A model for processing updates with inconsistent information on propositional databases*

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Abstract

In the present paper a model for information update on propositional databases is formulated using the standard format of Adaptive Logics. The core structure of the update procedure is represented by the abnormal expressions of the language that formalize received information contradicting previous contents. The strategy defined to restrict abnormalities works by establishing, at each stage of the process, the most recent and reliable information, updating constantly the base and removing older data.

1 Introduction

The notion of update appeared in the literature of theory change in relation to the operation of belief revision defined by the AGM paradigm. The distinction between revision and update was introduced in [17] and it was later formalized in [15]. It refers to the following intuitive idea: whereas revision formalizes changes due to new information in a static world, update refers to the changes that a knowledge base undergoes when the world of reference changes. An obvious extension of the notion of update has been given in terms of inconsistent knowledge bases, i.e. update by addition of inconsistent information. Processing inconsistent data is a crucial operation, both for knowledge representation and for database theory. The update by inconsistent data is typically a problem for relational databases, where sets of tuples are grouped by having the same attribute: conflicting information needs to be treated in appropriate ways to perform a correct grouping of data without loss of any relevant tuple. This kind of resolution procedures are also required by the integration of single autonomous propositional databases (heterogeneous databases), where the integration of different Integrity Constraints (IC) leads to the extraction of data in order to build consistent datasets.

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In the logical literature, the standard approach to update refers to a model-theoretic interpretation which aims at satisfying Gärdenfors postulates using a notion of minimality of difference between bases: in this way, one formulates an order on the updated bases by referring to minimal changes (see e.g. [14], [16], [4], [12]). A number of logical frameworks have been proposed to resolve the case of updates with inconsistent data: many-valued systems based on Belnap's logic from [5], see e.g. [13]; annotated logics in [10] and [18]; a paraconsistent logic in [6]; the logic of formal inconsistency in [7] and [8]. The present paper introduces a novel approach to formalize the process of updates with inconsistent data using the standard framework of adaptive logics. It provides a consistent database out of the update process without *overload*, that is avoiding explosion: its resolution strategy is based on the principle of preserving the most recent information.

The logic introduced in this paper is a Prioritized Adaptive Logic in standard format called **AIU**, for *Adaptive Informational Update*. Along with the paraconsistent flavour of providing a non-explosive treatment of a contradictory database update processing, the logic **AIU** has in common with the treatment from [7] that the update process consists of producing at each stage new integrity constraints (IC): this is a definitional property for *evolutionary databases* and it is preserved in our model by considering the incoming information to be the most recently updated IC. Inconsistencies arising with respect to the result of older updates are formally represented in the language and resolved by removing information already stored in the database. Assuming that updates are more reliable than the actual stored knowledge, each content within the database becomes defeasible in view of new incoming information. This means that the design of the system cannot be fixed up-front,[1] it rather evolves through the various temporal states of the system. In this way, **AIU** implements the structure of a system based on evolutionary integrity constraints: the most recent incoming information constraints at each stage the remaining information in the database. Because each update is the declaration of new integrity constraints for the system, **AIU** does not block the incoming new inconsistent data, rather it defines a `Resolve`-selection to determine the oldest data to be removed in order to restore consistency. This corresponds to the request that no update be illegal, rather that the theory be modified to accept the change, analogously to the definition of system with incomplete information in [9]. Such a system would obviously assume complete reliability on the set of sources for the database at each new temporal stage.

On the other hand, by each update procedure the informativeness of the system is extended, inducing a partial order among its stages. The resolution of inconsistencies is obtained by retracting the minimal number of updates performed, starting from those that are in the less informative states of the system. In this way, the following principle is satisfied:

Definition 1 (Principle of Information Economy) *Keep the loss of information to a minimum.*

AIU is defined as a multi-modal language with temporally indexed update operators, valuated as possibility operators in a standard semantics for the logic

[1]In software engineering, by "fixed up-front design" one understands the full description and complete explanation of the requirements that need to be implemented in the creation of a system before its actual construction or execution.

T. **AIU** is an adaptive logic in standard format, which means that it is defined by three elements:

1. a Lower Limit Logic (**LLL**): a monotonic, compact logic;
2. a set Ω of abnormal formulas characterized by a logical form;
3. an Adaptive Strategy (**AS**).

The lower limit logic is called **IU** and it is the stable part of the adaptive logic. Abnormalities are supposed to be false, "unless and until proven otherwise". Each abnormality expresses a non-monotonic information update, which makes a premise set inconsistent in view of the combination of an update with previously held contents. The information with the highest index (later information) has the highest priority; lower indices are ranked in decreasing order. Hence, in **IU** sets of such abnormalities are indexed on the basis of the time index and the prioritized adaptive logic **AIU** is obtained by the superposition of the various abnormal logics defined by the derivability of differently indexed abnormal formulas. The final knowledge base for **AIU** is formulated according to the strategy, which establishes admissible and avoidable updates.

The approach is also novel in that the strategy is formulated both in terms of a semantics and of a proof theory. Model-theoretically, one proceeds from the models of a premise set verified by the newer updates to the models of the older ones. At each step, older updates are considered and one stops at the first stage at which allowing a non-monotonic update makes the consequence set inconsistent. Proof-theoretically, this is obtained by determining the persistance of each update on the basis of later ones: the reliability of the update on A is depending on the falsity of any update with $\sim A$ obtained at some later stage; failing this condition, previous updates are rejected.

The paper is structured as follows. In section 2 the needed formal preliminaries are introduced, among them the definition of *updated database* and its *setup*. In section 3 the Lower Limit Logic is presented, followed in section 4 by the formalization of possible non-monotonic updates. In sections 5 and 6 the resolution strategy for updated databases with inconsistent information is presented respectively in the semantic and proof-theoretical formats, along with some examples. The conclusive section draws some connections with other research in the field of adaptive logics.

2 Preliminaries

Let $\mathcal{L}$ be the standard language of classical propositional logic (**CL**), formed from a finite set of atoms $\mathcal{P}$. The symbol $\mathcal{P}^{\pm}$ will stand for the set of literals, atoms and negations of atoms. The symbol $\mathcal{W}$ stands for the set of well-formed formulas of $\mathcal{L}$. Latin capital letters $A, B, \ldots$ are metavariables for members of $\mathcal{W}$. A *database* is a finite subset of $\mathcal{W}$. The letter Γ is used as metavariable for a database. A **CL**-model is a function from $\mathcal{P}$ to $\{0,1\}$; letters $M, M', \ldots$ are metavariables for **CL**-models, and $\mathcal{M}$ denotes the set of all **CL**-models. A model $M(\Gamma)$ is a model of a database Γ if and only if all the members of Γ are true in it; $M \vDash A$ denotes that M verifies A. $Mod(\Gamma)$ is used to denote the set of all models of a database Γ; $Cn(\Gamma)$ denotes the semantic consequence set of Γ. $M(\Gamma) \vDash A$ corresponds to the selection of valuations for A in all $M \in Mod(\Gamma)$ of the form $v_M(A) = 1$. The focus on the valuation functions is useful for mimicking a user performing a query about A in Γ, and it is necessary for the forthcoming description of the updating of a database; the general case $v_M(A) = 1$ or $v_M(A) = 0$ is used to formalize the response to a query operation concerning A which produces no result, meaning that there is no specification concerning A according to the given database.[2]

