Propensities and conditional probabilities

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Abstract

The present paper deals with the objection that Paul Humphreys raised against the propensity interpretation of probability – "Humphreys' paradox". An update on existing solutions is offered, and it is concluded that none of them is completely satisfactory in view of Humphreys' 2004 rejoinder. Positively, an original solution is formulated and discussed.

Keywords: conditionalization, propensity, Humphreys' paradox

Introduction

Since the calculus of probability was axiomatized, the question of how probabilities should be interpreted has been central to philosophy of probability. This question can be understood as follows: what do probabilities measure? According to the propensity answer that Popper introduced in the 1950s[1], probabilities measure propensities tending to produce possible singular events. These propensities are features of the physical world, and so are (therefore) probabilities of singular events under the propensity interpretation. This was the main motivation for the introduction of the propensity theory, and it remains the main appeal of the proposal. Propensities are features of the physical world, but more precisely they depend on a set of physical conditions. For example, the propensity tending to produce the occurrence of *six* on next throw of a given die depends on the physical properties of the throwing device (that is, mainly the physical properties of the die itself and of the surface on which it is to be thrown.) Therefore the probability of *six* on next throw of the die also depends on this set of physical conditions.

The propensity theory of probability has been confronted with many criticisms. Among them, the most robust as well as fundamental one is undoubtedly "Humphreys' paradox". According to this criticism, the propensity theory may well be an interpretation of absolute probabilities, yet it cannot serve as an interpretation of conditional probabilities. In other words, there cannot be a propensity interpretation of conditional probabilities. Two important consequences ensue. First, one has to give up the idea of a correspondence between subjective and physical probabilities that would conform to Lewis' Principal Principle.[2] Indeed, rational degrees of conditional belief provably are conditional probabilities[3] and physical conditional probabilities should be too if the

[1] [15] and [16].
[2] [8].
[3] [20].

Principal Principle were to be retained. Second, one has to give up the idea that probability theory as we know it is a theory of all aleatory events – of whatever kind they may be.

Against Humphreys, several authors have proposed propensity interpretations of conditional probabilities. However, none of them is commonly accepted as successfully overriding the difficulty. On the other hand, it is not clear whether all currently existing proposals fall under Humphreys' paradox as recently restated[4]. In this context, the present paper aims both at giving an update on Humphreys' paradox and the debate surrounding it, and at determining whether and along which lines it could be solved.

Humphreys' paradox is presented in section 1 and existing suggestions for a propensity interpretation of conditional probabilities are reviewed in section 2. It is concluded that one of these suggestions may be accepted although it is hardly intelligible. Consequently, the end of the paper is devoted to produce an interpretation that is both acceptable and undoubtedly intelligible. The originality and interest of the formulated proposal lie at least as much in the constructivist approach to its formulation as in the precise way it is specified. To be a little bit more precise, the solution is built out of an analysis of what it means to interpret conditional probabilities. This analysis is provided in section 3. The proposed propensity interpretation of conditional probabilities is formulated in section 4 and it is discussed in section 5.

1 Humphreys' paradox

Starting in the 1980s, Humphreys' paradox has been exposed in a variety of ways. First, Humphreys' seminal paper[5] offers an informal as well as a formal version of the paradox. Second, subsequent discussions of Humphreys' paper gave rise to many reformulations of the difficulty, both in formal and informal forms. In this opening section, I will stick to formal versions of the paradox because, as Humphreys points out, "for those, a satisfactory solution is required"[6]. Moreover, I will focus on official versions: first the paradox as exposed in [5] and second the generalization it is given in [6]. This implies presenting the example on which Humphreys' analysis runs.

Both Humphreys' original argument and its extension in [6] depend on considering a particular physical system:

> A source of spontaneously emitted photons allows the particles to impinge upon the mirror, but the system is so arranged that not all the photons emitted from the source hit the mirror... Let I_{t2} be the event of a photon impinging upon the mirror at time $t2$, and let T_{t3} be the event of a photon being transmitted through the mirror at time $t3$ later than $t2$. Now consider the single-case conditional propensity $Pr_{t1}(.|.)$ where $t1$ is earlier than $t2$.[7]

[4] In [6].

[5] [5]. The paper is seminal in the sense that it has been the first published paper Humphreys devoted to his objection against the propensity interpretation of probability. Yet Humphreys already had the objection and the objection was already known among philosophers of science by the end of the 1970s. The name "Humphreys' paradox" was introduced in [3].

[6] [6] 668.

[7] [5] 561.

This description implies three (in)equalities involving probabilities relative to the system under consideration[8]:

1a. $Pr_{t1}(T_{t3}|I_{t2}) = p > 0$

1b. $1 > Pr_{t1}(I_{t2}|B_{t1}) = q > 0$

1c. $Pr_{t1}(T_{t3}|not - I_{t2}) = 0$.

Beside those (in)equalities, Humphreys claims that:

2. $Pr_{t1}(I_{t2}|T_{t3}) = Pr_{t1}(I_{t2}|not - T_{t3}) = Pr_{t1}(I_{t2})$.

Unlike 1a. – 1c., double equality 2. does not stem from the description of the photon-emitting system. Rather, it is a substantial hypothesis concerning the way some particular probabilites should be evaluated. More precisely, $Pr_{t1}(I_{t2}|T_{t3})$ has the specificity to be an inverse conditional probability, that is a conditional probability with conditioning event (T_{t3} in the case in point) *posterior to* conditioned event (I_{t2}). Correlatively, it is not clear how it should be evaluated when given a propensity interpretation. Indeed, propensities are causal-like entities, and therefore what they become in time-reversed contexts is problematic. Humphreys' answer concerning the evaluation of $Pr_{t1}(I_{t2}|T_{t3})$ can be seen as an instantiation of a more general principle for evaluating inverse conditional probabilities:

Principle 1 (CI) *If $Pr(A|B)$ is an inverse conditional probability given a propensity interpretation, then $Pr(A|B) = Pr(A|not - B) = Pr(A)$.*

Humphreys' justification for (CI) is that posterior events do not (at least normally, and in the situations that he addresses and that I also will address in the present text) influence prior events, and hence cannot modify the propensity for the system to produce them. In particular, "the propensity for a particle to impinge upon the mirror is unaffected by whether the particle is transmitted or not"[9].

