Alleged assassins: realist and constructivist semantics for modal modification

Bjørn Jespersen[1]* and Giuseppe Primiero[2]**

[1] Institute of Philosophy, Department of Logic,
Czech Academy of Sciences, Prague;
Department of Computer Science,
Technical University of Ostrava (Czech Republic)
`bjornjespersen@gmail.com`
[2] Centre for Logic and Philosophy of Science,
Ghent University (Belgium)
`giuseppe.primiero@ugent.be`

Abstract. Modal modifiers such as *Alleged* oscillate between being subsective and being privative. If individual a is an alleged assassin (at some parameter of evaluation) then it is an open question whether a is an assassin (at that parameter). Standardly, modal modifiers are *negatively* defined, in terms of *failed* inferences or *non*-intersectivity or *non*-extensionality. Modal modifiers are in want of a positive definition and a worked-out logical semantics. This paper offers two positive definitions. The *realist* definition is elaborated within Tichý's Transparent Intensional Logic (TIL) and builds upon Montague's model-theoretic semantics for adjectives as representing mappings from properties to properties. The *constructivist* definition is based on an extension of Martin-Löf's Constructive Type Theory (CTT) so as to accommodate partial verification. We show that, and why, "a is an alleged assassin" and "Allegedly, a is an assassin" are equivalent in TIL and synonymous in CTT.

Keywords: Modal modification, property vs. propositional modification, *alleged*, *allegedly*, Transparent Intensional Logic, Constructive Type Theory.

1 Introduction & overview

Kamp's seminal [10] seeks to draw a line between those adjectives whose meaning is a property and those adjectives whose meaning is a function that maps properties to properties. Kamp agrees with Montague's typing of properties as a function $\langle s, \langle e, t \rangle \rangle$ from a world/time pair s to a function from entity e to truth-value t. Montague [13, p.211] suggests that all adjectives have a property-

* Research Fellow, Project GACR 401/10/0792: *Temporal Aspects of Knowledge and Information*.
** Postdoctoral Fellow of the Research Foundation - Flanders (FWO).

to-property function as their meaning.[3] Kamp [10, pp.147ff] suggests that most adjectives have a property as their meaning. He admits that it would seem that some adjectives must occur in attributive position and are incapable of occurring in predicative position. Their meaning is a property-to-property function. Kamp's prime example of such a recalcitrant adjective is 'alleged':

> Is *alleged* a predicate, even in the most diluted sense? It seems not. [...] The same can be said to be true [...] of adjectives such as *fake*, *skilful*, or *good*. Where precisely we should draw the boundaries of the class of adjectives to which the second theory [property-to-property function] applies I do not know. For example, does *skilful* belong to this class? Surely we must always ask 'skilful what?' before we can answer the question whether a certain thing or person is indeed skilful [...].[4]

We agree with Kamp's linguistic observations. Kamp is concerned with a demarcation among adjectives. We hypothesize that his demarcation is in effect a demarcation between those adjectives that represent *properties* and those that represent *property modifiers*. Thus, to use Kamp's own example from [10, p. 123],"Every alleged thief is a thief" is not an instance of predication of two properties, *being alleged* and *being a thief*. Instead it is an instance of predication of one modified property, *being an alleged thief*. We must always ask 'alleged what', for nothing and nobody can be alleged, pure and simple. Nor is the logical form of the sentence, in predicate logic, anything like

$$\forall x((Alleged\ x \wedge Thief\ x) \rightarrow Thief\ x)$$

This form would, erroneously, trade a possible falsehood for a logical truth ([10, p.123]).

Kamp distinguishes between four kinds of adjectives in terms of their logical behaviour. Roughly the same taxonomy is known from modification theory. Interestingly, *Alleged* falls outside Kamp's taxonomy. In fact, *Alleged* and its ilk remain to this day a dimly lit corner of the research into adjectives and modifiers.[5] For instance, Partee [14, p.9] says that

> *Nonsubsective* adjectives may be either *modal* – expressing possibility or some other modal meanings – or *privative*, entailing negation. [...] There is no meaning postulate for the *modal* adjectives, since they have no entailments – an *alleged murderer* may or may not be a murderer, and similarly for adjectives like *possible*, *proposed*, *expected*, *doubtful*.

---

[3] Beesely [1] argues, *contra* Montague, that also evaluative adjectives like 'good' and 'tall' have a property as their meaning. Beesley holds that "a is a good F" should be given the intersective, or conjunctive, analysis "a is good and a is an F". Beesley, however, does not extend his claim to 'alleged' and suchlike; nor is it obvious how to do so.

[4] [10, pp. 153-4].

[5] See [3], [11], [15], [22], [27].

Yet everyone who is competent with the predicate 'is an alleged assassin' knows that it applies to someone who has been alleged to be an assassin and that they may, or may not, actually be an assassin.[6] So there is some clearly circumscribed linguistic competence to account for. What has as yet not been established is how to provide a *positive definition* of the sort of modifier that *Alleged* typifies.

Modal modifiers such as *Alleged* are uniquely characterized by oscillating between being *subsective* and being *privative*. Formally, the rule of subsective modification (cf. Kamp's 'affirmative adjective') eliminates the modifier, while the rule of privative modification replaces the modifier by negation. Let a be an individual, F a property, M a property modifier, and $[MF]$ the property resulting from modifying F by M. Then a modifier is subsective if it validates this inference (in rudimentary predicate-logical notation, to begin):

$$\frac{[[M_s F]\ a]}{Fa}$$

For instance, if a is a *wine-drinking Georgian* then a is a Georgian, hence *Wine-drinking* is subsective. In extensional terms, a set of wine-drinking Georgians must be extracted from a set of Georgians. Hence if a belongs to a set of wine-drinking Georgians then a belongs *ipso facto* to a set of Georgians. Subsective modification is the simplest kind of modification and of little logical interest.[7]

A modifier is privative if it validates this inference:

$$\frac{[[M_p F]\ a]}{\neg Fa}$$

For instance, if a is a *fake banknote* then a is not a banknote, hence *Fake* is privative. In extensional terms, a set of fake banknotes must be extracted from the complement of a set of banknotes. Hence if a belongs to a set of fake banknotes then a belongs *ipso facto* to a set in the complement of a set of banknotes. Privative modification is logically much more delicate than subsective modification. As [2] shows, iterated privative modification cannot be modelled by iteration of propositional negation. It must be modelled by property negation. As a result, because a logic of multiple privation is a logic of *contraries*, a pair of privative modifiers is equivalent to a modal modifier. The above rule applies only to single privation.

[6] Partee (in personal communication at LOGICA 2012, Hejnice) points out that though she held that modal adjectives/modifiers lack a meaning postulate she did not hold that they lack meaning.

[7] Within subsective modification the simplest kind of modification is constituted by *trivial* modification: a is a lump of *genuine* gold iff a is a lump of gold. A trivial modifier returns the modified property unmodified, as it were. The polar contrary is privative modification. We note that our adoption of trivial modification is at variance with Kamp and Partee's *non-vacuity principle* [11, p.161].