The basis of the logic for Adaptive Informational Update is defined by extending $\mathcal{L}$ to a multi-modal language $\mathcal{L}^I$ including *first-degree update operators*. We call this logic **IU** and the set of wffs of $\mathcal{L}^I$ shall be denoted by $\mathcal{W}^I$. In the set of modal operators $\mathcal{I} = \{I_1, \dots, I_n\}$, each operator is indexed from a set $\mathcal{T} = \{1, 2 \dots n\}$ of temporal indices. The letters $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ are now used as metavariables for temporally ordered databases: by the attachment of temporal indices to formulas in $\mathcal{L}^I$, a database is temporally characterized. We shall now call an *updated database* a database Ψ such that in it every formula is of the form I_iA and there are in Ψ at least two subsets Γ_i, Γ_j, each called a *stage* of Ψ: $\Psi = \{\Gamma_1, \dots, \Gamma_n \mid \Gamma_{i-1} \subseteq \Gamma_i\}$ and $1 \leq i \leq n \in \mathcal{T}$. Formulas of the form I_iA represent updates of a database at time $i-1$; each database Γ_i will contain all formulas with temporal operators up to $i \in \mathcal{T} \mid 1 \leq i \leq n$. A formula $A \in \mathcal{W}^I$ contained in Ψ is said to be "at stage i of Ψ" iff it is introduced in Ψ by an update indexed by $i \in \mathcal{T}$. Further notation from the language $\mathcal{L}$ is adapted to $\mathcal{L}^I$ simply by attaching temporal indices were appropriate. The *setup* of an updated database Ψ is given by describing the operations that lead to each consecutive stage: these operations have an appropriate correspondence to a selection of models at the various stages i, j of the database; hence, $\mathcal{V}(\Gamma_i)$ denotes the setup of the database up to stage i, $\mathcal{V}(\Gamma_j)$ up to stage j and so on; the final result for Ψ corresponds to the set of all models of that database, $Mod(\Psi)$. A model $M_{IU}(\Gamma_i)$ is a model of a database Γ if and only if all the members of Γ are true in it at time i and $M(\Gamma_i) \vDash_{IU} A$ denotes that there is a model M of database Γ at stage i that verifies A according to the logic **IU**. $M_{IU}(\Psi)$ denotes a model in which all the members of an updated database Ψ are true and $Mod(\Psi)$ is intended for the set of all **IU**-models of an updated database Ψ; the set of all **IU**-models is in turn denoted by $\mathcal{M}_{IU}$. The set of models of a database according to the logic **IU** will correspond to the appropriate classical models, unless in the setup of that database contradictory information has been provided; in that case, the dynamics typical of the adaptive logic is used to gain the appropriate consistent subset of data.

The setup of an updated database is obtained via the update operation and analysed in view of the differences of the various setups at stages. The intuitive meaning of a formula of the form I_iA contained in an updated database Ψ is the following: for some updated database $\Psi = \{\Gamma_1, \dots, \Gamma_n\}$, the stage Γ_i of the database contains all propositions held true up to time $1 \leq i \leq n$; at time i the stage Γ_{i-1} is updated so that contents holding at that stage are preserved and moreover a new function $v_{M_{IU}}(I_iA) = 1$ holds, which correspondingly performs a selection on models. At time $i-1$ either there was no specification concerning the propositional content A, or $I_{i-1}{\sim}A$ was valid: in the former case, a normal

[2] An automated information retrieval process is started by entering a query into the system; queries are formal statements of information needs; in the propositional system at stake here, a query is a formal statement for checking the occurrence of a literal in a database; see also [13].

restriction on the valid models is performed, in the latter case the update will be inconsistent (extending again the set of valid models) and one will need to perform the appropriate consistency-restoring operations provided by the adaptive machinery. The semantic definition of the I_i operator is given as a possibility modality, in order to allow that the content A be true at the state valid at time $i-1$, and the content $\sim A$ be true at the state valid at time i (or viceversa).[3] The language $\mathcal{L}^I$ defining the logic **IU** is a multi-modal version of the modal logic **T**. **IU** shall be completely defined in the next section, whereas in section 4 the second element of the standard format of adaptive logics is introduced,[4] namely the form of abnormalities that correspond to non-monotonic updates.

3 The Lower Limit Logic IU

The first element of the standard format of an adaptive logic is its Lower Limit Logic (**LLL**). For the *Adaptive Logic of Information Update* the corresponding **LLL** is called **IU**, whose language has already been introduced in the preliminaries. A **IU**-model is obtained in a standard possible-worlds semantics as a quadruple $\langle W, w_0, \mathcal{R}, v\rangle$, defined as follows: W is a set of possible worlds, in which formulas from the language $\mathcal{L}^I$ are valuated; w_0 is the actual world; $\mathcal{R}$ is a set of temporal accessibility relations from the actual world to the set of possible worlds: $R_i : w_0 \to W(i \in \mathcal{T})$; v is the function valuating formulas in the worlds: $v : \mathcal{P} \times W \to \{1, 0\}$. The valuation of a model M_{IU} is characterized in the following way:

C1 where $A \in \mathcal{P}$, $v_{M_{IU}}(A, w) = v(A, w)$
C2 $v_{M_{IU}}(\sim A, w) = 1$ iff $v_{M_{IU}}(A, w) = 0$
C3 $v_{M_{IU}}(A \vee B, w) = 1$ iff $v_{M_{IU}}(A, w) = 1$ or $v_{M_{IU}}(B, w) = 1$
C4 $v_{M_{IU}}(A \wedge B, w) = 1$ iff $v_{M_{IU}}(A, w) = 1$ and $v_{M_{IU}}(B, w) = 1$
C5 $v_{M_{IU}}(A \supset B, w) = 1$ iff $v_{M_{IU}}(A, w) = 0$ or $v_{M_{IU}}(B, w) = 1$
C6 $v_{M_{IU}}(I_i A, w) = 1$ iff $v_{M_{IU}}(A, w') = 1$ for some w' such that $R_i ww'$

A **IU**-model M_{IU} verifies A iff $v_{M_{IU}}(A, w_0) = 1$; A is valid in **IU** ($\models_{\mathbf{IU}} A$) if it is verified by all its models; and A is a consequence of a premise set Ψ in **IU** ($\Psi \models_{\mathbf{IU}} A$) if A is true in every model of Ψ.

In addition to all **CL**-axioms, the logic **IU** validates for any $i \in \mathcal{I}$

- Necessitation Rule: if $\vdash_{\mathbf{CL}} A$ then $\vdash_{\mathbf{IU}} I_i A$;
- Distribution: $I_i(A \supset B) \supset (I_i A \supset I_i B)$;
- Consistency: $I_i A \supset \sim I_i \sim A$;
- Reflexivity: if $\vdash_{\mathbf{IU}} I_i A$ then $\vdash_{\mathbf{CL}} A$.

