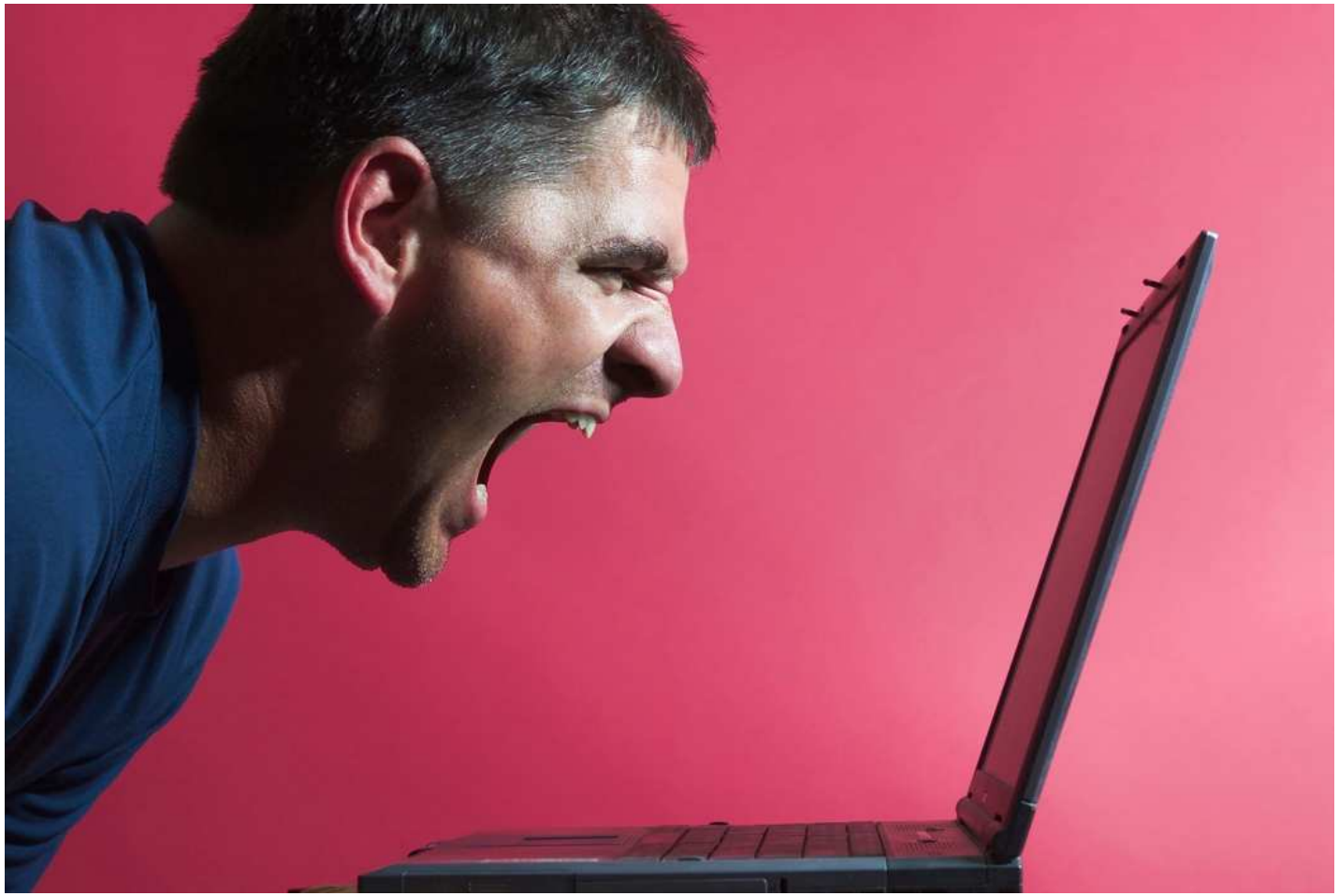
- Take computer *seriously* – as a **medium** (Kittler, 1985). The core issues become visible *through* computer *per se* and are hence shaped by it
- Philosophy of mathematical **practice(s)** that is *really* guided by that practice → Study "gory" details of (history of) computer-assisted math + no π-in-the-sky-phil-of-math

⇒ (Phil of Mat)

⇒ **Do** ask what computers in mathematics can do for philosophy of mathematics in order to see what philosophy of math can do for (computer-assisted) math.