In virtue of the oscillation between subsection and privation, if a is an alleged assassin then either a is an assassin or a is not an assassin. So the rule of inference defining modal modifiers would seem straightforward:[8]

$$\frac{[[M_m F]\ a]}{Fa \vee \neg Fa}$$

But, of course, this classical tautology is trivially satisfied by *all* modifiers. A subsective modifier will invariably validate the left-hand disjunct. A privative modifier will invariably validate the right-hand disjunct. What is non-trivial is that a modal modifier will sometimes validate the left-hand disjunct and sometimes the right-hand disjunct. For any one instance of $[M_m F]\ a$, the paucity of the informational value of $[M_m F]\ a$, when true, is such that it cannot be inferred which side of $Fa, \neg Fa$ truth will come down on. This is what we mean by modal modifiers oscillating between subsection and privation. It is obvious, then, why the 'conjunctive' analysis $((Alleged\ x \wedge Thief\ x) \rightarrow Thief\ x)$ is to no avail. It eliminates modifiers from the analysis. And it prejudges in favour of subsection at the expense of privation, thereby missing the unique feature of modal modifiers.

It is not immediately obvious what a *positive* definition of modal modification would amount to. It is easy enough to characterize modal modification *negatively*. First, as we saw, a modal modifier fails to validate either of $Fa, \neg Fa$ as the conclusion of an argument whose only premise is $[M_m F]\ a$. Second, a modal modifier is *non-intersective* for failure to validate this argument, M_i an intersective modifier (Kamp: 'predicative'):

$$\frac{[[M_i F]\ a]}{M^* a \wedge Fa}$$

For instance, if a is a wine-drinking Georgian then a is a wine-drinker and a is a Georgian. In extensional terms, a set of individuals with the property $[M_i F]$ is the intersection of a set of Fs and a set of M^*s. Notice that M_i is a modifier whereas M^* is a property. A modifier cannot be detached from a context in which it modifies a property and be predicated of an individual. Instead it can be *pseudo-detached* in the following manner: if a is an $[MF]$, M an arbitrary modifier, then there is a property p such that a is an $[Mp]$. M^* is the schematic property $[Mp]$. The conclusion of the rule of inference defining intersective modification is formed by means of the rules of pseudo-detachment, subsection, and $\wedge$-introduction.[9]

---

[8] [20] may have given the impression that the above inference was the rule we proposed at the time for modal modifiers. We intended no such impression, however. See [20, p.269]. See also [8].

[9] The need for a rule of *left subsectivity* such as the one of pseudo-detachment tends to be overlooked in the Montagovian tradition. [14, p.3], for one, puts forward a meaning postulate to regulate intersective adjectives: For each intersective meaning ADJ', it holds that $\exists P_{\langle e,t\rangle} \forall Q_{\langle s\langle e,t\rangle\rangle}[ADJ'(Q)(x) \leftrightarrow P(x) \wedge {}^{\vee}Q(x)]$. The meaning postulate

Third, a modal modifier is *non-extensional* (or *intensional*, in the pejorative sense of 'intensional') for failure to validate this argument (adapted from [10, p.125]):

$$\frac{Fa \leftrightarrow Ga}{[MF]\ a \leftrightarrow [MG]\ a}$$

For instance, even if it so happens that all and only kings are philosophers, it may not follow that all and only belligerent kings are belligerent philosophers. An individual who is both philosopher and king may be a belligerent king (waging war in his capacity as king) without being a belligerent philosopher (waging war in his capacity as philosopher). Hence *Belligerent* is a non-extensional modifier. Logically, non-extensional modifiers are those that do *not distribute*, because they are logically sensitive to whether a belongs to F or G, even though the extension of F happens to be identical to the extension of G.[10]

So modal modifiers are non-intersective, hence non-extensional, possibly (non-) subsective and possibly (non-) privative. The *actual truth* of $[M_m F]\ a$ entails that one of two *possibilities* is realized: a being an F; a not being an F. Thus there is a striking similarity between *modal modifiers* and *non-factive attitudes*. Assume that b *knows whether* a is an assassin. As is well-known, *knowing whether* is invariant under complementation: a knows whether $A \equiv a$ knows whether $\neg A$.[11] If a is an assassin then b knows that a is an assassin; if a is not an assassin then b knows that a is not an assassin. But the fact that b knows whether a is an assassin entails that one of the same two possibilities as above is true. The truth of a knowing whether b is an assassin is compatible with either disjunct being true.[12]

gets the truth-condition right, but fails to account for the transition from ADJ' to P. Besleys' theory in [1] makes left subsectivity trivial, in virtue of his conjunctive analysis. The left conjunct is simply obtained by conjunction elimination. Beesley's task, of course, is to make a case for 'good' denoting a property rather than a modifier. See [7] for further discussion.

[10] Since modal modifiers are non-intersective they must also be non-extensional. Kamp [10, pp. 125-6] states, correctly, that all intersective modifiers (predicative adjectives) are extensional, and wonders whether the converse holds. The jury is still out. What seems obvious is that most, or all, logically, semantically and philosophically interesting modifiers are going to be non-extensional, because it is interesting in each individual case why they fail to distribute. For further discussion, see [2, p.11]. [20, p.253] uses 'intensional' interchangeably with 'modal'; but they are better used as labels for two different kinds of modifiers.

[11] See [5, §5.1.4].

[12] The link between modal modifiers and non-factive attitudes probably runs deeper than we let on in the present paper. [15, p.152] provides the following list of 'plain nonsubsective' (in effect, modal) modifiers/adjectives: *potential*, *alleged*, *arguable*, *likely*, *predicted*, *putative*, *questionable*, *disputed*. With the exception of *potential*, they all have something attitudinal about them. And all of those attitudes are non-factive. A bold hypothesis would be that almost all modal modifiers are parasitic on non-factive attitudes. Modal modifiers should not be filed under 'nonsubsective', for

The similarity between modal modifiers and non-factive attitudes suggests to us that modal modification should be modelled in terms of *possibility*. One conception of possibility is in terms of *alethic possibility*: reality may turn out in one of two contrary ways. Another conception is as *epistemic possibility*: something rather than the opposite may be known. We shall develop both conceptions below. The former is based on Tichý's Transparent Intensional Logic (TIL). Formally, TIL is a hyperintensional, partial, typed λ-calculus, whose syntax is interpreted by means of a realist procedural semantics. The portion of TIL that concerns property modification is continuous with Montague's: a property modifier is a mapping from properties to properties. In TIL a property is logically a mapping from a logical space of possible worlds to a mapping from times to sets of individuals, where sets of individuals are characteristic functions. The latter conception, of possibility as epistemic possibility, is based on an extension of Martin-Löf's Constructive Type Theory (CTT). Formally, CTT is a typed calculus based on intuitionistic logic, endorsing the Curry-Howard isomorphism (propostions-as-types) and the equivalence between sets and propositions under a constructive syntax.[13] A modifier is obtained by interpreting in the appropriate way the assertion conditions under which a formula holds.

This paper builds on [20], which applies CTT and TIL to privative modification. Common features of TIL and CTT include:

– a functional approach based on the typed lambda calculus
– a typed universe
– an interpreted logical syntax
– a notion of meanings as constructions/procedures: a proof procedure for a proposition (CTT); a procedure for producing (in this case) a possible-world proposition/empirical truth-condition (TIL).

The key differences are that TIL offers a procedural semantics erected upon a model-theoretic structure for modifiers whereas CTT offers a proof-theoretic semantics.[14] In [20] a TIL property modifier is a function from properties to properties, whereas a CTT property modifier is a function from sets to sets. This latter difference is particularly important for our present purposes. A constructive set is a set of proof-objects for a proposition, and since constructive propositions are identified with their sets of proof-objects, a constructive property modifier is, in the final analysis, a function from propositions to propositions (hence a function between intensional entities). Modal modifiers require an interpretation of partially evaluated terms as the range of the function at hand. Therefore the

due to their oscillation between subsection and privation each modal modifier will be subsective on some occasions and privative on other occasions.