[3]The interpretation of the update operator by means of a possibility operator is crucial both from the conceptual and the formal point of view. Formally, it allows to maintain a defeasible notion of information, which can be rejected at later stage if new contradictory updates are obtained. Conceptually, it describes a notion of information for a dynamic process of becoming informed. For more on the debate on the nature of information contents and related epistemic states, see [11], [19].

[4]For the standard format for Adaptive Logics see [1] and [2]. For a formal and philosophical justification of the adaptive logic programme, see [3].

By Necessitation, for any valid sentence there is an update at some time by that sentence; and by Reflexivity, any update by a valid sentence provides a content holding at each information state of the database. Hence, our logic is defined by the set of all reflexive frames. But in our logic the axiom for transitivity $I_iA \supset I_jA$, for all $i \leq j \in \mathcal{T}$, does not hold, that is the information satisfied at one state is not necessarily still satisfied at all later states.

The analogy with an updated database as introduced in the preliminaries can be explained as follows. The set W contains all the informational stages that the updated database can be extended to at various times from the database actual stage, indicated by w_0. Such actual stage w_0 is built up by all the different informational stages already reached at previous times. The set $\mathcal{R}$ contains all the update relations, that is all the possible accessibility relations to newer informational stages of the system. Hence, there is a correspondence between the informational states of an updated database Ψ and its temporal stages Γ_i, Γ_j ($i < j \in \mathcal{T}$), because the updates provide new stages of the system at different times. For the notion of an updated database to be general enough, one needs to introduce the case of an *empty database* and *empty updates*: a formula I_1A is an update performed on an empty stage, i.e. $\Gamma = \{\emptyset\}, \Gamma_1 = \{I_1A\}$; moreover, a new informational state of an updated database Ψ can also collect all and only the contents contained at its previous stage, that is an empty update consists in confirming previous contents at a later stage. The general formulation of an updated database $\Psi = \{\Gamma_1, \ldots, \Gamma_n\}$ can now be given with respect to the set of valuation functions that build its setup $\mathcal{V}(\Psi)$:

Definition 2 (The setup of updated databases) *For any updated database $\Psi = \{\Gamma_i, \Gamma_j\}$ and all models $M_{IU}(\Gamma_i)$ and $M'_{IU}(\Gamma_j)$:*

1. *either for any $< v, v' >$ and any $A \in \mathcal{P}^{\pm}$, $v_{M_{IU}(\Gamma_i)}(A) = v'_{M'_{IU}(\Gamma_j)}(A)$;*
2. *or for all $< v, v' >$ and $A \in \mathcal{P}^{\pm}$, $v_{M_{IU}(\Gamma_i)}(I_iA) = 1$ or $v_{M_{IU}(\Gamma_i)}(I_i{\sim}A) = 1$ and either $v'_{M'_{IU}(\Gamma_j)}(I_jA) = 1$ and $v'_{M'_{IU}(\Gamma_j)}(I_j{\sim}A) = 0$; or $v'_{M'_{IU}(\Gamma_j)}(I_jA) = 0$ and $v'_{M'_{IU}(\Gamma_j)}(I_j{\sim}A) = 1$;*
3. *or, for some $< v, v' >$ and $A \in \mathcal{P}^{\pm}$, either $v_{M_{IU}(\Gamma_i)}(I_iA) = 1$ and $v'_{M'_{IU}(\Gamma_j)}(I_j{\sim}A) = 1$ or $v_{M_{IU}(\Gamma_i)}(I_i{\sim}A) = 1$ and $v'_{M'_{IU}(\Gamma_j)}(I_jA) = 1$.*

By the first clause, updates can be empty, that is at a given temporal stage no new content is provided in view of a previous stage and the set of models valid according to Γ_j does not change in view of Γ_i; by the second clause, an update to stage Γ_j brings new consistent information with respect to the previous stage where both possibilities where still accounted for, that is a further restriction on the valid models is performed; by the third clause, two valuations conflict giving contradictory information updates, so that at Γ_j an update with information A is provided, where ${\sim}A$ was the information given at a previous stage Γ_i, or viceversa. In this latter case, the setup enlarges again the set of valid models, in a way that leads classically to triviality. The third clause takes care of what we shall call *conflicting valutations*:

Definition 3 (Conflicting Valuations) *Given valuations $v, v' \in \mathcal{V}(\Psi)$, the valuation v' is conflicting w.r.t. v if and only if for some $A \in \mathcal{P}^{\pm}$ it holds that $v_{M_{IU}}(I_iA) = 1$ and $v'_{M'_{IU}}(I_j{\sim}A) = 1$, and $i \neq j \in \mathcal{T}$.*

Let now $\mathbf{V}^{\neq}(\Psi)$ be the subset of $\mathcal{V}(\Psi)$ that contains all and only the conflicting valuations contained in an updated database Ψ. Let us call $\mathbf{V}^{\neq}(\Psi)$ *the conflicting setup of* Ψ. Then the following strict order is defined on $\mathbf{V}^{\neq}(\Psi)$:

Definition 4 (Minimal Conflicting Valuation) *A valuation v for some $A \in \mathcal{P}^{\pm}$ is said minimal in $\mathbf{V}^{\neq}(\Psi)$ if and only if for every valuation v' such that $< v, v' > \in \mathbf{V}^{\neq}(\Psi)$, v is the valuation function at stage Γ_i of Ψ, and v' is a valuation function at some stage Γ_j of Ψ such that $i < j \in \mathcal{T}$.*

This means that the minimal conflicting valuation in $\mathbf{V}^{\neq}(\Psi)$ corresponds to the verification function of the oldest non-monotonic update in Ψ. It is possible now to provide an appropriate definition for the informativeness of stages Γ_i, Γ_j of an updated database Ψ, based on the definition of setup:

Definition 5 (Informativeness of States) *For any updated database $\Psi = \{\Gamma_i, \Gamma_j\}$ and corresponding setups $\mathcal{V}(\Gamma_i)$ and $\mathcal{V}(\Gamma_j)$, a partial order $\Gamma_i \sqsubseteq \Gamma_j$ holds iff $\mathcal{V}(\Gamma_j)$ is obtained by $\mathcal{V}(\Gamma_i)$ by one of the appropriate operations as by Definition 2. If a partial order $\Gamma_i \sqsubseteq \Gamma_j$ holds, then Γ_j is a more informative stage of Ψ with respect to Γ_i.*

The more informativeness of increasing stages is explained as follows: by an empty update one allows new models verifying a literal already valid in models with lower indices (formally, this amounts to the literal becoming more persisting than before; informally, the information is more recent – in both cases this is a relevant description in a context of defeasible information); by a consistent udpdate, one makes valid a certain set of models (those for example validating I_jB) and thus eliminates the models that validate the contradictory formula (respectively, $I_j{\sim}B$), operation that produces hence new information; finally, by a non-montonic update a certain set of models is restored as valid (for example, those validating again $I_j{\sim}B$) which makes again needed a certain (adaptive) selection that will lead to invalidating models where $I_iB \mid i < j$ hold.