Humphreys' justification for (CI) seems to be sound, since it essentially appeals to the acknowledged causal features of propensities. The matter is that (CI) turns out to be incompatible with usual properties of probabilities. More exactly, Humphreys derives two contradictions. The premises of the first one are: 1a. – 1c., 2. and the law of total probability, while the premises of the second ones are: 1a. – 1c., 2. and Bayes' theorem. Now, both the law of total probability and Bayes' theorem are fundamental results concerning standard conditional probabilities. Therefore, the existence of the two derivations just mentioned inescapably leads to conclude that the propensity theory is not an interpretation of probability theory considered as a theory of both absolute and conditional probabilities. "Propensities cannot be probabilities"[10], and this is the result known as "Humphreys' paradox".

Since 1a. – 1c. stem naturally from the description of the photon-emitting system considered by Humphreys, the only strategy out of the formal paradox consists in denying 2., which implies rejecting (CI). In [6], Humphreys identifies in the literature two principles that compete with (CI):

[8] [5] 561. For reasons that should become clear later on, my notations are slightly different from Humphreys' ones. Moreover, $not - E$ refers to the event of E not occurring.

[9] [5] 561.

[10] [5] 557.

Principle 2 (ZI) *If $Pr(A|B)$ is an inverse conditional probability given a propensity interpretation, then $Pr(A|B) = 0$*

and:

Principle 3 (FP) *If $Pr(A|B)$ is an inverse conditional probability given a propensity interpretation, then $Pr(A|B) = 1$ or 0.*

Principle (ZI) is supported in particular by Fetzer[11] as a consequence of posterior events (normally) having zero causal influence on prior events. Principle (FP) is most noticeably advocated in [13] and expresses the fact that at $t3$ posterior to $t2$, E_{t2} has definitely occurred or failed to occur: the inverse conditional probability takes value 1 in the first case and value 0 in the second one. It could be debated which one – if any – of (CI), (ZI) and (FP) is adequate in the propensity framework. However, the debate is useless as far as Humphreys' paradox is concerned. Indeed, it is shown in [6] that both (ZI) and (FP) lead to formal paradoxes analogous to the one concerning (CI). More precisely, the paradox then stems from some inverse conditional probabilities (all of them, actually, in the case of (ZI)) being given value 0 by principle. Humphreys' paradox is thus generalized.

At that point, the situation may look quite desperate. Still, from what has been stated hitherto I would like to draw two remarks that may eventually allow for some optimism. To begin, it must be emphasized that none of the proponents of (CI), (ZI) or (FP) explicitly raises the question of how conditional probabilities should be interpreted in the propensity framework. To put it another way, they implicitly agree on the idea that the propensity interpretation of absolute probabilities analytically contains an interpretation of conditional probabilities: they consider that what conditional probabilities measure stems from what absolute probabilities measure. But – and this is the first of my remarks – the very debate reveals that this assumption is false. Diverging principles for the evaluation of inverse conditional probabilities indeed rely on different conceptions of the way conditional probabilities should be interpreted in the propensity framework. More precisely, Humphreys' (CI) cannot be separated from the idea that $Pr(A|B)$ measures the propensity tending to realize A in as much as it is possibly physically modified by the occurrence of B; (ZI) presupposes that $Pr(A|B)$ measures the causal influence of B on A; and (FP) stems from the idea that this probability measures the propensity tending to realize A as it stands at the moment which is characteristic of B. Such a disagreement would not happen if it were true that the propensity theory of absolute probabilities analytically contains an interpretation of conditional probabilities. As a consequence, it should be considered an open question how conditional probabilities could be interpreted in a propensity framework.

In this context, what [5] and [6] show is only that a certain number of answers will not do because they lead to principles for the evaluation of inverse conditional probabilities that are incompatible with standard probability theory. Still, one must notice that Humphreys does not give a general argument against the very idea of producing a propensity interpretation of conditional probabilities. If the question of how conditional probabilities should be interpreted in a propensity framework is indeed open, [5] and [6] do not rule out

[11] [3].

the possibility of this question receiving a satisfactory answer. They do not even rule out the possibility that the question already received a satisfactory answer. Indeed, there currently exist proposals for the propensity interpretation of conditional probabilities that are not addressed by Humphreys – whether these proposals were formulated posterior to [6], or Humphreys does not take them into account properly. I will now turn to these proposals, and examine whether one of them provides a satisfactory answer to the open question of how conditional probabilities could be interpreted in the propensity framework.

2 Proposals not addressed by Humphreys

Examination of the proposals that are unaddressed in [6] aims primarily at determining whether one of them is satisfactory. Consequently, it will be useful to start with some precisions as to what it is for a proposed propensity interpretation of conditional probabilities to be satisfactory. Here, I will take it as uncontroversial that a necessary (but for sure not sufficient) condition for this is that the following two criteria are met:

- Interpretation: the proposal should be an interpretation of conditional probabilities, that is it should tell us what conditional probabilities measure;
- Admissibility[12]: the proposal should account for the standard properties of conditional probabilities.

[6] shows that a necessary condition for a proposal to satisfy Admissibility is that it does not lead to any of principles (CI), (ZI) or (FP) for the evaluation of inverse conditional probabilities. The proposals that are not addressed in this paper and that must be now confronted with Interpretation and Admissibility fall out into three types: co-production proposals, what I will call "ratio proposals", and Milne's conditional-event proposal.

2.1 Co-production proposals

The phrase "co-production interpretation" is introduced in [6][13], where it is defined as follows:

> A *co-production interpretation* considers the conditional propensity to be located in structural conditions present at an initial time t, with $Pr_t(.|.)$ being a propensity at t to produce the events which serve as the two arguments of the conditional propensity.[14]

Co-production proposals for the propensity interpretation of conditional probability are supposedly addressed by the generalized version of Humphreys' argument. However, it is my contention that they are not addressed *properly* by Humphreys. More precisely, my claim is that [6] does not succeed in establishing that co-production interpretations of conditional probabilities fail to satisfy Admissibility. I shall here start with a justification of this claim, and go on with an

[12]The term is introduced by Salmon as one of his criteria of adequacy for the interpretations of probability. See [19] 63–64.

[13][6] 671.

[14][6] 671.

examination of whether co-production proposals indeed meet the Interpretation and Admissibility requirements.