[13] For a recent introduction to the Curry-Howard isomorphism, see [25].

[14] The modal logic of TIL is $S5$ with a constant domain. See [5, ch.4]. The standard modal interpretation of Martin-Löf's Type Theory refers to $S4$: see [16]. For semantic considerations related to the possibility operator underlying the extension of CTT used in this paper, see the relations to usually modally defined knowledge operators in [21].

CTT analysis of "a is an alleged assassin" is in effect an analysis of "Allegedly, a is an assassin", *Allegedly* being a propositional modifier. We point out below the equivalence (though not synonymy) of "a is an alleged assassin" and "Allegedly, a is an assassin" in TIL.

The rest of this paper is organized as follows. Section 2 presents the TIL explanation of modal modification. Section 3 presents the CTT explanation of modal modification. Section 4 compares the main results.

2 TIL: types and constructions

TIL comes with a ramified type hierarchy embedding a simple type theory.

Definition 1 (Type of order 1).
Let B be a base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:*

i) Every member of B is an elementary type of order 1 over *B;*
ii) Let $\alpha, \beta_1, \ldots, \beta_m (m > 0)$ be types of order 1 over B. Then the collection $(\alpha\beta_1 \ldots \beta_m)$ of all m-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into α is a functional type of order 1 over *B;*
iii) Nothing is a type of order 1 over *B unless it so follows from (i) and (ii).*

Remark. For the purposes of natural-language analysis, the following base of ground types is currently assumed:

- o: the set of truth-values $\{\mathbf{T}, \mathbf{F}\}$
- ι: the set of individuals (a constant universe of discourse)
- τ: the set of real numbers (doubling as temporal continuum)
- ω: the set of logical possible worlds (the logical space)

Functional types are defined over those ground types in the standard manner. A functional type with domain in possible worlds is an intensional type as known from possible-world semantics. The simple type theory suffices to type properties, propositions, property and propositional modifiers:

- property: $(((o\iota)\tau)\omega)$, abbreviated as '$(o\iota)_{\tau\omega}$'
- property modifier: $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$
- proposition: $(((o)\tau)\omega)$, abbreviated as '$o_{\tau\omega}$'
- propositional modifier: $(o_{\tau\omega}o_{\tau\omega})$

We model those empirical conditions as *possible-world intensions*. Formally, intensions are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type β. The type β is frequently the type of the *chronology* of α-objects, i.e. a mapping of type $(\alpha\tau)$. Thus α-intensions are frequently functions of type $((\alpha\tau)\omega)$, abbreviated as '$\alpha_{\tau\omega}$'. We shall typically say that an index of evaluation is a world/time pair $\langle w, t\rangle$. *Extensional entities* are entities of some type α where $\alpha \neq (\beta\omega)$ for any type β.

Logical procedures are so-called *constructions*. There are six different kinds of constructions, three of which are defined inductively below.[15] The basic idea is that functional abstraction is the very procedure of forming or presenting or *constructing* a function (rather than the resulting function); that functional application is the very procedure of applying function to argument (rather than the resulting functional value); and that variables provide input for those procedures to operate on.

Definition 2 (Construction).

(i) *The* Variable x *is a* construction *that constructs an object* O *of the respective type dependently on a valuation* v*; it* v*-constructs* O*;*
(ii) *The* Composition $[XY_1 \dots Y_m]$ *is the following* construction. *If* X v-constructs *a function* f *of a type* $(\alpha\beta_1 \dots \beta_m)$*, and* $Y_1, \dots, Y_m$ v-construct *entities* $B_1, \dots, B_m$ *of types* $\beta_1, \dots, \beta_m$*, respectively, then the* Composition $[XY_1 \dots Y_m]$ v-constructs *the value (an entity, if any, of type* α*) of* f *on the tuple-argument* $\langle B_1, \dots, B_m \rangle$*. Otherwise, the* Composition $[XY_1 \dots Y_m]$ *does not* v-construct *anything and so is* v-improper*;*
(iv) *The* Closure $[\lambda x_1 \dots x_m Y]$ *is the following* construction. *Let* $x_1, x_2, \dots, x_m$ *be pairwise distinct variables* v*-constructing entities of types* $\beta_1, \dots, \beta_m$ *and* Y *a construction* v*-constructing an entity of type* α*. Then* $[\lambda x_1 \dots x_m Y]$ *is the* construction λ-Closure *(or* Closure*). It* v-constructs *the following function* f *of type* $(\alpha\beta_1 \dots \beta_m)$*. Let* $v(B_1/x_1, \dots, B_m/x_m)$ *be a valuation identical with* v *at least up to assigning objects* $B_1, \dots, B_m$ *of types* $\beta_1, \dots, \beta_m$*, respectively, to variables* $x_1, \dots, x_m$*. If* Y *is* $v(B_1/x_1, \dots, B_m/x_m)$*-improper (see iii), then* f *is undefined on* $\langle B_1, \dots, B_m \rangle$*. Otherwise, the value of* f *on* $\langle B_1, \dots, B_m \rangle$ *is the entity of type* α $v(B_1/x_1, \dots, B_m/x_m)$*-constructed by* Y*;*
(v) *Nothing is a* construction*, unless it so follows from (i) through (iv).*

Explicit intensionalization and temporalization enables TIL to encode constructions of possible-world intensions, by means of terms for possible-world variables and times, directly in the logical syntax.[16] Where w ranges over ω and t over τ, the following logical form essentially characterizes the logical syntax of an empirical sentence:

$$\lambda w \lambda t[\dots w \dots t \dots]$$

Any instance of this schematic Closure constructs the set of $\langle w, t \rangle$ pairs at which the empirical truth-condition constructed by $[\dots w \dots t \dots]$ is satisfied.

Thus the Closure

$$\lambda w \lambda t[Assassin_{wt}\ a]$$

[15] The constructions *Trivialization*, *Single* and *Double Execution* are not needed for present purposes. However, see [5, §1.3.2].

[16] See [5, §2.4.3] for a comparison between TIL and Montague Grammar.

constructs the set of $\langle w, t\rangle$ pairs at which a is an assassin. The extensionalized property $[Assassin]_{wt}$ is applied to a to obtain a truth-value, which is subsequently abstracted over to obtain a possible-world proposition. $\lambda w \lambda t[Assassin_{wt}\ a]$ is a *hyperproposition* whose procedural product is a possible-world proposition.

To construct the modified property *being an alleged assassin*, the modifier *Alleged* is applied to the property *Assassin*, and the resulting property is extensionalized for application to a. Thus the meaning of "a is an alleged assassin" is the Closure

$$\lambda w \lambda t[[Alleged\ Assassin]_{wt}\ a]$$

2.1 TIL: definitions of modifiers

Kamp's definitions of his respective kinds of adjectives (predicative, affirmative, etc.) are model-theoretic. An adjective is, for instance, predicative, provided it satisfies the definition of predicative adjectives on all admissible interpretations. Kamp then proceeds to note that he doubts that at least any adjective that is privative on some admissible interpretation is privative on all of them [10, p.125]. The parallel phenomenon for modifiers is discussed in [9, §2.5]. For instance, *Nordic* gold is not gold (but a copper alloy) whereas a Nordic salmon is a salmon. And a *false* friend is not a friend (though pretending to be one) whereas a false proposition is still a proposition.[17] One could even argue that *Alleged* is not exclusively modal. If an *alleged proposition* is a proposition that someone alleges to be true then that occurrence of *Alleged* is as a subsective modifier. In general, we must not succumb to the naïve assumption that every modifier would be only intersective, only privative, etc.