⇒ We **do** need a philosophy of the computer (in mathematics)

Taking the computer seriously....



Taking the computer seriously – two classical "myths"

- "*Another argument that continually arises is that* **machines can do nothing we cannot do ourselves**, *though it is admitted that they can do many things faster and more accurately. The statement is true, but also false. It is like the statement that, regarded solely as a form of transportation, modern automobiles and aeroplanes are no different than walking.* **[T]hus the change by six orders of magitude in computing have produced many fundamentally new effects that are being simply ignored when the statement is made that computers can only do what we could do ourselves if we wished to take the time**" (Hamming, 1965)

- " *'computers can only do what they are told to do'. True, but that is like saying that, insofar as mathematics is deductive, once the postulates are given all the rest is trivial. [...]The truth is that in moderately complex situations, such as the postulates of geometry or a complicated program for a computer,* **it is not possible on a practical level to foresee all of the consequences**" (Hamming, 1965)

Taking the computer seriously....

⇒ Taking into account "material" and "social" changes of computer (changes in architecture, programming techniques, etc) in a study of computer-assisted math to detect global changes

⇒ Four (intrinsically related) core features of CaM *per se*:

- Time-squeezing
- Space-squeezing
- Internalization
- Mathematician-computer interactions

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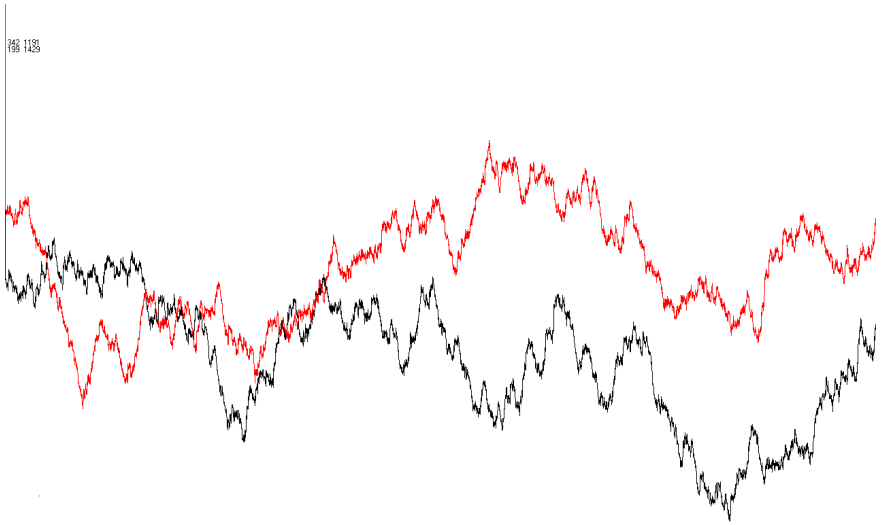
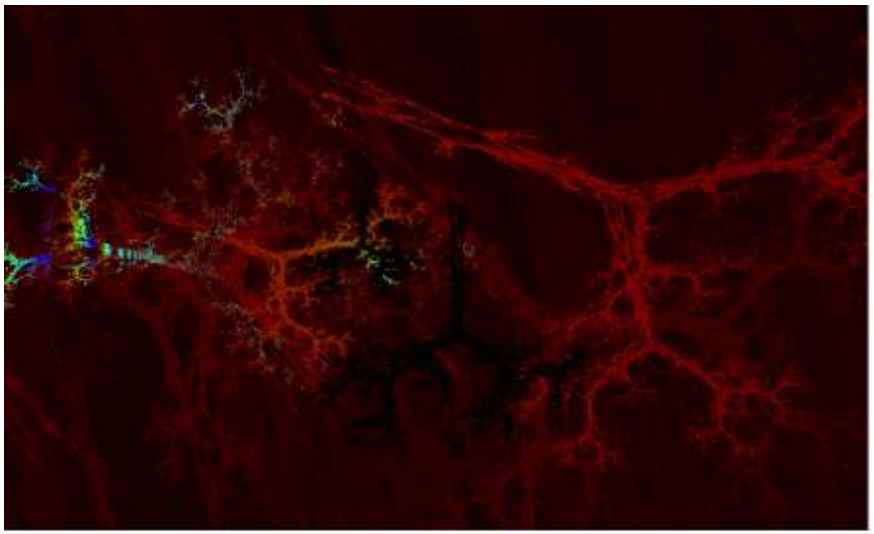
Time and space squeezing





Time and space squeezing: A "toy" example I....