As previously mentioned, transitivity is invalidated in the logic **IU**, which allows in section 4 for the formal description of inconsistencies, restricted to the case of conflicting valuations between older and newer information; this also means that at each state only consistent information is allowed. The adaptive selection of such inconsistencies is presented in the next two sections. For the semantic formulation of the adaptive selection, the costruction of abnormal formulas in **IU** on the basis of the notion of conflicting valuations and their minimality property is introduced. For the syntactic version, a restriction on the derivability of abnormal formulas is formulated. In sections 5 and 6, the adaptive strategy nicknamed `Resolve` is defined respectively semantically and proof-theoretically. For the first format, the correspondence between the temporal structure and the informativeness of the system is used to explain a selection procedure on the models of a given updated database Ψ, eliminating the models that satisfy a minimal conflicting valuation; consistency is restored with the minimal loss of informativeness in the system. In the proof-theory, special rules and few crucial principles are defined, and the adaptive notions of derivability at stage and final derivability provide corresponding results for updates valid at a given stage and updates finally valid according to **AIU**. As shown in [2], a dynamic proof-theory for any adaptive logic in standard format is sound and complete with respect to the static semantics. The strategy nick-named `Resolve` is nothing else than an application of the standard *Minimal Abnormality* strategy for adaptive logics.

4 Possible non-monotonic updates

Admitting complete monotonicity over updates means to stipulate that any such operation is consistent with previously obtained contents: for every $v_{M_{IU}}(A, w_0) = 1$, there is no $R_i w_0 w'$ according to which $v'_{M'_{IU}}(\sim A, w') = 1$. In such a system no information update ever contradicts a given content. The logic **AIU** is obtained by formulating a procedure to stabilize the **IU**-consequence set of some premise set in which this principle does not hold. The localization of the contents for which it is the case that $v_{M_{IU}}(A, w_0) = 1$ and there is a $R_i w_0 w'$ according to which $v'_{M'_{IU}}(\sim A, w') = 1$, is given in **IU** by the formulation of the set of *abnormalities*. The restriction of their validity is the aim of the *adaptive strategy* in the following sections.

In **IU**, abnormalities are all formulas whose logical form expresses a non-monotonic informational update:

Definition 6 (Set of Abnormalities) $\Omega_i = \{I_i A \wedge \sim A \mid A \in \mathcal{P}^{\pm}\}$.

By this definition, a formula in Ω_i expresses the occurrence at temporal stage i of a non-monotonic update for some Ψ. This means that for some $\Psi = \{\Gamma_i, \Gamma_j\}$, in their setups $\mathcal{V}(\Gamma_i), \mathcal{V}(\Gamma_j)$ there are conflicting valuations v and v'.

The set of *all* abnormalities for a given Ψ will be denoted simply by Ω and it is obtained as the union of all indexed sets of abnormalities: $\Omega_1 \cup \Omega_2, \ldots, \cup\, \Omega_k$, where k is the highest $i \in \mathcal{T}$ for Ψ.

When a database is updated at consecutive stages with updates $I_i A$ and $I_j {\sim} A$, the resulting inconsistent state can be described by the formulation of the conflicting valuations in terms of different ordered pairs:

$$< v_{M_{IU}}(A, w_0) = 1, v'_{M'_{IU}}(\sim A, w') = 1 >;$$
$$< v'_{M'_{IU}}(\sim A, w') = 1, v_{M_{IU}}(A, w_0) = 1 > .$$

Intuitively, this means that one can consider at stage j of the "history" of the database, the update with $\sim A$ holding at state w' that is conflicting with A holding at state w; or, one can consider the update with A (happened at time i) holding at state w that is now conflicting with the new information $\sim A$ holding at state w'. This has a nice correspondence in the behaviour of the logic **IU**: from a premise set including formulas $I_i A$ and $I_j {\sim} A$, two distinct abnormal formulas can be formulated, namely $I_i A \wedge \sim A$ and $I_j {\sim} A \wedge A$. Let us call these the *possible non-monotonic updates*. Consider for example a database $\Psi = \{I_1(p \vee q), I_2 {\sim} p, I_3 {\sim} q\}$: it stands for an empty database updated respectively at time 1 with the information $p \vee q$, at time 2 with $\sim p$, and at time 3 with $\sim q$. By this latter step the information conveyed by Ψ is inconsistent. According to Definition 6, the following indexed sets of abnormalities can be formulated for Ψ:

$$\Omega_1 = \{I_1 p \wedge \sim p; I_1 q \wedge \sim q\};$$
$$\Omega_2 = \{I_2 {\sim} p \wedge p\};$$
$$\Omega_3 = \{I_3 {\sim} q \wedge q\}.$$

The consequence set $Cn_{IU}(\Psi)$ will not contain (at any stage) any of these possible non-monotonic updates, because none is **IU**-derivable from Ψ. But from this updated database the *possibility* of valid non-monotonic updates is derivable according to the monotonic logic **IU**. In other words, *disjunctions* of such abnormalities are derivable, whereas the disjuncts are not. This is the case of the formula $(I_1 p \wedge \sim p) \vee (I_1 q \wedge \sim q)$, derivable according to **IU** from the mentioned Ψ.

In the standard format of an adaptive logic one defines $Dab(\Delta)$ to stand for the disjunction of members of Δ, where Δ is a finite subset of Ω:

Definition 7 (*Dab*-Formula) *$Dab(\Delta)$ stands for $\bigvee(\Delta)$ where $\Delta \subseteq \Omega$.*

If Δ is a singleton, $Dab(\Delta)$ is simply an abnormality $(A \vee Dab(\emptyset))$, i.e. a member of Ω; if Δ is empty, $Dab(\Delta)$ is empty as well.

It is in view of the validity of *Dab*-formulas for a given premise set that the *adaptive strategy* is needed. The adaptive strategy specifies what it means, in the case of disjunctions of abnormalities, that the abnormalities are false *unless and until proven otherwise.* Given the same lower limit logic and the same set of abnormalities, there are different ways to interpret a set of premises *as normally as possible.* The precise meaning is given by the formal presentation of the strategy in the following sections.

5 The semantic approach to `Resolve`

The semantics of **AIU** consists in a selection on each group of **IU**-models of a premise set. We shall nickname this selection `Resolve`, which is nothing else than the sandard *Reliability Strategy* for adaptive logics: this strategy selects those **IU**-models of a premise set Ψ that are not more abnormal than what required by the premises. The standard of abnormality is in this case determined by the temporal index attached to non-monotonic updates: in general, formulas in Ω_i are less abnormal than formulas in Ω_{i-1}, which means that later non-monotonic updates are preferred to older ones. The strategy selects only those models verifying the abnormalities which cannot be avoided by the premise set (provided that all premises are true).