Two major advocates of a co-production interpretation of conditional probabilities are Christopher McCurdy (in [9]) and David Miller (in [10] and [11]). For three, non-independent, reasons, I will focus on McCurdy's proposal. First, [9] both most directly addresses the formal version of Humphreys' paradox that I am interested in, and offers the most elaborate version of a co-production proposal. Second, Humphreys' 2004 response to co-production proposals focuses on McCurdy's 1996 proposal. As for Miller, Humphreys essentially refers the reader to the criticism he has just raised at McCurdy.[15] Third, it cannot be questioned that [9] is central in the field of co-production positions concerning conditional propensities. In particular, Miller himself refers to [9] as "a paper that I am largely in agreement with (though it is suggested incorrectly on 106 that I regard propensities as fundamentally propensities to generate frequencies)"[16].

According to McCurdy,

> the "conditional" propensity $Pr_{t1}(T_{t3}|I_{t2}B_{t1})$ is interpreted as the propensity at $t1$ (for a system satisfying conditions B_{t1}) to produce a photon that is transmitted at $t3$ conditional upon its producing a photon that impinges upon the mirror at $t2$. On this account, a conditional propensity such as $Pr_{t1}(T_{t3}|I_{t2}B_{t1})$ is interpreted as the propensity at $t1$ for the system to produce the event I_{t2}, given that the event T_{t3} is also produced. [...] Furthermore, the values assigned to conditional and inverse conditional propensities are intended to provide a measure of the strength of the propensity for the system to produce the two future events in the manner specified.[17]

Let me ignore B_{t1} for a while, and state how the conception exposed by McCurdy is meant to solve Humphreys' paradox. Focusing on the photon-emitting device introduced by Humphreys, the solution essentially resides in McCurdy's claim that his views on conditional propensities lead to the conclusion that the inverse conditional probability $Pr_{t1}(I_{t2}|T_{t3})$ has value 1. In McCurdy's words:

> the value of $Pr_{t1}(I_{t2}|T_{t3}B_{t1})$ must be one since the description of the system indicates that the system is arranged in such a manner that if the system produces a photon that is transmitted at $t3$, then the system must also produce a photon that impinges upon the mirror at $t2$.[18]

This makes McCurdy's conception immune to Humphreys' original paradox and to its 2004 generalizations relying on some inverse conditional probabilities taking value 0 by principle. Before I turn to Humphreys' analysis of McCurdy's position, let me quote McCurdy's further explanation of the failure of (CI) for the photon example:

> the events I_{t2}, T_{t3}, and $not - T_{t3}$ share common causal factors that are effective between $t0$ and $t2$. Specifically, the photon transmission arrangement itself (described by B_{t1}) provides a host of common causal factors.

[15] [6] 677.
[16] [11] 111.
[17] [9] 109.
[18] [9] 110–111.

> This fact is responsible for the failure of principle (CI): if the system produces event T_{t3}, then it must have exhibited certain causal factors, some of which have an influence on event I_{t2} [and this influence, it is argued, is such that I_{t2} cannot have but occurred].[19]

In [6], Humphreys claims that McCurdy's application of his own analysis to the photon example is flawed, and that co-production proposals in fact lead to (CI). His argumentation goes along the following lines:

1. he analyzes McCurdy's mistake in terms of the photon example "misleadingly suggesting some quasi-deterministic aspects of the fundamentally indeterministic propensity at $t1$, $Pr_{t1}(I_{t2}|B_{t1})$"[20];
2. he introduces an alternative example involving radioactive decay as a source of indisputable indeterminism. This new example is analogous to the initial one, but the formal role of impingement of the emitted photon against the mirror is now played by an episode of radioactive decay;
3. from the indisputable indeterminism of the new example, he concludes that no argument involving common causal factors is available in that case and that (CI) becomes "evidently true"[21] – or, more rigorously, that it evidently applies in the new case;
4. from the fact that (CI) should apply in the newly introduced example, he infers that co-production proposals lead to (CI) as a general principle for the evaluation of inverse conditional probabilities.[22]

This line of argumentation suffers from two important weaknesses. First, Humphreys does not give any reason why (CI) being adequate for the new, indisputably indeterministic example should imply its being adequate for the old photon example – let alone its being correct as a general principle for the evaluation of inverse conditional probabilities. In particular, he does not explain why the difference between the "quasi-indeterministic aspects"[23] of the initial system and the "irreducibly indeterministic nature"[24] of the new one should not lead to different principles for the evaluation of inverse conditional propensities. Second, (CI) is not given a satisfactory justification even for the indisputably indeterministic case. More precisely, Humphreys does not give any new positive justification of (CI), but only argues that McCurdy's line of reasoning cannot be followed concerning the new example. Moreover, his justification for this relies on the claim that radioactive decay, being indisputably indeterministic, does not have any causes. But it is not at all clear that the envisaged system cannot count as a cause of radioactive decay, very much in the same way as "the photon transmission arrangement itself (described by B_{t1}) provides a host of [...] causal factors"[25] for I_{t2} and T_{t3} according to McCurdy. My conclusion, then, is that Humphreys fails to establish that co-production proposals lead to (CI). In other words, co-production proposals are not correctly addressed in

[19] [9] 116.
[20] [6] 674. For the sake of coherence, I slightly modify Humphreys' notations.
[21] [6] 675.
[22] [6] Table 1 677.
[23] [6] 674.
[24] [6] 675.
[25] [9] 116.

[6]. Therefore, it remains possible that co-production proposals constitute a satisfactory answer to the question of how conditional probabilities should be interpreted in a propensity framework.

Let me now explain, still focusing on McCurdy's proposal, why I think that co-production proposals do not in fact constitute such a satisfactory answer. I have already quoted McCurdy's proposal for the interpretation of conditional probabilities: "a conditional propensity such that $Pr_{t1}(T_{t3}|I_{t2}B_{t1})$ is interpreted as the propensity at $t1$ for the system to produce the event I_{t2}, given that the event T_{t3} was also produced"[26]. My contention is that the conception presented in this excerpt does not meet the Interpretation criterion for a satisfactory interpretation of conditional probabilities. Use of the phrase "given that" is crucial here: "given that" is characteristic of conditional probability statements and that it is central in McCurdy's proposal reveals that the proposal fails to give insight into how conditional probabilities should be understood and in particular into what they measure.