....tag systems (De Mol, 2010)

Let T_{Post} be defined by $\Sigma = \{0, 1\}, v = 3, 1 \rightarrow 1101, 0 \rightarrow 00$

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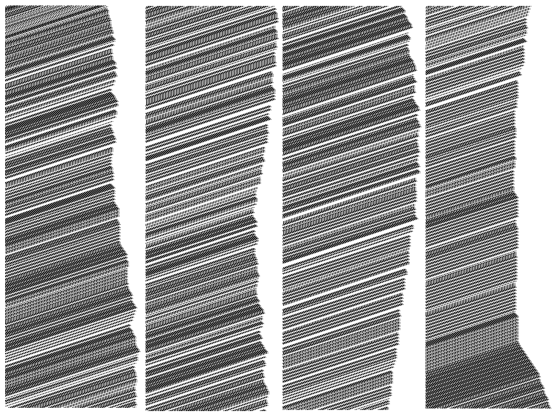

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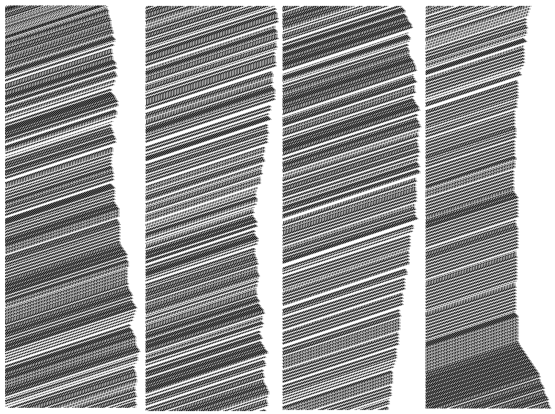


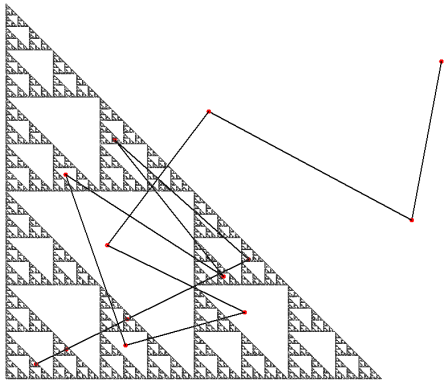
⇒ (Space) Squeeze 1s and 0s on screen to observe behavior

⇒ (Time) Squeeze computational process(es) itself into a (humanly) observable process – zooming out on the process

⇒ (Time) Confrontation with unpredictability computational process

⇒ (Time+Space) Not one or 10 but hundreds of tag systems (internal memory)

Time and space squeezing: A "toy" example II....
....fractals and the chaos game (De Mol, 2007)



Sierpinski triangle

$f_1(x_i, y_i) = (0.5x_{i-1}, 0.5y_{i-1})$

$f_2(x_i, y_i) = (0.5x_{i-1} + 0.5, 0.5y_{i-1})$

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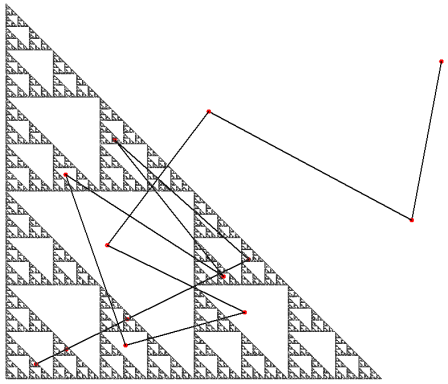