In the procedural semantics of TIL we define first the respective *sets* of subsective, privative, intersective, and modal modifiers. Then we issue this conditional: *if* modifier M is intersective (etc.) *then* so-and-so follows. It is left up to a given interpretation to decide whether M is in fact intersective (etc). Any such interpretation will ideally be sensitive to the actual, extra-model status of M.[18] It is too strong to demand that M must be privative on all admissible interpretations in order to qualify as privative on one admissible interpretation.

[17] See also [14, p. 9]. Of course, Partee famously wishes to reduce all modifiers to subsective ones, leaving room only for (some) modal modifiers as not being necessarily subsective. See [24] for objections to Partee's proposal. In TIL the categories of subsective, privative, intersective, and modal modifiers are mutually irreducible. Yet in a derived or indirect sense privation is a variant of subsection. For example, extensionalize the property of being a banknote to obtain a set, then generate its complement, and apply comprehension to that set to extract its subset (perhaps empty, perhaps non-empty) of fake banknotes. In CTT, privative modifiers are an extreme kind of subsective modifiers: where the latter generate proper subsets, the former take the improper subset of functions mapping to the empty set. See [20] for details.

[18] It falls to linguistic analysis to decide what that status of M is. There is no logical mechanism for deciding it. (Thanks to Petr Šimon.)

So the logical status of the modifiers *Nordic*, *False*, etc, is a function of their respective arguments. E.g. the definition of *privative modifier* is a definition of the set of modifiers that are privative with respect to the argument property F. The type of a set of property modifiers is $(o((o\iota)_{\tau\omega}(o\iota)_{\tau\omega}))$. Modifiers will be defined in terms of *requisite properties*.[19] A requisite of a property G is a property F such that anything with property G must also have F.

Definition 3 (Requisite relation over $(o\iota)_{\tau\omega}$). *Let $X, Y/(o\iota)_{\tau\omega}$, and let x range over ι. Then*

$$[Req\ YX] = \forall w \forall t[\forall x[X_{wt}\ x] \supset [Y_{wt}\ x]]$$

Gloss *definiendum* as "Y is a requisite of X", and *definiens* as "Necessarily, any x instantiating X at any $\langle w,t\rangle$ also instantiates Y at $\langle w,t\rangle$." For instance, if the property *being a mammal* is a requisite of the property *being a whale* then if a happens to be a whale at $\langle w,t\rangle$ it is necessary that a also be a mammal at $\langle w,t\rangle$. Or if at $\langle w,t\rangle$ a has the modified property $[M_s F]$ then it is necessary that a also be an F at $\langle w,t\rangle$. The reason is because F is a requisite of $[M_s F]$: necessarily, whatever is an $[M_s F]$ is an F.

Definition 4 (Subsective, privative, modal, intersective modifiers). *Let g, g', g'', g''' range over $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; let g'''^{*} range over $(o\iota)_{\tau\omega}$; let x range over ι; $F/(o\iota)_{\tau\omega}$; $\exists/(o(o\iota))$; $\wedge/(ooo)$; $\neg/(oo)$. Then:*

Subsective w.r.t. F: $\lambda g[Req\ F[g\ F]]$
Privative w.r.t. F: $\lambda g'[Req[\lambda w \lambda t \lambda x \neg[F_{wt}x]][g'\ F]]$
Modal w.r.t. F: $\lambda g''[Req[\lambda w \lambda t[\lambda x[[\exists \lambda w'[[[\exists \lambda t'[[[M_m\ F]_{wt}\ x] \supset [F_{w't'}\ x]]]] \wedge [\exists \lambda w''[\exists \lambda t''[[[M_m\ F]_{wt}x] \supset \neg[F_{w''t''}x]]]]]]][g''\ F]]]]$
Intersective w.r.t. F: $\lambda g'''[Req[\lambda w \lambda t \lambda x[[g'''^{*}_{wt}x] \wedge [F_{wt}x]]][g'''\ F]]$.

A modal modifier behaves with respect to one and the same property F as subsective at one $\langle w',t'\rangle$ and as privative at another $\langle w'',t''\rangle$. No other modifier has the feature that its status (here, subsective vs. privative) depends on the given $\langle w,t\rangle$ of evaluation. The definition of modal modifiers defines the set of modifiers g'' that are modal with respect to F, such that if a is a $[g''\ F]$ at $\langle w,t\rangle$ then at $\langle w,t\rangle$, possibly, a is an F and, possibly, a is not an F. Put differently, whenever a has the property $[g''\ F]$ then a also has the property of being such that at one $\langle w',t'\rangle$ a is an F and at another $\langle w'',t''\rangle$ a is not an F. To compare subsective, privative and modal modifiers, every $\langle w,t\rangle$ is such that if x is an $[M_s F]$ then x is an F, and if x is an $[M_p F]$ then x is not an F. Furthermore, every $\langle w,t\rangle$ is such that if x is an $[M_m F]$ then it is possible that x be an F and it is possible that x not be an F. This last inference does not apply to $[M_s F]\ x$ or $[M_p F]\ x$. If x is an $[M_s F]$ then it is not possible that x not be an F (for it is necessary that x be an F). And if x is an $[M_p F]$ then it is not possible that x be an F (for it is necessary that x not be an F).

[19] See [5, Ch. 4, esp. §4.1] and [9, §2.5ff] on the notion of requisite. *Privative modifier* was also defined by means of requisites in [20, §4.2].

2.2 TIL: *Alleged*

To enable a direct comparison between TIL and CTT, TIL will also state an introduction rule and an elimination rule. The *definition* of modal modifiers, however, is the one provided in terms of requisites in Def.4.

What is required to acquire the property of being an alleged assassin? That somebody performs the *speech act* of alleging that a is an assassin.[20] Let f range over $(o\iota)_{\tau\omega}$; $Alleges/((o\iota o_{\tau\omega})_{\tau\omega})$: a relation-in-intension between individuals and propositions they allege to be true.[21]

Then the *introduction rule* for *Alleged* is

$$\frac{\exists\lambda x[Alleges_{wt}\ x\lambda w\lambda t[f_{wt}\ a]]}{[[Alleged\ f]_{wt}\ a]}$$

Gloss: "If somebody alleges that a is an f then a is an alleged f."

In fact, the set of properties f such that somebody alleges that a has f is identical to the set of properties f such that a is an alleged f:

$$\lambda f[\exists\lambda x[Alleges_{wt}\ x\ \lambda w\lambda t[f_{wt}\ a]]] = \lambda f[[Alleged\ f]_{wt}\ a]$$

However, it is not obvious how to generalize from this particular introduction rule to an introduction rule for any modal modifier. Sometimes a speech act is required, and sometimes an attitude, and sometimes something else. For instance, it is not obvious what the introduction rule for *Possible* would be, as soon as we want more than *ab esse ad posse*. As a hypothesis, however, I propose this general type-theoretic pattern underlying an introduction rule: where the premise has an object of type $(o\iota o_{\tau\omega})_{\tau\omega}$ the conclusion must have an object of type $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$.