It could be, however, that I was unfair to McCurdy and that the quotation I picked out does not do full justice to his conception. To fix this, let me now come to the most elaborate aspect of McCurdy's proposal: the rule for "updating" probabilities. Indeed, McCurdy devotes a long passage to explaining how probabilities given a propensity interpretation evolve in time:

> the propensities that systems possess, and the values of those propensities change over time. In the photon example system, if the event I_{t2} occurs at $t2$, then it is possible to update the dispositional nature of the "new" system as it exists at $t2$. Updating the dispositional system of that system requires the definition of a new propensity function for the propensity system at $t2$; call this new function Pr_{t2}. This function is conditioned on a set of background conditions B_{t2} which consists of the conditions expressed in B_{t1} as well as the additional condition that the event I_{t2} occurred at $t2$. [...] The assignments made by the new propensity function are defined as follows: $Pr_{t2}(T_{t3}|B_{t2}) = Pr_{t1}(T_{t3}|I_{t2}B_{t1}) = p$, and $Pr_{t2}(not - T_{t3}|B_{t2}) = Pr_{t1}(not - T_{t3}|I_{t2}B_{t1}) = 1 - p$.[27]

This passage makes clear that conditioning events B_{ti} have a particular status: they stand for sets of physical conditions to which propensities are relative. To this extent, and as noticed by McCurdy[28], probabilities with form $Pr_{ti}(E_{tj}|B_{ti})$ should be considered as absolute probabilities rather that conditional ones. Now, McCurdy claims that B_{ti}s are modified as time passes by and (usual) conditioning events occur or fail to occur. More precisely, the modification is such that B_{t2} is "the conditions expressed in B_{t1} *as well as*[29] the additional condition that the event I_{t2} occurred at $t2$". Conditional probabilities measure propensities relative to these updated sets of physical conditions, and the fact that it conveys this claim seems to be how McCurdy's proposal meets Interpretation.

Still, what, exactly, are those sets of physical conditions? In particular, how should one understand "as well as" in this context? The notation introduced by McCurdy at the end of the passage suggests understanding it as a conjunction. This, however, will not do: there is no straightforward sense in which a set of

[26] [9] 109.
[27] [9] 112.
[28] [9] 110.
[29] My emphasis.

physical conditions and an event may conjoined. And even if one considers the *proposition* describing B_{t1} and the *proposition* stating that I_{t2} occurred, it is far from evident why their conjunction should describe a new set of physical conditions and what this new set should be. In the end, McCurdy's proposal is at best incomplete, more precisely lacking an analysis of how propensities-at-$t1$ are related to propensities-at-$t2$-given-that-I_{t2}-occurs. But this, precisely, is lacking an explanation of the phrase "given that" that is central to conditional probability statements. Therefore, the conclusion that McCurdy's and, more generally, co-production proposals fail to meet the Interpretation criterion cannot be escaped.

2.2 Ratio proposals

An analogous criticism can be raised at another type of proposals for the interpretation of conditional probabilities in the propensity framework. These proposals are not addressed by Humphreys, neither in [5] nor in [6]. I call them "ratio proposals" because they essentially consist in coming back to the usual definition of conditional probabilities as ratios of absolute probabilities – $P(A|B$ being equal to $P(AB)/P(B)$ for $P(B) \neq 0$. In other words, the idea is to get an interpretation of conditional probabilities out of the propensity interpretation of absolute probabilities together with the definition of conditional probabilities as ratios of absolute probabilities. Two major advocates of ratio proposals are Max Albert:

> the interpretation of the two conditional propensities $Pr_{t1}(T_{t3}|I_{t2}B_{t1})$ and $Pr_{t1}(I_{t2}|T_{t3}B_{t1})$ is unproblematic. The conditional propensity $Pr_{t1}(T_{t3}|I_{t2}B_{t1}) =_{def} Pr_{t1}(T_{t3}I_{t2}|B_{t1})/Pr_{t1}(I_{t2}|B_{t1})$, for instance, just gives the fraction of the causal pressure exerted by B_{t1} towards I_{t2} that is also exerted towards T_{t3}.[30]

and Nuel Belnap:

> $Pr_{t1}(T_{t3}|I_{t2}B_{t1})$ should be read as the proportions of cases (histories) through B_{t1} in which both T_{t3} and I_{t2} occur among all the cases (histories, courses of events) in which I_{t2} occurs.[31]

Although the reference to histories is an essential aspect of Belnap's paper as a whole, it is *not* an essential aspect of the way he proposes to conceive of conditional probabilities and claims to solve Humphreys' paradox. Correlatively, Belnap and Albert propose the same answer to the question of how conditional probabilities should be interpreted in the propensity framework: they are ratios of absolute probabilities each of which measures a propensity.

Ratio proposals have the important advantage of undoubtedly accounting for the standard properties of conditional probabilities: because they are defined out of the standard formal characterization of conditional probabilities, they meet Admissibility by definition. Still, my contention is that ratio proposals do not fare better than co-production ones as far as Interpretation is concerned.

[30] [1] 13. I change Albert's notations in order to stick to the ones that I have been using hitherto. Recall that, under these notations, probabilities of the form $Pr_{ti}(E_{tj}|B_{ti})$ can be considered as absolute probabilities (more on this below).

[31] [2] 606, same remark as previously concerning the notations.

Indeed, the proposal to conceive of conditional probabilities as ratios, or "fractions", of absolute probabilities does not amount to more than recalling how standard conditional probabilities considered as numerical magnitudes relate to other numerical magnitudes. In other words, it does not amount to more than recalling how standard conditional probabilities can be *computed* out of absolute probabilities. But it does not tell what conditional propensities *are* or (to put it in a slightly different way) what conditional probabilities *measure* in the propensity framework. Ratio proposals, thus, fail to meet the Interpretation criterion.

2.3 Milne's proposal

Let me now turn to a last proposal that is not taken into account in [6]. It was formulated in [14] and consists in assuming that conditional probabilities are probabilities of conditional events and in extending to conditional probabilities thus understood the classical propensity interpretation of probabilities as measures of propensities. Conditional probabilities then measure propensities tending to produce conditional events – and the proposal does not have any problem meeting Interpretation.