It was explained in the previous section how a non-monotonic update can be described by two different ordered pairs, each composed by the related conflicting valuations. Provided the mentioned time-based standard of abnormality, for any update on a given premise set there will be an *admissible* and an *avoidable* abnormality: the admissible abnormality corresponds to the ordered pair whose first element is the valuation function for the literal provided by the newest update (among those referred to by the involved conflicting valuations); the avoidable abnormality corresponds to the ordered pair whose first element is the valuation function for the literal provided by the oldest update. Intuitively, provided abnormalities have decreasing relevance determined by their index, the selection prefers abnormalities with higher index.

The *Dab*-formulas valid by **IU** from a premise set Ψ will be called the *Dab*-consequences of Ψ:

Definition 8 (*Dab*-Consequence) *$Dab(\Delta)$ is a Dab-consequence of a set Ψ iff $\Psi \vDash_{\mathbf{IU}} Dab(\Delta)$.*

If $Dab(\Delta)$ is a Dab-consequence of a set Ψ, then so is any $Dab(\Delta')$ such that $\Delta' \supset \Delta$. This is the case of the example from the previous section: where $\Psi = \{I_1(p \vee q), I_2{\sim}p, I_3{\sim}q\}$, one can derive longer disjunctions of abnormalities, namely $(I_1 p \wedge {\sim}p) \vee (I_1 q \wedge {\sim}q) \vee (I_2{\sim}p \wedge p)$ and $(I_1 p \wedge {\sim}p) \vee (I_1 q \wedge {\sim}q) \vee (I_3{\sim}q \wedge q)$, where both include the shorter $(I_1 p \wedge {\sim}p) \vee (I_1 q \wedge {\sim}q)$ which is also derivable. This is why a further definition is needed:

Definition 9 (Minimal Dab-Consequence) *A disjunction of abnormalities $Dab(\Delta)$ is a* minimal *Dab-consequence of a premise set Ψ iff $\Psi \vDash_{\mathbf{IU}} Dab(\Delta)$ and there is no $\Delta' \subset \Delta$ such that $\Psi \vDash_{\mathbf{IU}} Dab(\Delta')$.*

For any index $i \in \mathcal{T}$ contained in an updated database Ψ, a set of non-monotonic updates will correspond. The various sets of abnormalities from Ω_1 to Ω_k – for the highest $k \in \mathcal{T}$ occurring in Ψ – form the structure of the prioritized consequence set of Ψ according to **IU**. Each of these consequence sets is determined on the basis of the same **LLL** and they differ precisely with respect to the set of abnormalities (in terms of the index). The consequence set of the combined adaptive logic **AIU** is then obtained by (where k is the highest index for which I_k occurs in the premise set):

$$Cn_{AIU}(\Psi) = Cn_{IU_1}(Cn_{IU_2}(\ldots Cn_{IU_k}))$$

where each inclusion within the consequence set with lower index is determined according to a selection procedure. The set $Mod(\Psi)$ contains then all **IU**-models, each verifying some Dab-formula; for any such model, the abnormal part of degree i of model M is the set of abnormalities of degree i satisfied in M:

Definition 10 (Abnormal model) *Provided M is a* **IU***-model, $Ab^i(M) = \{A \in \Omega_i \mid M \models A\}$.*

The selection procedure by the Reliability Strategy, requires that the consequence set of a premise set Ψ be formulated assuming that all the abnormalities in a minimal Dab-formula are unreliable formulas of that premise set. The set of unreliable formulas is then determined at each degree by the corresponding indexed set of abnormalities:

Definition 11 (Set of unreliable formulas) *Where $Dab(\Delta_1), \ldots, Dab(\Delta_n)$ are all the minimal Dab-consequences of Ψ, $U^i(\Psi) = \Delta_1 \cup \ldots \cup \Delta_n$ is the set of unreliable formulas of degree i, where $i \in \mathcal{T}$ occurs in Ψ.*

A **IU**-model is said reliable at a given degree if ad only if its abnormal part is set-theoretically in the set of unreliable formulas for Ψ at that degree:

Definition 12 (Reliable Model of Ψ at degree) *A* **IU***-model M is a reliable model of a premise set Ψ at degree i iff $Ab^i(M) \subseteq U^i(\Psi)$.*

Consider now the various consequence sets of Ψ at the different degrees up to the highest one k ($\mathbf{Cn_{IU_1}} - \mathbf{Cn_{IU_k}}$). The set of all **IU**-models of Ψ at degree k will contain all abnormalities Δ at the various degrees up to k: $\Delta \subseteq \Omega_1, \ldots, \Delta \subseteq \Omega_k$. A reliable model of Ψ at degree k will verify no abnormalities which are not within $U^k(\Gamma)$, so that at this stage all the abnormal models which are set-theoretical part of the set of unreliable formulas at the highest degree k are considered, and all the other excluded. The selection proceeds on the consequence set of Ψ at the next step in view of all the abnormal models that are set-theoretical part of the set of unreliable formulas at the highest degree k and at degree $k-1$; and so it goes on up to include the abnormalities of the lower degree:

1. $sel_0(\mathcal{M}_{IU})$ iff $Ab^k(M) \subseteq U^k(\Psi)$, and there is no index in $\mathcal{T}$ greater than k occurring in Ψ;
2. $sel_{i+1}(\mathcal{M}_{IU})$ iff $sel_i(\mathcal{M}_{IU})$ and $Ab^{k-(n+1)}(M) \subseteq U^{k-(n+1)}(\Psi)$.

In this way, at any next step of the selection procedure the abnormal models of the next lower degree are considered and the selection stops when the first avoidable abnormality is reached.

Let us consider again our simple example $\Psi = \{I_1(p \vee q), I_2{\sim}p, I_3{\sim}q\}$. The selection goes as follows:

$$sel_0(\mathcal{M}_{IU}) = Ab^3(M);$$
$$sel_1(\mathcal{M}_{IU}) = Ab^3(M) \cup Ab^2(M);$$

by these two steps all **IU**-models in $Mod(\Psi)$ including those that verify respectively abnormality $(I_3{\sim}q \wedge q)$ by the first selection step, and abnormality $(I_2{\sim}p \wedge p)$ by the second step, are included. A further selection step would include the abnormalities of degree 1, turning the consequence set into inconsistency. Having lowest index, abnormalities of degree 1 are considered the avoidable ones.

A model M is then a reliable model of Ψ if M is in the intersection of models provided by all the selection steps: $M_{IU_1} \cap M_{IU_2} \cap \ldots \cap M_{IU_k}$. By this, one obtains the definition of consequence for **AIU**:

Definition 13 (AIU-Consequence) $\Psi \vDash_{AIU} A$ *iff* A *is verified by all reliable models of* Ψ.

Among the reliable models of $\Psi = \{I_1(p \vee q), I_2{\sim}p, I_3{\sim}q\}$ we count the models selected by sel_0, sel_1 from the previous example; it follows that $\Psi \vDash_{AIU} {\sim}p, {\sim}q$.

It is now possible to carachterize the result of the adaptive selection procedure in terms of the ordering on the members of $\mathcal{V}(\Psi)$ and the ordering on the informativeness of stages of Ψ.