The *elimination* rule for M_m can be stated in full generality, though. From Definition 2 we obtain the following rule, f ranging over properties:

$$\frac{[M_m f]_{wt}\ x]}{\exists\lambda w'[\exists\lambda t'[[[M_m\ f]_{wt}\ x] \supset [f_{w't'}\ x]]] \wedge \exists\lambda w''[\exists\lambda t''[[[M_m f]_{wt}\ x] \supset \neg[f_{w''t''}\ x]]]}$$

[20] The introduction rule assumes that any existential presupposition pertaining to the premise should be satisfied. If it is true that x alleges that the King of France is bald then it will be neither true nor false that the King of France has the property of being alleged to be bald, for there is currently no King of France around to instantiate that property. Thus the rule would not take a truth to a truth and so be invalid. We are suppressing the issues of existential presupposition, partiality, and truth-value gaps to keep the basic exposition as simple as possible.

[21] This typing is another simplification. TIL would tend to type *Alleges* as an empirical relation to a *hyperproposition*. But in this paper we have not introduced the ramified type hierarchy required to type hyperintensions. The simplification saves us from having to explain the descent from a hyperproposition that has been alleged to the proposition it constructs.

Gloss: "From x being an $[M_m\ f]$ at $\langle w,t\rangle$, infer that there is a $\langle w',t'\rangle$ such that if x is an $[M_m\ f]$ at $\langle w,t\rangle$ then x is an f at $\langle w',t'\rangle$ and that there is a different $\langle w'',t''\rangle$ such that if x is an $[M_m\ f]$ at $\langle w,t\rangle$ then x is not an f at $\langle w'',t''\rangle$."

Absolute elimination of M_m in the conclusion is impossible due to the oscillation between subsection and privation, so the rule must be restricted to conditional elimination.

The set-theoretic counterpart of modal modification is the *union* of two disjoint sets. In the example of *alleged assassin*, the relevant union is the union of the set of assassins at $\langle w',t'\rangle$ and the set of non-assassins at $\langle w'',t''\rangle$. It is logically trivial that an alleged assassin, just like any other individual, is a member of that union, but it is not epistemically trivial which of its two subsets a given alleged assassin, or any other individual, belongs to.

2.3 TIL: *Allegedly*

In [5, p.506] it is argued that *Allegedly*, as referred to in "Allegedly, a is an assassin", must be a *propositional property* – of type $(oo_{\tau\omega})_{\tau\omega}$ – rather than a *propositional modifier*, of type $(o_{\tau\omega}\ o_{\tau\omega})$. The argument is that a propositional property can, as a propositional modifier cannot, preserve the contingency of the proposition denoted by "Allegedly, ...". This argument is misconceived. The adverb 'allegedly' may well be analyzed as denoting a propositional modifier.

The reason is straightforward. The meaning of the above sentence is

$$[\mathit{Allegedly}\ \lambda w\lambda t[\mathit{Assassin}_{wt}\ a]]$$

The result of applying *Allegedly* to the proposition constructed by $\lambda w\lambda t[\mathit{Assassin}_{wt}\ a]$ is another proposition, which is true at all those $\langle w,t\rangle$ where it is alleged that a is an assassin. The proposition constructed by $[\mathit{Allegedly}\ \lambda w\lambda t[\mathit{Assassin}_{wt}\ a]]$ is as contingent as anything. Where p ranges over propositions, the *introduction rule* for *Allegedly* is

$$\frac{\exists\lambda x[\mathit{Alleges}_{wt}x\ p]}{[\mathit{Allegedly}\ p]_{wt}}$$

Gloss: "If somebody alleges that p, then p is alleged (is allegedly true)."

Let M'_m be a propositional modifier. Then the *elimination rule* for M'_m can be stated in full generality:

$$\frac{[M'_m\ p]_{wt}}{\exists\lambda w'[\exists\lambda t'[[M'_m\ p]_{wt}\supset p_{w't'}\ x]]\wedge\exists\lambda w''[\exists\lambda t''[[M'_m\ p]_{wt}\supset\neg p_{w''t''}]]}$$

Gloss: "From p being modified by $[M'_m]$ at $\langle w,t\rangle$, infer that there is a $\langle w',t'\rangle$ such that if p is modified by $[M'_m]$ at $\langle w,t\rangle$ then p is true at $\langle w',t'\rangle$ and that there is a different $\langle w'',t''\rangle$ such that if p is modified by $[M'_m]$ at $\langle w,t\rangle$ then $\neg p$ is true at $\langle w'',t''\rangle$."

As with *Alleged*, the open question is whether the $\langle w, t\rangle$ of the premise is the $\langle w', t'\rangle$ or else the $\langle w'', t''\rangle$ of the conclusion. This is simply to say that an allegation may, or may not, be true. It is readily seen from the respective introduction and elimination rules for *Alleged* and *Allegedly* that the Composition

$$[\mathit{Allegedly}\ \lambda w \lambda t[F_{wt}\ a]]$$

and the Closure

$$\lambda w \lambda t[[\mathit{Alleged}\ F]_{wt}\ a]$$

are equivalent, but not synonymous, constructions of the same proposition. Those two different sentential meanings converge in the same truth-condition.[22]

3 CTT on modal modification

In the spirit of anti-realist semantics, a constructivist understanding of modal modification can be given by directly representing the conditions for the assertion of a modified judgement. Accordingly, we shall consider a modal modifier M_m as an operator applying to a predication Fa, where a property F is predicated of an individual a. The predication of a modally modified property will be abbreviated as '$M_m[Fa]$'. Notice the shift of position from the previously considered '$[M_m F]\ a$', a shift motivated by the application of the modifier to the predication as a whole, leading in the following to our analysis of *Alleged* as *Allegedly*.

In Section 1 we stressed that M_m oscillates between being privative and being subsective, thereby making it impossible to infer which of Fa and $\neg Fa$ holds when the premise is $M_m[Fa]$. Subsection for CTT is standardly given by proper subset formation; privative modification, as introduced in [20], is defined as a mapping to the empty set; the oscillation of modal modification between the two corresponds precisely to the contingent satisfaction of either the maximal proper subset satisfying the property involved by the predication, or of the contradictory construction. The only constructive way to understand this oscillation is to formulate Fa's truth value in terms of *contingently* satisfiable conditions, thus leaving open the question of which conditions are *actually* satisfied. For a constructivist semantics, where truth is defined by constructors and refutations are given by implication to contradiction, this is no obvious task. On such a strong understanding of proven and refuted contents, no space is left for a formal representation of *contingent* truths.

A way to formulate assertion conditions for contingent truths is offered by the formal distinction between *refuted contents* and *missing constructions*. This was

---

[22] See [5, def. 2.3, p.154] for a definition of the individuation of meaning in terms of *procedural isomorphism*. In [4] we present a neo-Churchian Alternative $\frac{1}{2}$ that defines procedural isomorphism in terms of α- and η-conversion and an Alternative $\frac{3}{4}$ that adds a rule of restricted β-conversion.

already exploited in [12], where classical formulas were reduced to intuitionistic 'pseudo-truths' by double negation, the implication from $\neg\neg Fa$ to Fa being valid in a finite domain. In the same vein, the meaning of a valid judgement Fa *true* justifies the further conclusion that no construction for $\neg Fa$ *true* is possible. Then, a weaker form of predication is justified by inferring from $\neg\neg Fa$ the new judgement Fx, where the constructor x stands for a *refutable assumption*, corresponding to a place-holder for a (yet) missing, though admissible, construction of truth. In other words: no proof that Fa is a valid predication has been given; but, provided no refutation has been performed either, one such assumption can be made, up to refutation or confirmation being provided. We shall further explore this direction in order to provide appropriate rules for modal modification in a constructivist setting.

3.1 Modal types for contingent truth

We rely on the previous work [18] for the complete presentation of a type theory with refutable assumptions. We shall consider briefly the modal fragment of that language, which is here adapted for the interpretation of modal modifiers. In the following, we shall revert to the standard type-theoretic notation that expresses a predication Fa in the form of an object of type $a:F$.