Conditional events, however, cannot be ordinary events, on pain of the kind of triviality that Lewis pointed out in [7].[32] In this paper, Lewis shows that, except for trivial cases, there does not exist a connective $\Rightarrow$ that would 1) have the usual properties of conditionals and 2) be such that conditional probabilities $Pr(A|C)$ have the same values as absolute probabilities $Pr(C \Rightarrow A)$.[33] Accordingly, the bulk of Milne's paper is devoted to spell out what conditional events may be if they are to satisfy a very natural criterion for identity[34] and their probabilities are to be conditional probabilities. Following a formal analysis, Milne identifies some features of conditional event b:a:

> *b:a definitely occurs* when *a* and *b both occur*; *b:a definitely fails to occur* when *a occurs* and *b fails to occur*; *b:a* neither *definitely occurs* nor *definitely fails to occur*, when *a fails to occur*.[35]

Thus, it is possible for Milne's conditional events to neither occur nor fail to occur. To this exact extent, they are not ordinary events.

The first worry one might have about Milne's proposal is that it does not seem to bring with it any good reason why probabilities of conditional events characterized along the suggested lines should actually be conditional probabilities. In other words, Milne does not give any good reason why his proposal should meet Admissibility. Still, there is also no obvious reason why it should not meet it. In particular, Milne's account leads to an apparently symmetrical treatment of inverse and non inverse conditional probabilities, which should not commit to any of the principles for the evaluation of inverse conditional probabilities which are dismissed by [6]. Moreover, Milne's proposal does not lead to any general principle for the evaluation of inverse conditional probabilities and, hence, cannot be a target for a further generalization of Humphreys' paradox.

[32] [7] 300–303.

[33] [7] 300–303. More precision about Lewis' results, and in particular about the distinction between the two results he has, is not needed here.

[34] This criterion is formulated at the beginning of the paper: [14] 319.

[35] [14] 324.

To finish, it should be underlined that this (the absence of a good reason why Milne's proposal should meet Admissibility) is only as far as I understand the proposal correctly.

But precisely here lies the main difficulty with Milne's proposal. More explicitly, I must admit that the passage I have just quoted does not give me a very firm grasp on what Milne's conditional events are. And things become even trickier when one comes to propensities tending to produce conditional events. Indeed, they cannot be propensities to make conditional events occur in the same way as propensities to produce usual events are propensities to make them occur. The reason for this is that conditional event b:a (definitely) occurs only when both a and b occur and occurrence of a conditional event cannot be reducible to co-occurrence of two usual events. Then, one has to admit that:

> Propensities are propensities to produce events/outcomes and in the case of conditional events we just do not seem to have the events to produce.[36]

All this, I claim, is far from making clear and straightforward sense, and it is my contention that Milne's proposal is not properly intelligible.

Acknowledgedly, the criticism I have just leveled against Milne's proposal is not fatal. First, it crucially depends on both my ability to understand Milne's proposal and my own standards for intelligibility. Second, it must be clearly stated that Milne successfully shows that if there are conditional events at all, then they must be understood along the lines that he draws. However, he also acknowledges that nothing commits us to consider conditional probabilities as probabilities of conditional events: "we don't *have* to do this, nothing *forces* us to do this, but we can"[37]. My point, then, is best understood as follows: before embracing Milne's interpretation, one should make sure that the assumption according to which conditional probabilities are probabilities of conditional events is necessary in order to get a proposal that 1) fares as well as Milne's as far as Admissibility and Interpretation are concerned and 2) is easily and undoubtedly intelligible. This is examined in the sequel of the text. More specifically, I will now leave out Milne's assumption that conditional probabilities are probabilities of conditional events and will try to construct a propensity interpretation of conditional probabilities having features 1) and 2). To this effect, I will first discuss the very notion of an interpretation of conditional probabilities and examine how conditional probabilities can interpreted once the assumption that conditional probabilities are probabilities of conditional events has been left out.

3 Interpreting conditional probabilities

Let me take the following observation as a starting point: both the frequentist and subjectivist accounts of probability provably succeed in providing an interpretation of both absolute and conditional probabilities[38] and yet none of them appeals to Milne's mysterious conditional "things" – which would be, more precisely, conditional propositions in the subjectivist case and conditional properties in the frequentist one[39]. This observation suggests to first examine the

[36] [14] 327.
[37] [14] 319.
[38] In the case of the frequentist theory, the result is immediate. In the case of the subjectivist theory, it is established in [18] for the absolute case and in [20] in the conditional one.
[39] [14] 324.

way subjectivists and frequentists interpret conditional probabilities. This will enable me to propose a general analysis of what it is to interpret conditional probabilities.

Under the frequentist interpretation, probabilities are relative frequencies in sequences of events.[40] More precisely, frequentists start with the idea that absolute probabilities are relative frequencies in a sequence of events. For example, the probability of *six* in a sequence of throws of one given die is the relative frequency, in this sequence, of those throws that give *six*. Now, the conditional probability of *six* given[41], say, *even* is the relative frequency of throws that give *six* among those that give an even result. In other words, conditionalizing on *even* amounts to switching from probabilities as relative frequencies in the original sequence, to probabilities as relatives frequencies in one of its subsequences. What this subsequence is depends on what one conditionalizes upon.

In the subjectivist case, absolute probabilities are interpreted as degrees of rational belief. Probability functions, then, measure rational belief in the propositions they take as arguments. Specifically, let Pr be the function measuring rational belief of individual I under stock of information K. This means that, say, $Pr(A)$ is the degree to which I rationally believes in A under stock of information K. Now $Pr(A|B)$, the subjectivist claims, is the degree to which I rationally believes in A under stock of information $K \cup \{B\}$. More generally, $Pr(.|B)$ is the function measuring I's rational beliefs under information $K \cup \{B\}$. Thus, conditionalizing on B amounts to adding B to I's initial stock of information. As in the frequency case, it is switching from one probability function to another one, and the exact nature of the switch depends on the proposition one conditionalizes upon.

The frequentist and subjectivist pictures corroborate one conclusion that was already drawn from the interpretation of the propensity theory alone: interpretations of conditional probabilities are not analytically contained in interpretations of absolute probabilities. To put it in a slightly different way, conditionalization has to be interpreted. Moreover, the frequentist and subjectivist pictures suggest that interpreting conditionalization can be viewed as explicating how probability functions are modified by what one conditionalizes upon.