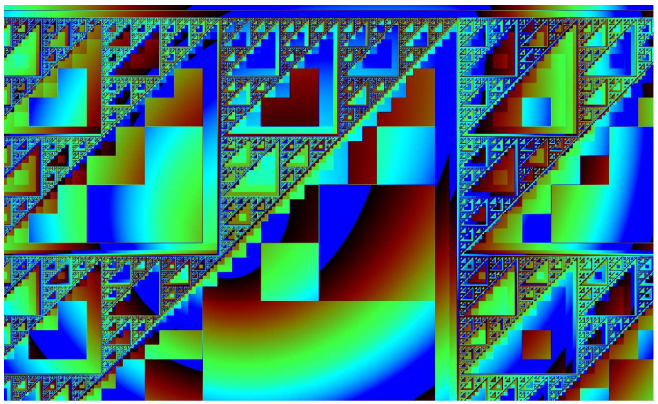

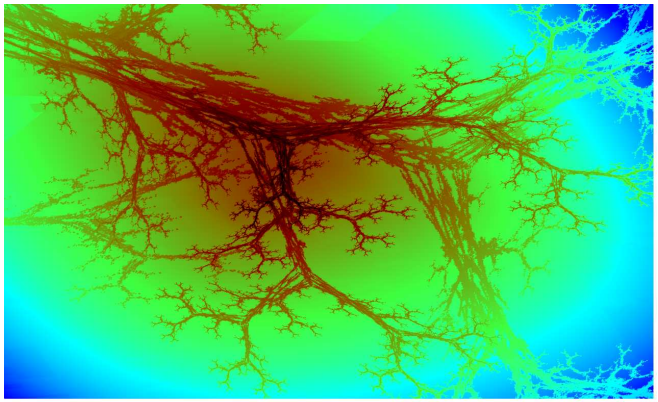

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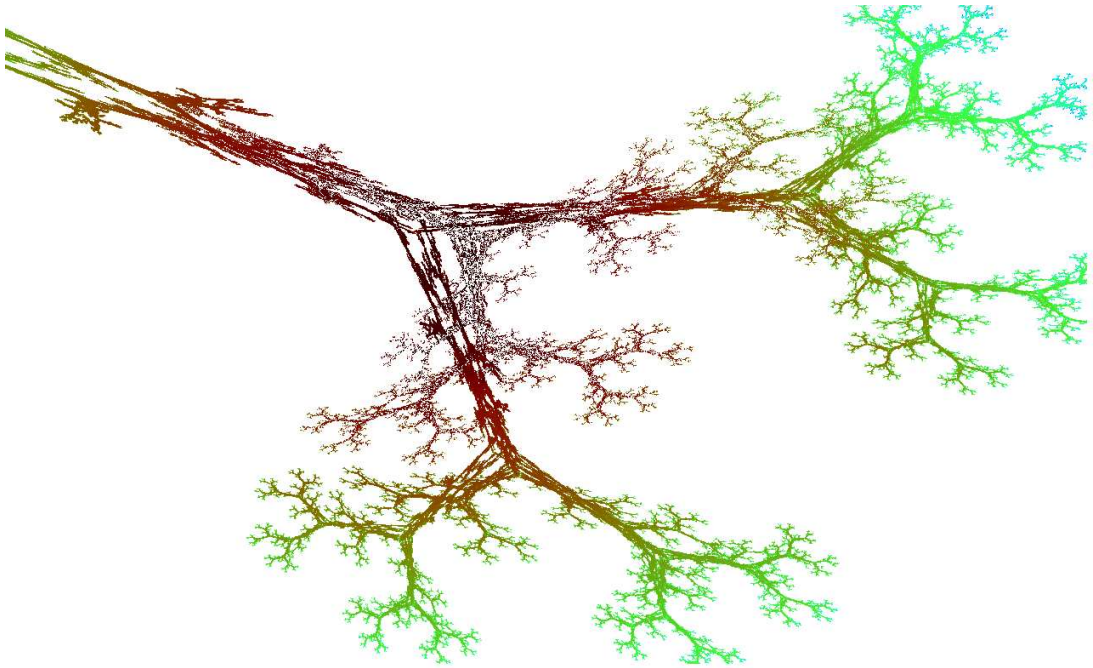