Let us start by considering a polymorphism of both types and constructors: we have one kind of expressions F *type*, by which we collect propositions *justified* by appropriate verifications $a, b, \ldots$; another kind of expressions F $type_{inf}$ collects propositions which are *assumed* to be true, as their constructions are not refuted and only their negation is not yet refuted. For these expressions a constructor is thus a variable $x, y, \ldots$, induced from a judgement $\neg(F \rightarrow \bot)$. Judgements of the first sort induce a constructive notion of truth (*true*); the second ones, a weaker predicate of *contingent* truth ($true^*$). Identity of terms holds within *type*, and its constructors are composed in the standard manner by way of listing, application, abstraction and pairing to define connectives (conjunction and implication) and quantifiers. Conversion rules are defined over terms of the $type_{inf}$ fragment, β-reduction of $type_{inf}$ terms to corresponding *type* terms (evaluation) and α-term equality.

Respecting the usual convention of distinguishing true from valid assumptions, we shall refer to a set of valid assumptions Δ as a set of constructions of the *type* kind used to infer another construction; a set of true assumptions Γ is a set of constructions of the kind $type_{inf}$, used to infer another construction. Contextual judgements are thus built by derivability from judgements $\Gamma = \{F_1\ true^*, \ldots, F_n\ true^*\}$, which establishes the truth of F under refutable assumptions $F_1, \ldots, F_n$. When those $F_1, \ldots, F_n$ are fully verified (computationally, by reducing them by β-conversion to appropriate terms in *type*) the validity of F *true* is established. Derivability from refutable assumptions defines a truth predicate at a particular stage, depending on possible further states of knowledge. Derivability from valid assumptions expresses validity preserved under any further condition. In this way, we have extended the usual conceptual description of types in terms of two semantic notions of truth and derivability, respectively, for provable and refutable contents. To preserve this distinction two start rules are defined:

$$\frac{}{\Gamma, a\!:\!F, \Delta \vdash F\ true}\ \text{Premise Rule}$$

$$\frac{}{\Gamma, x\!:\!F, \Delta \vdash F\ true^*}\ \text{Hypothesis Rule}$$

The premise rule allows us to derive explicitly verified contents. The hypothesis rule reflects the derivation of contents that are only assumed to be true. The *true* predicate can be understood as validity (that is, truth in every situation) and it corresponds to truth by verification, whereas the predicate $true^*$ corresponds to truth in a context of (true) assumptions.

To express such a distinction in the object language of the type system, we extend our analysis to a modal language.[23] This also allows restoring a unique semantic predicate. The modal operators are informally introduced following the previous explanation of truth and validity: provided that the conditions for having the right to express a judgement are satisfied, the notion of judgemental necessity $\Box(F\ true)$ corresponds to that of an *apodictic judgement*: what is known to be thus and so cannot be known to be otherwise. The constructive interpretation identifies provability, truth and knowledge. The basic condition for the truth of Fa is thus the individual (construction, proof) a that makes F true ($a\!:\!F$); when F presupposes further types (propositions) to be valid (true), these represent the context in which F is formulated (instantiated, known). Conditions in such a context Γ can be seen as contextual or background knowledge. Hence, $\Box(F\ true)$ is knowledge for which no further contextual conditions are needed ($\Gamma = \emptyset$). The corresponding interpretation of a judgemental possibility operator starts from the propositional equivalence $\Box F \leftrightarrow \neg\Diamond\neg F$; this leads in [26] to the other equivalence: $\Diamond(F\ true) \Leftrightarrow \neg\Box(\neg F\ true)$. If conditions needed for the knowledge of $F\ true$ can all be satisfied only with Γ empty, then this formula reduces to the conditions for $\Box(F\ true)$. Otherwise, truth is preserved in certain knowledge states in which appropriate conditions $\Gamma = (F_1\ true^*, \ldots, F_n\ true^*), n \geq 1$ are formulated. The latter amounts in our language to $x_i\!:\!F_i$ as a condition for $F\ true$. Hence, we infer modalities directly from our polymorphic constructors:

$$\frac{a\!:\!F}{\Box(F\ true)}\ \Box\text{-formation} \qquad \frac{x\!:\!F}{\Diamond(F\ true)}\ \Diamond\text{-formation}$$

The inference to the truth of contextual judgements requires generalization to contextual formulae:

Definition 5 (Necessitation Context). *For any context Γ, $\Box\Gamma$ is given by $\bigcup\{\Box F_i\ true \mid \text{ for all } F_i \in \Gamma\}$.*

Definition 6 (Normal Context). *For any context Γ, $\Diamond\Gamma$ is given by $\bigcup\{\circ F_i\ true \mid \circ = \{\Box, \Diamond\}$ and $\Diamond F_i\ true$ for at least one $F_i \in \Gamma\}$.*

[23] For more on the following explanation of epistemic modalities, see [17].

Then a judgement valid under assumptions becomes a possibility judgement if its context remains normal, that is, at least one its assumptions is $true^*$.[24] For the use of contingent truths as the key to modelling modal modification, we are interested here in the rule that characterizes the use of Normal Contexts. Local validity (or derivability under true assumptions) is defined by introduction and elimination rules for the $\Diamond$-operator:[25]

$$\frac{\Gamma, x_i : F_i \vdash F\ true^*}{\Box\Gamma, \Diamond(F_i\ true) \vdash \Diamond(F\ true)}\ \text{I-}\Diamond$$

This rule has a corresponding elimination rule that returns the $true^*$ predicate from a $\Diamond(F\ true)$ judgement occurring in the second premise.[26] For current purposes, we formulate the elimination rule to generate explicitly two predications resulting, respectively, from verifying or refuting the conditions:

$$\frac{\Box\Gamma, \Diamond(F_i\ true) \vdash \Diamond(F\ true) \qquad [x_i/a_i] : F_i}{\Box\Gamma, \Box(F_i\ true) \vdash \Box(F\ true)}\ \text{E-}\Diamond(1)$$

$$\frac{\Box\Gamma, \Diamond(F_i\ true) \vdash \Diamond(F\ true) \qquad F_i \rightarrow \bot}{\Box\Gamma, \Box(\neg(F_i\ true)) \vdash \Box(\neg(F\ true))}\ \text{E-}\Diamond(2)$$

3.2 CTT: ***Alleged*** and ***Allegedly***

The basic idea informing our simulation of a modal modifier is to interpret it as a modal operator producing a non-terminating set of terms to provide assertion conditions for a *contingent* truth. Our judgemental $\Diamond$ operator and its introduction and elimination rules regulate precisely such epistemic conditions.

Let us start by reconsidering the example "a is an assassin". We start with a an individual constructor in the kind *type*, and F the property (*assassin*) predicated of a. A valid predication Fa corresponds, in our language, to a judgement of the form $a : F$, expressing the (proven) fact that there is an individual a who is an assassin:[27]

$[Fa]$: "(It is true that) a is an assassin"

[24] *Necessitation* and *Normal Context* are equivalent to *Global* and *Local Context* as known from the literature in modal logic. Cf. [6].

[25] Structural rules such as Weakening, Contraction, Exchange hold in the form of theorems; also, a rule of substitution for truth predicates and terms can be proved, plus the local inversion of these modal rules with the appropriate $\Box$-counterparts, corresponding to their soundness and completeness. See [18].

[26] See [18].