Examination of the frequentist and subjectivist accounts in fact enables to go further in the analysis of what it is to conditionalize and, therefore, to interpret conditionalization. Indeed, it is clear from the presentation given above that the frequentist interpretation makes probability functions dependent on sequences of events, whereas the subjectivist interpretation makes them dependent on individuals together with the stock of information they have. Sequences of events on the one hand and individuals endowed with stocks of information on the other hand determine probability functions. Consequently, I propose to call them "determinants" of probability functions. They differ in nature from the arguments of probability functions – which are properties in the frequentist case and propositions in the subjectivist one. Consequently, I propose that

[40] Whether these sequences are finite or infinite and actual or hypothetical does not matter here. Therefore, I will not take these distinctions into account, and I will discuss frequentist interpretations in general.

[41] This ("given") is the reading I will stick to for the conditionalization bar. This is debatable, but what the outcome of the debate would be does not make any difference to the point I want to make.

determinants of probability functions be displayed as indexes of the Pr probability symbol whereas arguments appear inside the brackets that follow it. For instance, $Pr_D(A)$ will be the image of argument A for the probability function whose determinant is D.

Under the distinction between determinants and arguments of probability functions, the idea according to which interpreting conditionalization is telling how probability functions are modified by what one conditionalizes upon can be refined. More explicitly, one is led to conceive of an interpretation of conditionalization as an explication of how the argument of a probability function modifies its determinant. Rigorously, the idea is that conditionalization should be interpreted as a function that associates a new determinant to each pair composed of an initial determinant and an argument[42]. Let us put this suggestion formally, beginning with the frequentist case. To that effect, let $\mathbf{S}$ be the set of sequences of events and $\mathbf{P}$ the set of properties that the events in these sequences may satisfy. With this notation, the frequency interpretation of conditionalization can be represented in the following way:

$$\begin{array}{lllll} c_f & : & \mathbf{S} \times \mathbf{P} & \longrightarrow & \mathbf{S} \\ & & (S, P) & \longmapsto & S' \end{array}$$

with S' the sequence that you get when retaining from S only the events that have property P. Coming now to the subjectivist case, it has been claimed that a probability function is determined by an individual together with a stock of information, or (equivalently) by a couple composed of one individual and of the set of propositions he is informed of. Letting $\mathbf{I}$ be the set of individuals, $\mathbf{Po}$ the set of propositions, and $\mathbf{K}$ the powerset of $\mathbf{Po}$, this amounts to the claim that determinants of probability functions are elements of $\mathbf{I} \times \mathbf{K}$. I have also claimed that, in the subjectivist context, conditionalization amounts to adding the conditioning proposition to the stock of information of the individual under consideration. Following the notations that have just been introduced, this leads to consider that the subjectivist interpretation of conditionalization is by the following function c_s:

$$\begin{array}{lllll} c_s & : & (\mathbf{I} \times \mathbf{K}) \times \mathbf{Po} & \longrightarrow & \mathbf{I} \times \mathbf{K} \\ & & ((I, K), P) & \longmapsto & (I, K') = (I, K \cup \{P\}). \end{array}$$

Having left out Milne's assumption that conditional probabilities are probabilities of conditional "things" and examined the frequentist and subjectivist accounts, I come to support the following view: what one has to interpret is conditionalization (rather than conditional probabilities) and it has to be interpreted as a function associating a new determinant for a probability function, to an initial determinant together with an argument. This leads to the wider view of an interpretation of probability as consisting of:

1. an interpretation of absolute probabilities. This must specify in particular :
   (a) what kind of objects arguments of probability functions are;
   (b) what kind of objects determinants of probability functions are;

[42]Or, more exactly: to each pair composed of an initial determinant and an argument having probability different from 0 relative to that determinant. This point, however, is not tackled in the present paper and is left for future examination. Only pairs having the aforementioned property are considered in the rest of the paper.

2. an interpretation of conditionalization as a function from the cartesian product of the set of arguments and the set of determinants, to the set of determinants.

Armed with this conception, I now come back to the propensity theory of probability. More precisely, I will try to follow the lines that I have just drawn in order to construct a propensity interpretation of conditional probabilities.

4 Proposal for a propensity interpretation of conditionalization

Under the propensity theory as it was presented at the beginning of the paper:

1. absolute probabilities measure propensities of sets of physical conditions to realize singular events. Hence,
   (a) arguments of probability functions are elements of the set **E** of singular events;
   (b) determinants of probability functions are elements of the set **PC** of sets of physical conditions.

   Consequently,

2. an interpretation of conditionalization is a function c_p from the cartesian product $\mathbf{PC} \times \mathbf{E}$, into **PC**.

To begin with, notice that the first part of this account throws light on the status of the B_{ti}s that appeared above in the discussion. More precisely, it throws light on the way B_{ti}s differ from other conditioning events, and on McCurdy's mysterious claim that $Pr_{ti}(E_{tj}|B_{ti})$ probabilities should be considered as absolute rather than conditional[43]. Indeed, it is now clear that B_{ti}s are determinants rather than arguments of probability functions interpreted along the propensity lines.

More important, the second part of the proposed account amounts to restate the question of providing a propensity interpretation of conditional probabilities as the question of defining c_p. The task I aim to perform is more precisely as follows: defining c_p in such a way as to satisfy the criteria set out at the beginning of section 2 and without appealing to Milne's problematic notion of conditional event. In order to do so, and because one can imagine so many functions from $\mathbf{PC} \times \mathbf{E}$ into **PC**, I shall consider the following property of Bayesian conditionalization: for any probability function Pr and any E such that $Pr(E) \neq 0$, $Pr(E|E) = 1$. I will use this property of Bayesian conditionalization as a constraint on c_p. Specifically, c_p must have the following property:

Property 1 *For any set of physical conditions PC and any singular event E such the $Pr_{PC}(E) \neq 0$, $Pr_{c_p(PC,E)}(E) = 1$.*

In the end of the current section, I will first envisage two straightforward ideas for defining c_p in such a way as Property 1 is satisfied and explain why these

[43] [9] 110.

ideas cannot be endorsed. Elaborating on this discussion, my own proposal will finally be stated.