Theorem 1 $\Psi \vDash_{AIU} A$ *iff* $v_{M_{IU}}(I_iA)$ *is either not conflicting or, if so, then is not* minimal *in* $\mathbf{V}^{\neq}(\Psi)$.

Proof. By Definition of the **AIU** consequence set and that of the set $\mathbf{V}^{\neq}(\Psi)$, the only interesting case is when there are conflicting valuations such that both A and ${\sim}A$ are updates at different stages of Ψ.

1. By Definition 13, $\Psi \vDash_{AIU} A$ iff there is a model M_{IU} s.t. $M \vDash_{IU} A$ and M is reliable for Ψ, i.e. M_{IU} is reliable at any degree. Hence the valuation $v_{M_{IU}}(I_iA) = 1$ holds for some reliable model of Ψ, whereas by assumption there is a conflicting valuation such that an update with ${\sim}A$ holds and the model M'_{IU} that validates it cannot be reliable. Then by

the selection steps, the reliable model M_{IU} is chosen and by Definitions 10 and 12, the model M_{IU} will satisfy abnormalities of higher degree then M'_{IU}. Hence, the update with $\sim A$ will be the minimal among the two. On the other hand, by the explanation of abnormalities as ordered pairs of conflicting valuations, the abnormality with lowest index among those making the consequence set inconsistent corresponds to the *minimal valuation* in $\mathbf{V}^{\neq}(\Psi)$. Hence the valuation $v_{M_{IU}}(I_iA) = 1$ cannot be at the same time the one selected by **AIU** and also the minimal in $\mathbf{V}^{\neq}(\Psi)$.

2. Consider a valuation $v_{M_{IU}}(I_iA) = 1$ minimal in $\mathbf{V}^{\neq}(\Psi)$, then there is a conflicting valuation $v'_{M'_{IU}}(I_j\sim A) = 1$ such that $i < j \in \mathcal{T}$. Then two different ordered pairs of valuations are defined, $< v, v' >$ and $< v', v >$: by construction of these pairs, the pair whose first element is the valuation function for the newest update corresponds to the the admissible abnormality by **AIU**-selection; and the ordered pair whose first element is the valuation function for the literal provided by the oldest update corresponds to the avoidable abnormality. Then by Definition 4 if $v_{M_{IU}}(I_iA)$ is minimal, the conflicting valuation $v'_{M'_{IU}}(I_j\sim A)$ has to refer to an abnormality with higher index, it is not minimal and it has to occur among the models selected by the adaptive strategy.

$\square$

Theorem 2 *$\Psi \vDash_{AIU} A$ iff $\Psi = \{\Gamma_i, \Gamma_j\}$ and $M(\Gamma_j) \vDash_{IU} A$.*

Proof. On the basis of Definition 2, we have three cases:

1. for every $A \in \mathcal{P}^{\pm}$, $v_{M_{IU}(\Gamma_i)}(A, w_0) = v'_{M'_{IU}(\Gamma_j)}(A, w')$: by hypothesis, $Mod(\Psi) \vDash_{AIU} A$, therefore, provided that $M(\Gamma_i) \in Mod(\Psi)$, for some $\Gamma_j \supset \Gamma_i$ in Ψ if $M(\Gamma_i) \vDash_{IU} A$ then also $M'(\Gamma_j) \vDash_{IU} A$, for every Γ_j;

2. for some $A \in \mathcal{P}^{\pm}$, $v_{M_{IU}(\Gamma_i)}(A, w_0) = 1$ or $v_{M_{IU}(\Gamma_i)}(\sim A, w_0) = 1$ and either $v'_{M'_{IU}(\Gamma_j)}(A, w') = 1$ and $v'_{M'_{IU}(\Gamma_j)}(\sim A, w') = 0$; or $v'_{M'_{IU}(\Gamma_j)}(A, w') = 0$ and $v'_{M'_{IU}(\Gamma_j)}(\sim A, w') = 1$: similar to 1.;

3. for some $A \in \mathcal{P}^{\pm}$, $v_{M_{IU}(\Gamma_i)}(A, w_0) = 1$ and $v'_{M'_{IU}(\Gamma_j)}(\sim A, w') = 1$ or viceversa (i.e. v, v' are conflicting valuations):

   [3*a*.] if $\Psi \vDash_{AIU} A$, then $v_{M_{IU}(\Gamma_i)}(A, w_0)$ cannot be minimal in $\mathbf{V}^{\neq}(\Psi)$ by the previous Theorem and the model in which this valuation holds has to be $M'(\Gamma_j)$ for some $\Gamma_j \supset \Gamma_i$ stages of Ψ;

   [3*b*.] if $M'(\Gamma_j) \vDash_{IU} A$ and $\Psi = \{\Gamma_i, \Gamma_j\}$, then the model in of valuation $v'_{M'_{IU}(\Gamma_j)}(A, w')$ has to have its corresponding model for a minimal conflicting valuation, and this has to be $M(\Gamma_i)$; then $M(\Gamma_i) \vDash_{IU} \sim A$ and by the adaptive selection the models of Γ_j are the first to be selected, $M(\Gamma_i)$ will not be selected, and thus $\Psi \vDash_{AIU} A$.

$\square$

6 The syntactic approach to `Resolve`

The nature of updates of the knowledge base and their localization on a temporal (i.e. information-based) order is essential for the formulation of our proof theory. The standard structure of a line in an adaptive derivation (see e.g. [1]) contains the following elements:

(i) a line number;

(ii) the derived formula;

(iii) the line numbers of the formulas from which the element in (ii) is derived;

(iv) the name of the rule(s) applied to derive the formula from previous lines;

(v) the condition on which the second element is derived.

The unusal element is represented by the condition. It is the element on the basis of whose falsity a new content is derived. This means that the adaptive frame is able to describe in a peculiar way the dynamic of update: if a database containing A is updated at some new stage with the information that $\sim A$, whatever was true at any previous stage on the condition of $\sim A$ being false, shall now be rejected. The proof theory of **AIU** formalizes therefore the way in which updates and their consequences are accepted.

The rules for the logic **AIU** are the following:

PREM at any stage of a proof, for any $A \in \Psi$, one may add to the proof a line consisting of:
(i) an appropriate line number;
(ii) A;
(iii) a dash;
(iv) PREM;
(v) $\emptyset$;
the *premise rule* establishes that premises are introduced in a line on the empty condition;

RU at any stage of a proof, for any $B \in \mathcal{P}$, if $A_1, \ldots A_n \vdash_{IU} B$, and $\Delta_1, \ldots, \Delta_n$ are the conditions respectively for $A_1, \ldots A_n$, then a line may be added consisting of:
(i) an appropriate line number;
(ii) B;
(iii) the line numbers of the $A_1, \ldots A_n$;
(iv) RU;
(v) $\Delta_1 \cup \ldots \cup \Delta_n$;
the *unconditional rule* refers to derivability in the **LLL**: it allows to add a line containing a formula already occurring in the proof, without any new condition but (if any) the conditions of the formulas to which the rule is applied;

RC at any stage of a proof, for any $B \in \mathcal{P}$, if $A_1, \ldots A_n \vdash_{IU} B \vee Dab(\Delta)$, and $\Delta_1, \ldots, \Delta_n$ are the conditions respectively for $A_1, \ldots A_n$, then a line may be added consisting of:
(i) an appropriate line number;
(ii) B;

(iii) the line numbers of the $A_1, \ldots A_n$;
(iv) RC;
(v) $\Delta_1 \cup \ldots \cup \Delta_n \cup (\Delta)$;
the last rule refers to the derivability in the **LLL** of a formula of the form $B \vee Dab(\Delta)$, which is transformed in the derivation of B on the assumption that some member of $Dab(\Delta)$ is false; in the new line such a member will be introduced as a new condition.