[27] This would, ideally, be the individual a caught in the act of killing someone. As explained below, this rather unrealistic representation is replaced by the requirement that the true predication "John is an assassin" satisfies all the conditions that make John an assassin.

with the truth predicate hidden by the formalism. It is crucial for our construction to unveil the nature of such a truth predicate, i.e. to establish which rules it obeys. When M is a modal modifier like *Alleged*, $[MF]\ a$ should be unpacked as

$[MF]\ a$: "(It is true that) a is an alleged assassin"

Our claim is that the modal modifier M over F applied to a can be simulated in terms of our I-$\Diamond$ rule by analytically defining the conditions under which the truth of Fa is asserted. The meaning of $[MF]\ a$ is in turn equivalent to $\Diamond(F\ true)$.

To get started, let us note the following. The constructivist epistemology underlying the present interpretation of modal modifiers rests entirely on the *perspectivalist* view of performing acts of judging propositions, according to which acts of judging (hence of knowing) are always acts performed from within a *first-person* perspective by an epistemic subject in an appropriate knowing context.[28] This means that judgements ground speech acts, which may remain implicit. Hence, the above-mentioned reading of *Alleged* readily transforms into the following:

$M[Fa]$: "Allegedly, (it is true that) a is an assassin"

In fact, where the apodictic form of judgement expressed by "a is an assassin" is grounded in a proof object independent of unsatisfied conditions, the modally modified judgement expressed by "Allegedly, a is an assassin" is grounded in a proof object dependent on refutable conditions, as explained above.

We thus exploit the nature of the predication as dependently defined. Every assertion can be formulated as depending on (possibly implicit) conditions. The most obvious conditions can be made evident in terms of an analytic deconstruction of the predicate, as e.g. by saying that "John is an assassin, provided it is true that he killed a human being on purpose". Besides this analytical form, a judgement can be turned into a dependent one by referring to conditions dictated by the *perspective* from which the act of judging occurs. As an example, let us assume that the proposition expressed by "John is an assassin" is asserted from within the perspective of a legal system where to be convicted as an assassin requires that the individual has been found guilty by the lowest to the highest courts. Then, an obvious formulation of our dependency relation would be of the form: "John is an assassin, provided it is true that he has been found guilty of killing a human being by each of the required courts". The list of conditions can be further modified by adding, for example, "under no mitigating circumstances".[29] If and when such conditions are proved to hold, we shall declare

[28] On the perspectivalist epistemology underlying Martin-Löf's Type Theory, see e.g. [23]. For the use of such an approach in the epistemological debate on information and knowledge, see [19].

[29] The perspective can be easily changed so that also conditions change. For example, for someone who considers hunting an act of violence, the following might hold: "John

John an assassin. The modified form, predicating of John that he is an alleged assassin, holds in so far as it cannot be decided that all the conditions that make John an assassin hold.

In fact, $\Diamond(F\ true)$ expresses precisely the impossibility of reducing the constructor for F to *type*. This means that at least one of the assumptions under which $a\!:\!F$ is constructed remains unverified, hence making it impossible to assert that a has property F. Then, one cannot judge either Fa or $\neg Fa$ to be true. Notice that our $\Diamond$ rules prevent ill-behaving inferences. In particular, it is not possible to infer that John has allegedly killed someone from the fact that John is not an assassin: the latter has its own set of (satisfied, valid) assumptions, falsifying the open conditions for being an *alleged assassin*. It is also impossible to infer that John is an alleged assassin from *not knowing* that John has killed someone: the latter means that there is no predication at all with respect to $a\!:\!F$, hence also no account of the conditions for considering its truth. As a result, no judgement candidate has been laid down for acceptance or refutation.

The validity of the dependent judgement corresponds to the reduction of the construction to the *type* fragment and hence of its assertion conditions to $\Box\Gamma$, so as to finally execute an inference $\Box\Gamma \vdash \Box(F\ true)$. To do so, we require that none of the conditions under which $F\ true$ holds be falsifiable. The construction $\Diamond(F\ true)$ expresses instead the validity of $type_{inf}$ for at least one condition which does not reduce.

$M[Fa]$ is the modally modified predication of F of a. It is constructively expressed as a function $M(x)[x\!:\!F]$, saying that for at least one $F[x_i\!:\!F_i]$ it holds that $F_i\ type_{inf}$, and hence $F\ true^*$.[30] To preserve the functional aspect of M in the constructive notation, we will refer to $M(x)[x:F]$ as the type satisfied by some $f\!:\!F$ modified by having a judgement of the form $f\!:\!F$, for which at least one $f_i\!:\!F_i$ cannot be shown to reduce:

$$\frac{F\ type[\Gamma] \qquad F_i\ type_{inf} \in \Gamma \qquad M(x)[x\!:\!F]}{\Box\Gamma, (x_i\!:\!F_i)f\!:\!F \vdash F\ type[(x_i(f))(f_i)\!:\!F_i]}\ \text{Modal Modification}$$

Gloss: "Let there be an object type F that is satisfied, provided that all the object types in $\Gamma = \{F_1, \ldots, F_n\}$ have appropriate type constructors; let it be the case that for $F_i \in \Gamma$ a constructor is admissible but no reduction is provided, so that $F_i\ type_{inf}$ holds; then it is the case that, provided all the constructors in Γ apart from (at least) F_i are satisfied, a modifier M holds for F such that F is an object *type* if and only if F_i has an appropriate β-contractum, and it does not hold if F_i does not reduce." This rule is nothing other than an analytic definition of $type_{inf}$, inducing immediately the judgement $\Diamond(F\ true)$.

is an assassin, for he killed a beast". Our concern is here only to assess the role of dependent judging in the formulation of a construction for modal modifiers. Hence we shall restrict ourselves to the more evident and less problematic formulation of such meanings.

[30] Because M applies to an a already predicated in F, CTT has no need for a counterpart of pseudo-detachment or any other rule of left subsectivity.

Three remarks are in place to appreciate why such a construction qualifies as one of a modal modifier:

1. the modal operator expresses the separation between terminating and non-terminating terms, a property which is not available in the standard format of CTT; by presenting the constructor $\Box\Gamma, (x_i : F_i) f : F \vdash F\ type[(x_i(f))(f_i) : F_i]$, we refer to a term f that is modified by the missing reduction for a term f_i on which it depends;
2. given the admissibility of f_i, this construction simulates a derivability relation that satisfies the *tertium non datur* of the non-modal alternatives $Fa \vee \neg Fa$, though the language does not allow its formal derivability from $\Diamond(F\ true)$. The appropriate way of expressing the meaning of a modal modifier such as *Alleged* in a sentence like "(It is true that) John is an alleged assassin" is to say: "(It is true that) John is an assassin if x is true" or "(It is false that) John is an assassin if x is not true";
3. the resolution of the contingency of the modal predication is possible by one of the two elimination rules.

4 Conclusion

The uniquely defining feature that any theory of modal modifiers must accommodate is their oscillation between subsection and privation. It is relative to a particular context of evaluation whether a particular modal modifier is subsective or else privative. No other modifier – be it subsective, privative or intersective – shares this feature of context-sensitivity.

We suggested above an intimate connection between modal modifiers and non-factive attitudes. From b knowing whether a is an F it follows only that it is possible that a be an F and that it is possible that a not be an F. If a is an alleged assassin, the same two possibilites hold. We worked out an account of modal modifiers in two different directions. Within TIL we worked out an account of alethic possibility. The truth of a being an alleged assassin is logically compatible with one of two possible states-of-affairs obtaining: a being an assassin, a failing to be an assassin. Within CTT we worked out an account of epistemic possibility. The knowledge that a is an alleged assassin is compatible with either of two possible pieces of knowledge: knowing that a is an assassin, knowing that a is not an assassin. Neither has been refuted or verified.