Let me first assume that a set PC of physical conditions can be considered as a pair composed of a physical system S and a moment t in time. Under this assumption, a first straightforward idea is to interpret conditionalization as a "temporal jump" from the initial moment t to a moment t' when the probability of the conditioning event – say, E – in S is 1. In other words, the idea is to have $c_p((S,t),E) = (S,t')$ with t' such that $Pr_{(S,t')}(E) = 1$. Property 1, thus, is clearly satisfied. However, a suitable t' may fail to exist: there may very well be systems S and events E such that the probability of E *never* equals 1 in S. As a consequence, c_p cannot modify only the temporal component of the pairs to which it applies, but has to change systems too.

A simple such change would be from an initial system to the system the most similar to it *among those in which the conditioning event occurs.* This proposal does not encounter the same difficulty as the previous one. Indeed, by definition of the proposal, E occurs in S' that is the first component of $c_p((S,t),E)$. Therefore there exists t' such that the probability of E at t' in S' is 1. Correlatively, one can complete the definition of c_p in such a way as to obtain a general proposal for which Property 1 is satisfied. There are two reasons why this proposal is appealing. First, it allows an analogy between the propensity theory on the one hand, and the frequentist and subjectivist interpretations of probability on the other. In the same way as frequentist conditionalization consists in taking into account only those events that instantiate the conditioning property, and in the same way as subjectivist conditionalization consists in learning that what the conditioning proposition describes occurred, conditionalization under the propensity view would consist in moving to a system in which the conditioning event occurs. Second, the proposed c_p is economical in the sense that the change it makes in systems is as small as compatible with the feature here central: occurrence of the conditioning event.

Still, there also is a sense in which the proposal is *not* economical: it implies, for Property 1 to be satisfied, that conditionalization modifies *both* the system and the moment that one considers. Indeed, given that E occurs in S', it is not the case in general that its probability is 1 at initial time t. Or, at least, it is not the case in general in the indeterministic contexts that the propensity interpretation was precisely meant to deal with. More generally, in these contexts it is not the case in general that "the system which is the more similar to S among those in which E occurs" has a referent at moments preceding the moment when E actually occurs. A consequence is that the proposal under examination cannot be completed in such a way that $c_p((S,t))$ is defined at t. But I think it is not acceptable: it seems a fair requirement that an interpretation of conditionalization be such that the way a given conditional probability is interpreted is defined by the time the conditional probability itself is.

Following the above discussions, I suggest defining c_p as:

$$\begin{array}{cccccc} c_p & : & \mathbf{PC} \times \mathbf{E} & \longrightarrow & \mathbf{PC} \\ & & ((S,t),E) & \longmapsto & (S^E,t) \end{array}$$

with S^E the system that is the most similar to S among those giving probability 1 to E at t. In words, the proposal is to interpret conditionalization as the function that makes to systems the slightest difference compatible with the conditioning event having probability 1 at the moment initially under consideration. Conditionalization, then, is interpreted as the change in systems that is minimal among those compatible with a certain way of satisfying Property 1 (namely by considering a system that gives probability 1 to the conditioning event at the initial moment). The proposed interpretation, thus, is economical in the same way as the interpretation discussed in last paragraph. Moreover, it is such that the way a conditional probability should be interpreted is completely defined by the time the conditional probability itself is. This makes my suggestion immune from any of the difficulties that led me to reject the two more straightforward proposals that I have discussed. Positively, the proposal has the property of being economical in the sense that the proposal discussed in last paragraph was not: under my proposal, conditionalization modifies systems only, not moments in time. All in all, the proposed interpretation has important prima-facie appeal. Still, it should be more closely discussed whether it can be accepted as an interpretation of conditional probabilities. Next section is devoted to this discussion.

5 Discussion

Two kinds of things have to be discussed here: whether the proposed interpretation meets Interpretation and Admissibility and whether it is immune from specific difficulties turning out to be diriment. These two questions are tackled in subsections 5.1 and 5.2 respectively. Concerning more precisely Admissibility, what I have to say is not conclusive: I give reasons to believe that the proposed interpretation satisfies Admissibility, but do not properly show that it does satisfy it. This explains why it has been possible for David Miller to come with an argument to the effect that the proposed interpretation *does not* satisfy Admissibility. Miller's objection and the answer he himself proposed are presented in subsection 5.3.

5.1 Interpretation and Admissibility

Let me discuss first whether the interpretation I propose meets the Interpretation criterion. Here, a comparison with co-production proposals is in order. Indeed, following [6], I took as characteristic of these proposals the fact that they locate propensities in the physical conditions at the time initially considered. Under this characterization, my proposal can be considered a co-production proposal. But I argued earlier that co-production proposals fail with regard to Interpretation. Let me, therefore, be a little bit more precise and insist on what I argued being precisely that co-production proposals as they find their best elaboration in [9] lack a proper account of how an initial set of physical conditions and a conditioning event together define a new set of physical conditions. Now, this is exactly what my proposal provides, following the analysis of section 3. Consequently, although the propensity interpretation for conditional probabilities that is here proposed can rightfully be considered a co-production proposal, my contention is that it differs from previous co-production proposals to exactly the extent that it meets Interpretation.

Coming now to Admissibility, let me first reiterate that I cannot offer a conclusive reason why the proposed interpretation meets it. Still, before Miller formulated his objection, it also seemed to me that – as was the case with Milne's proposal – there was no good reason why it should fail to meet it. Moreover, I identified several reasons to believe that it may meet Admissibility and could be accepted. These reasons are stated in the end of current subsection, whereas Miller's analysis is presented in subsection 5.3.

To begin with, it should be noticed that the proposed interpretation shares with Milne's the property of apparently not leading to a general principle for evaluating inverse conditional probabilities. If it indeed does not, then it is immune from any further generalization of Humphreys' paradox in its formal version. Moreover, and now slightly departing from Milne's proposal, the interpretation I propose was constructed so as to satisfy Property 1. Since the emphasis on Property 1 is exclusive of taking into account the other (numerous) properties of Bayesian conditionalization, the fact that my proposal was designed to satisfy Property 1 could be taken as indicating that there is no good reason why this proposal should satisfy Admissibility. However, it must be noticed that Property 1 is fundamental to Bayesian conditionalization.[44] Correlatively, my proposal being constructed in order to satisfy Property 1 may not be considered merely as a symptom of the absence of a conclusive reason why it should satisfy Admissibility. It may be seen also as a point in favor of the proposal actually satisfying Admissibility – and as a point contributing to distinguish my proposal from Milne's.