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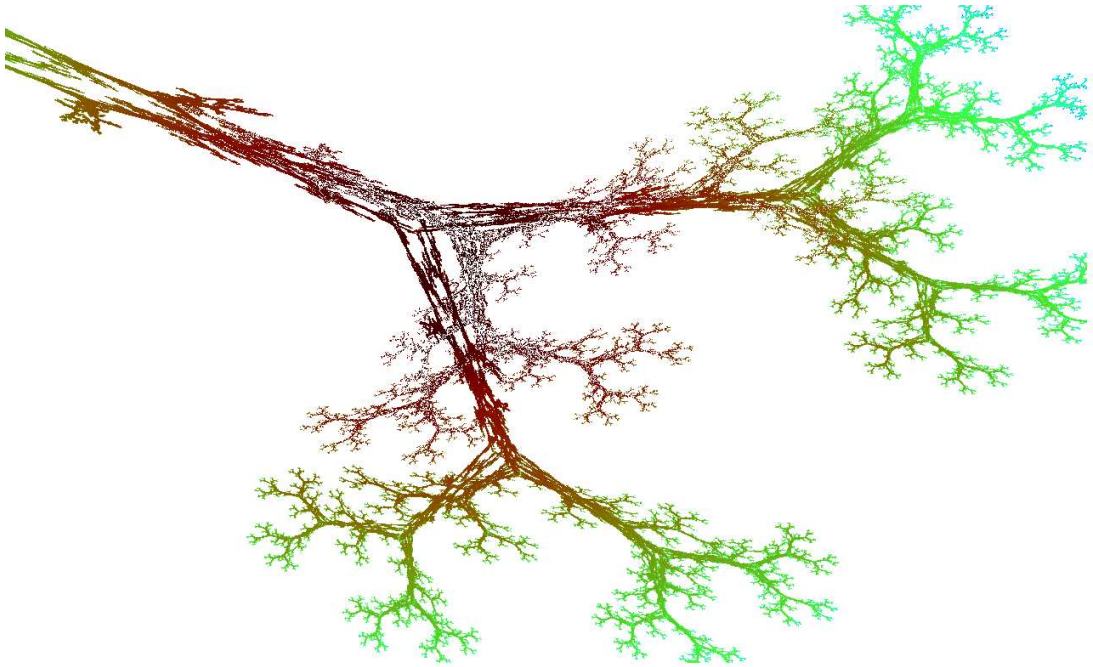

⇒ (Time) Millions of computations to generate the object

⇒ (Space) Squeezing objects and their properties in humanly observable space (zooming-out)

⇒ (Time + Space) "*When seeking new insights, I look, look, look and play with many pictures (One picture is* never enough)" (Mandelbrot, 2004)

Time and space squeezing: Changes in (phil of) math??

- 'Direct' observation of and access to (unfinished) objects and processes – zooming out and in
- 'Illusion' of 'exploring' a (computer-generated) world of mathematics within **physical constraints** of our world!

$\Rightarrow$ Tension between finite approximations of infinite objects

$\Rightarrow$ The 'proof' is in the process and/or in the visual *approximation* of an object

$\Rightarrow$ Necessity to think algorithmically, to translate "problems" into algorithms

$\Rightarrow$ Confrontation with unpredictability and processes of construction; not "stable", "eternal" objects but finite and dynamical objects ("live " math): **Reinjection of time in math?**

Discussion

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- Taking the machine serious: fundamental impact not only on math proper but also on its philosophy
- Time and space squeezing, internalization and human-computer interaction as interrelated core issues of CAM
- *Also* the philosophy of math changes according to material and social changes in math, or, should at least account for these changes (there is no such thing as a stable concept in philosophy)

$\Rightarrow$ We *do* need a good theory of computer-assisted mathematics and, ultimately, the machine itself (just as we need a better phil. of math.)

By way of a challenge.....

"*If computers are the first machines to reduce the contingency or incomputability of some, though not all futures to a finite degree, its own contingency should remain as open as possible. [...] If somebody went and wrote all the programmes hitherto running under the name of philosophy into hardware, that would be the goal itself.*" (Kittler 1987)