The notion of *Dab*-formula introduced in the previous section is now formulated for the prioritized procedure of update at stage of a derivation. In this syntactic version, the notion of *degree* of a *Dab*-formula is needed (the syntactic counterpart to the indexed abnormal model of Definition 10):

Definition 14 (*Dab*-Formula at degree) *$Dab(\Delta)$ is called a Dab-formula of degree i of iff it is a disjunction of members of Δ where $\Delta \subseteq \Omega_i$.*

The condition to accept a monotonic update in the form of a *Dab*-formula at a given degree is given as a regularity condition:

Definition 15 (Regular *Dab*-formulas) *Given a* **AIU** *proof, $Dab(\Delta)$ is a regular Dab-formula of degree i iff (i) $Dab(\Delta)$ is derived on condition Θ on an unmarked line, (ii) $\Delta \subseteq \Omega_i$, and (iii) $\Theta \subseteq \Omega_{i+1}, \ldots, \Omega_k$.*

This definition says that a disjunction of abnormalities is regular at a certain degree if and only if it is asserted conditionally on the falsity of updates happening at later stages. This guarantees the priority for updates at any higher index: on this basis **AIU** selects the valid **IU**-consequences of a premise set assuming that the later the update, the more reliable the information received. The regularity condition is embedded into the already given principle of minimality for disjunctions of abnormalities:

Definition 16 (Minimal *Dab*-formulas) *Given a* **AIU** *proof, $Dab(\Delta)$ is a minimal Dab-formula of degree i iff $Dab(\Delta)$ is a* regular *Dab-formula of degree i and there is no $\Delta' \subset \Delta$ for which $Dab(\Delta')$ is a regular Dab-formula of degree i.*

The dynamic aspect of the proof theory is based on the ability of the adaptive frame to derive a formula at a certain stage of a proof, and to *mark* it at a later one, i.e. to suspend its derivability, when the falsity of its conditions can no longer be assumed.[5] The dynamics of updates relies therefore on a non-monotonic update being considered false as long as this is possible. Whenever such an update turns out to be derivable, the content obtained on the basis of its falsity shall be retracted.[6]

The rules holding for the combined logic **AIU** are given therefore by:

(i) the rules **RU** and **RC** defined for the logics $IU_1 - IU_k$, for the highest $k \in \mathcal{T}$ occurring in a premise set Ψ: the unconditional rule is identical

[5]The notion of derivability at stage of a proof, formally introduced in this section, has an obvious intuitive correspondence with the validity of a formula at a stage Γ_i of an updated database Ψ, introduced in the semantic formulation of the `Resolve`-strategy.

[6]In the final section more is said on a standard Adaptive Logic that explicitely defines a retraction operator on informational updates.

for each IU_i because depending on the **LLL** which is the same for any indexed consequence set; the conditional rule has different applications for each index, because it depends on the abnormalities at degree;

(ii) the combined marking definitions, which determine the syntactic version of the `Resolve`-strategy.

The Marking Definition defines the procedure to establish which contents at a certain line in a proof are withdrawn, because derived on condition of the falisity of an update which later turns out to be derivable. In the following definition, the notion of *unreliable formulas at degree* from Definition 11 is used:

Definition 17 (Marking for Reliability) *A line l is marked at stage s of an* **AIU***-proof from a premise set Ψ iff:*

1. *Δ is the condition at line l with degree $i \in \mathcal{T} = \{1, \dots, k\}$ occurring in Ψ;*

2. *Δ is part of the set of unreliable formulas of Ψ at degree i; i.e. $\Delta \cap U^i(\Gamma) \neq \emptyset$, or $\Delta \cap U^{k-(i+n)}(\Gamma) \neq \emptyset$, ..., or $\Delta \cap U^k(\Gamma) \neq \emptyset$.*

According to the previous definition, a formula derived at one stage can be marked at a later one if its condition is derived at that stage. This establishes an unstable notion of derivability:

Definition 18 (Derivability at stage) *A formula A is derived from Ψ at stage s of an* **AIU***-proof iff A is the second element of a line whose condition Δ is not part of any set of unreliable formulas for Ψ at stage s.*

Correspondingly, a stable notion of derivability is defined, which intuitively says that a content is unmarked at a certain stage of the proof and it will stay unmarked at any extension of the proof from the very same premise set:

Definition 19 (Final Derivability) *A formula A is finally derived in a* **AIU***-proof from Ψ iff*

1. *A is the second element of an unmarked line l at stage s derived at condition Δ of degree i;*

2. *and line l stays unmarked with respect to sets of unreliable formulas of any higher degree $U^{i+1}(\Psi)$.*

Under these conditions, the operation of information update for **AIU** can be defined as follows:

Definition 20 (Final Update according to AIU) *An information update by $I_iA \mid A \in \mathcal{P}^{\pm}$ is finally valid in an* **AIU***-proof from a premise set Ψ iff*

1. *at stage s of that proof a formula of the form $I_iA \wedge {\sim}A$ is derived at line l on a condition Δ of degree $i + n$;*

2. *any formula ${\sim}A$ derived from Ψ at some line $l - n$ on a condition Δ of degree i is marked at stage s of the same proof;*

3. *and none of the Dab-formulas of degree $i+n$ that are conditions of $I_i A \wedge \sim A$ are derivable at stage $s+n$ of the same proof, which means that line l stays unmarked.*

This definition implies a final form of information update with respect to the updated database represented by the premise set Ψ. A content is therefore stable if it persists under any update within a given Ψ. An extension on Ψ produced by an external dynamics on the basis of new information leads to a logic **IU** of higher degree, which also means a different set of unreliable formulas and therefore a different selection for marking.