The TIL definition of modal modifiers M_m says that, necessarily, if x has the property $[M_m\ F]$ at $\langle w, t\rangle$ then, possibly, x is an F and, possibly, x is not an F. Possibly, there is a $\langle w', t'\rangle$ such that if x is an $[M_m F]$ then x is an F, and possibly, there is an alternative $\langle w'', t''\rangle$ such that if x is an $[M_m F]$ then x fails to be an F. It falls to empirical inquiry to establish whether $\langle w, t\rangle$ is like $\langle w', t'\rangle$ or like $\langle w'', t''\rangle$. The meaning of an adjective denoting a modal modifier is a procedure whose product is a mapping from properties to properties. From the definition of modal modifiers we obtained a conditional elimination rule for M_m. We also provided an introduction rule for *Alleged*, while pointing out that it may not generalize to all other modal modifiers.

The CTT definition of modal modifiers M_m is given in terms of rules for a modal operator $\Diamond$ that applies to a judgement of the form $(F\ true)$. The introduction rule spells out how to form such a judgement $\Diamond(F\ true)$ from laying down its assertion conditions, which are neither verified nor refuted. The elimination can be given in one of two forms: by verifying or by refuting conditions. The rule of Modal Modification expresses this dependency from open assumptions in the form of a function from constructions to constructions.

It follows readily from the TIL introduction and elimination rules for the property modifier M_m that the sentence "a is an alleged assassin" is equivalent with the sentence "Allegedly, a is an assassin", *Allegedly* a propositional modifier. Those are not synonymous sentences, however, since their respective meanings are two different procedures that produce the same truth-condition (possible-world proposition). In CTT the relationship between those two sentences is synonymy, because one is the logical analysis of the other. The logical form of "a is an alleged assassin" is, in the final analysis, $M_m[Fa]$ and not $[M_m\ F]a$. $M_m[Fa]$ is the modally modified judgement that the proposition that a is an F is true. This judgement can be made, defeasibly, as long as it has not yet been established whether a is an assassin.

Acknowledgements

The authors wish to thank Marie Duží and Petr Šimon for helpful comments and remarks. The two authors of this paper and of [20] contributed equally to both papers. Giuseppe Primiero wishes to acknowledge the support of the Isaac Newton Institute for Mathematical Science, Cambridge (UK), where he was a Visiting Fellow for the scientific programme *Semantics and Syntax: a Legacy of Alan Turing* during the final drafting of this paper.

References

1. Beesley, K.R. Evaluative adjectives as one-place predicates in Montague Grammar. *Journal of Semantics*, vol. 1, pp. 195-249, 1982.
2. Carrara, M. and B. Jespersen. Double privation. In submission.
3. Cresswell, M. Prepositions and points of view. *Linguistics and Philosophy*, vol. 2, pp. 1-41, 1978.
4. Duží, M. and B. Jespersen. Procedural isomorphism and restricted β-conversion by value. Revised and resubmitted.
5. Duží, M., Jespersen, B. and P. Materna. *Procedural Semantics for Hyperintensional Logic. Foundations and Applications of Transparent Intensional Logic.* Logic, Epistemology and the Unity of Science, vol.17. Springer, Berlin (2010)
6. Fitting, M. Basic modal logic. In vol. 4 of *Handbook of Logic in Artificial Intelligence and Logic Programming*, pp.368-449. Oxford University Press, 1994.
7. Jespersen, B. Left subsectivity. In submission.
8. Jespersen, B. and M. Carrara. Two conceptions of technical malfunction. *Theoria*, vol. 77, pp. 117-38, 2011.

9. Jespersen, B. and M. Carrara. A new logic of technical malfunction. *Studia Logica*. DOI:10.1007/s11225 -012-9397-8.
10. Kamp, H. Two theories of adjectives. In Keenan, E. (ed.), *Formal Semantics of Natural Language*, pp. 123-155. Cambridge: Cambridge University Press, 1975.
11. Kamp, H. and B. Partee. Prototype theory and compositionality. *Cognition*, vol. 57, pp. 129–191, 1995.
12. Kolmogorov, A. On the principle of excluded middle. In J. van Heijenoort (ed.), *From Frege to Gödel: a Source Book in Mathematical Logic 1879-1931*, pp. 414–437. Harvard University Press, 1967.
13. Montague, R.. English as a formal language. In B. Visentini et al. (eds.), *Linguaggi nella societá e nella tecnica*. Milan, pp. 189-224, 1970. Reprinted in R.H. Thomasson (ed.), *Formal Philosophy*. New Haven, London: Yale University Press, 1974.
14. Partee, B. Privative adjectives: subsective plus coercion. In R. Bäuerle, U. Reyle, and Thomas E. Zimmermann (eds.), *Presuppositions and Discourse*, Amsterdam: Elsevier, 2001. `http://people.umass.edu/partee/docs/ParteeInPressKampFest.pdf`.
15. Partee, B. Compositionality and coercion in semantics: the dynamics of adjective meaning. In Gerlof Bouma et al. (eds.), *Cognitive Foundations of Interpretation*, pp. 145-161. Amsterdam: Royal Netherlands Academy of Arts and Sciences. `https://udrive.oit.umass.edu/partee/Partee2007CompositionalityCoercionAdjs.pdf`
16. Pfenning, F. and R. Davies. A judgemental reconstruction of modal logic. *Mathematical Structures in Computer Science*, vol. 11, pp. 511–540, 2001.
17. Primiero, G. Epistemic modalities. in G. Primiero, S. Rahman (eds.), *Acts of Knowledge - History, Philosophy and Logic*, pp.207-232. College Publications, London, 2009,
18. Primiero, G. Constructive contextual modal judgements for reasoning from open assumptions. *Logique & Analyse*, vol. 220, 2012.
19. Primiero, G. Offline and online data: on upgrading functional information to knowledge. *Philosophical Studies*, 2012. DOI: 10.1007/s11098-012-9860-4.
20. Primiero, G. and B. Jespersen. Two kinds of procedural semantics for privative modification. *Lecture Notes in Artificial Intelligence*, vol. 6284, pp. 252-271, 2010.
21. Primiero, G. and M. Taddeo. A modal type theory for formalizing trusted communications. *Journal of Applied Logic*, vol. 10, issue 1, pp. 92-114, 2012.
22. Rotstein, C. and Y. Winter. Total adjectives vs. partial adjectives: scale structure and higher-order modifiers. *Natural Language Semantics*, 12:259–88, 2004.
23. Schaar, M. v.d. The cognitive act and the first-person perspective: an epistemology for constructive type theory. *Synthese*, vol. 180 (3), pp. 391-417, 2011.
24. Schaar, M. v.d. Wooden horses and false friends; on the logic of adjectives. To appear.
25. Sørensen, M.H. and Urzyczyn, P. *Lectures on the Curry-Howard Isomorphism*. Studies in Logic and the Foundations of Mathematics, vol. 149. Elsevier, 2006.
26. Sundholm, B.G. Mind your P's and Q's. On the proper interpretation of modal logic. In T. Childers, O. Majer (eds.), *The Logica 2002 Yearbook*, pp. 101–111. Filosofia, Czech Academy of Science, 2003.
27. Winter, Y. Cross-categorical restrictions on measure phrase modification. *Linguistics and Philosophy*, vol. 28, pp.233-267, 2005.