6.1 Some Examples

A first simple example runs as follows from the premise set $\Psi = \{I_1(p \vee q), I_2{\sim}p\}$:

1	$I_1(p \vee q)$	PREM	$\emptyset$
2	$I_2{\sim}p$	PREM	$\emptyset$
3	$p \vee q$	1; RU	$\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q\}$
4	${\sim}p$	2; RC	$\{I_2{\sim}p \wedge p\}$
5	q	3, 4; RC	$\{I_1 q \wedge {\sim}q, I_2{\sim}p \wedge p, I_1 p \wedge {\sim}p\}$

According to this derivation nothing is marked and lines 3–5 are derivable as desired. Consider the following premise set $\Psi = \{I_1(p \vee q), I_2{\sim}p, I_3{\sim}q\}$:

1	$I_1(p \vee q)$	PREM	$\emptyset$	
2	$I_2{\sim}p$	PREM	$\emptyset$	
3	$I_3{\sim}q$	PREM	$\emptyset$	
4	$p \vee q$	1; RU	$\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q\}$	$\surd^{9}$
5	${\sim}p$	2; RC	$\{I_2{\sim}p \wedge p\}$	
6	q	4, 5; RC	$\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q, I_2{\sim}p \wedge p\}$	$\surd^{9}$
7	${\sim}q$	3; RC	$\{I_3{\sim}q \wedge q\}$	
8	p	4, 7; RU	$\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q, I_3{\sim}q \wedge q\}$	$\surd^{9}$
9	$(I_1 p \wedge {\sim}p) \vee (I_1 q \wedge {\sim}q)$	1, 2, 3; RC	$\{I_2{\sim}p \wedge p, I_3{\sim}q \wedge q\}$	

lines 5 and 7 stay unmarked, which means that updates with ${\sim}p$ and ${\sim}q$ are accepted; on the other hand, lines 4, 6 and 8 are marked and therefore their contents rejected. Consider moreover that the derivation could be extended by deriving the following *Dab*-formula

10	$(I_2{\sim}p \wedge p) \vee (I_3{\sim}q \wedge q)$	1, 2, 3; RC	$\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q\}$

but this is not a regular (and therefore not minimal) *Dab*-formula of degree 3 according to Definitions 15 and 16, because its condition is of a lower degree; this means that $\{I_1 p \wedge {\sim}p, I_1 q \wedge {\sim}q\}$ are not part of the set of unreliable formulas of Ψ and they do not allow for any further marking, according to Definition 17.

Modify the previous premise set as $\Psi = \{I_1{\sim}p, I_2(p \vee q), I_3{\sim}q\}$:

1	$I_1{\sim}p$	PREM	$\emptyset$	
2	$I_2(p \vee q)$	PREM	$\emptyset$	
3	$I_3{\sim}q$	PREM	$\emptyset$	
4	${\sim}p$	1; RU	$\{I_1{\sim}p \wedge p\}$	$\surd^{10}$

5	$\sim q$	3; RC	$\{I_2 p \wedge \sim p, I_3 \sim q \wedge q\}$	
6	$p \vee q$	2; RC	$\{I_2 p \wedge \sim p, I_3 \sim q \wedge q\}$	
7	p	5; 6; RC	$\{I_2 p \wedge \sim p, I_3 \sim q \wedge q\}$	
8	q	4, 6; RC	$\{I_1 \sim p \wedge p, I_2 p \wedge \sim p, I_2 q \wedge \sim q\}$	$\surd^{10}$
9	$(I_1 \sim p \wedge p) \vee (I_2 q \wedge \sim q)$	1, 2, 3; RC	$\{I_2 p \wedge \sim p, I_3 \sim q \wedge q\}$	
10	$I_1 \sim p \wedge p$	1, 2, 3; 9 RC	$\{I_2 q \wedge \sim q, I_2 p \wedge \sim p, I_3 \sim q \wedge q\}$	

according to which the contents at lines 4 and 8 are rejected, which means that $\sim q$ and p (and obviously $p \vee q$) are derivable as desired. As for the previous example, also this derivation can be extended further by the following *Dab*-formula:

11	$(I_2 p \wedge \sim p) \vee (I_3 \sim q \wedge q)$	1, 2, 3; RC	$\{I_1 \sim p \wedge p, I_2 q \wedge \sim q\}$

which, as in the previous case, is not minimal and therefore does not allow for any further marking.

Consider now the following derivation from the premise set $\Psi = \{I_1(p \vee q), I_2 \sim p, I_3 \sim q, I_4 p\}$

1	$I_1(p \vee q)$	PREM	$\emptyset$	
2	$I_2 \sim p$	PREM	$\emptyset$	
3	$I_3 \sim q$	PREM	$\emptyset$	
4	$I_4 p$	PREM	$\emptyset$	
5	$p \vee q$	1; RU	$\{I_1 p \wedge \sim p, I_1 q \wedge \sim q\}$	
6	$\sim p$	2; RC	$\{I_2 \sim p \wedge p\}$	$\surd^{12}$
7	q	5, 6; RC	$\{I_1 p \wedge \sim p, I_1 q \wedge \sim q, I_2 \sim p \wedge p\}$	$\surd^{12}$
8	$\sim q$	3; RC	$\{I_3 \sim q \wedge q\}$	
9	p	5, 8; RC	$\{I_1 p \wedge \sim p, I_1 q \wedge \sim q, I_3 \sim q \wedge q\}$	
10	$(I_1 p \wedge \sim p) \vee (I_1 q \wedge \sim q)$	1, 2, 3; RC	$\{I_2 \sim p \wedge p, I_3 \sim q \wedge q\}$	$\surd^{12}$
11	p	4; RC	$\{I_4 p \wedge \sim p\}$	
12	$I_2 \sim p \wedge p$	2, 4; RC	$\{I_4 p \wedge \sim p\}$	

According to this derivation lines 6, 7 and 10 are marked, whereas p and $\sim q$ (and obviously $p \vee q$) are derived as desired. Also in this case one more *Dab*-formula can be derived:

13	$(I_2 \sim p \wedge p) \vee (I_3 \sim q \wedge q)$	1, 2, 3; RC	$\{I_1 p \wedge \sim p, I_1 q \wedge \sim q\}$

which once again is neither regular nor minimal, therefore it does not allow any further marking.

7 Conclusions

The logic **AIU** defines an adaptive resolution method for inconsistent updates on propositional bases. This logic has been formulated with an intuitive application to database theory, and it seems a fruitful method for the application to integrated databases. It can also be thought as an evolutionary system of constraints for static databases. The adaptive selection restores consistency by eliminating the older of the non-monotonic updates. By the ordering on the stages of the updated database, the removed content is also part of the less informative of such stages, satisfying the Principle of Informational Economy. By reference to the setups of the various informative/temporal stages of an updated database, the notion of semantic consequence for **AIU** is also characterized by the notion of minimal conflicting valuation.

In [20] an Adaptive Logic called **AIUR** for the corresponding retraction function on non-monotonic updates has been introduced. Its procedure restores consistency on a belief set by bringing it back to the time before the last update was performed. It models therefore a procedure which is the inverse with respect to `Restore`. The adaptive retraction satisfies the Inclusion Postulate in a preferential structure and, assuming logical closure, it also satisfies the Recovery Postulate. As a result, the consistent base will contain the oldest and more reliable data.

This work on the notion of update represents an alternative view on standard models of belief change A foreseeable extension of the present framework can be given in terms of an appropriate generalization to the predicative case, in order to provide more realistic applications, especially for relational databases. Such extension is very easy to reach in view of the standard format of Adaptive logics. An open problem is the formulation of extended integrity constraints for completeness requirements. Finally, a desirable implementation is the formulation of metadata sets, such as the explicit formulation of a `null`-answer for a query operation on missing data.

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