To conclude this initial discussion concerning Admissibility, let me put forward a comparative argument. Here the comparison will not be with alternative interpretations of probability, but with the propensity interpretation of *absolute* probabilities. Indeed, there does not exist any conclusive argument to the effect that the propensity interpretation of absolute probabilities is admissible – let alone a procedure for measuring propensities. As a consequence, it has to be *postulated* that it is. Now, one may have an analogous postulate in the conditional case. In other words, until proved that the proposed interpretation fails to account for the properties of Bayesian conditionalization, merely postulating that it does is a strategy that is available for a proponent of the propensity interpretation of absolute probabilities.

Even one who refuses this postulating strategy should at this point be convinced that my proposal fares at least as well as (and probably better than) Milne's with regard to the Interpretation and Admissibility criteria. On the other hand, I take it that this proposal is indisputably intelligible. Thus, the only reason I can now see for rejecting this proposal and sticking to Milne's one would be that it faces diriment specific difficulties. I now discuss what I take as the most serious candidates for being such difficulties.

5.2 Diriment specific difficulties?

I will discuss three specific reasons that might lead to reject the interpretation I have proposed for conditional probabilities. The first one is that the proposal appeals to the notion of similarity between physical systems. This seems problematic for both conceptual and consequent epistemic reasons. More precisely, at first sight it is clear neither whether this notion can be given a precise meaning and what such a precise meaning could be, nor how one is to identify the system the most similar to a given one among those that belong to a certain

[44] [7] 311.

class. As an answer to this double objection, I will essentially argue that it is at least seriously weakened by widening the theoretical view. More explicitly, the commonest approach to counterfactual conditionals is through the notion of similarity between possible worlds. Now, I take this to make two (non independent) points in favor of similarity between physical systems. First, similarity between physical systems looks much less problematic than similarity between possible worlds. More precisely, similarity between physical systems significantly differs from similarity between possible worlds in that giving the first one a precise definition does not seem to be precluded by the nature and ontological status of the *relata*. Positively, Paul Humphreys suggested that the notion of state phase may be given a central place in the definition of similarity between physical systems, while David Miller proposed to consider that the least the amount of energy needed to turn one physical system into another one, the more similar these systems are. Whether or not these ideas may be accepted, it is at least clear that, contrary to similarity between possible worlds, similarity between physical systems may receive a precise, physical definition. Correlatively, accepting similarity between systems is no great deal for one who has accepted similarity between possible worlds. Reciprocally, rejecting similarity between systems commits to reject similarity between possible worlds. Consequently, it commits to reject our commonest and probably best analysis of counterfactuals – and along with it an important tool for the analysis of causality. All in all, it seems to me that resorting to similarity between physical systems cannot be taken as invalidating the proposed interpretation of conditionalization.

The second worry that one might have concerning this interpretation is that it seems to imply that, given a time t, a system S and an event E that (S, t) can produce, there always exists S^E that is similar to S and such that $Pr_{(S^E,t)}(E) = 1$. In other words, the proposal apparently implies the existence of a slight modification of S that is sufficient to give E probability 1 at considered time t. However seriously this worry should be taken in the indeterministic contexts at which propensities are directed, it can be properly answered in the framework here proposed. Indeed, in this framework, similarity is not taken as a categorical, but rather as a gradual concept. More explicitly, the idea is not to pick up a (or the) system similar to an initial one, but the system *the most* similar to it among those having a given property. This system, however, does not need to be (categorically) similar to the initial one. As a consequence, the proposed interpretation does not have the unpleasant implication that I presented at the beginning of the paragraph.

A third difficulty for the proposed interpretation has to do with the status of conditioning events. Under the propensity account, arguments of probability functions, and hence in particular conditioning events, are singular events. Now, the concept of a singular event is a fine-grained one, generally construed in such a way that the identity of a singular event is sensitive in particular to the circumstances under which the event is produced. This is the case in particular under the classic Quinean characterization of events as contents of portions of space-time[45]. Under such a characterization, moving from the initial system S to S^E (as is required by the proposed interpretation of conditionalization) is not compatible with the initial conditioning event E being identical to the one having probability 1 relative to (S^E, t). But this means that fundamental

[45] [17] 171.

Property 1 of Bayesian conditionalization is not in fact properly accounted for: this property requires that the probability of E conditional on *itself* is 1. My answer to this objection is as follows: the propensity account of probability does not commit to a notion of singular event under which identity of singular events is sensitive to the circumstances under which they are produced. To make it clear, let me come back to what I consider as essential to the propensity theory – which Miller calls the "crucial move"[46] away from frequentism: the idea that propensities are dispositions exercised in every single case, rather than dispositions to produce relative frequencies. In other terms (but still following [12]), the notion here central is that of a "propensity of the arrangement at a single time to do something in a single case"[47]. Now, this does not imply that the "thing" the arrangement has a propensity to do is defined beforehand – that is, before it actually occurs (if it does) – in such a way that its identity is sensitive to any modification in the conditions of its possible production. Attaching probabilities to singular instances does not commit to any specific definition of singular events. In particular, it is perfectly compatible with the definition of a singular event in the rather coarse-grained terms of an individual instantiating a given property within the limits of a given period of time. Such a conception in fact endorsed by both Humphreys and his disputants, as shown by the notations they adopt. Moreover, I would claim that a rather coarse-grained notion of singular event is not only allowed by the propensity theory as it "officially" stands, but also required for the very notion of probability of a singular event to make interesting sense. On the one hand, I cannot really see how the probability of a very fine-grained singular event could be different from 0 before the time it actually occurs. On the other hand, attributing a probability to such an event has neither practical nor theoretical interest. We are interested in whether an individual instantiates a given property in a given period of time, but not in the exact and exhaustive circumstances of this instantiation. As a consequence, I will consider that the objection discussed in the present paragraph is irrelevant, and does not constitute a diriment specific difficulty for the interpretation of conditionalization that was proposed at the end of section 4.

5.3 Miller's objection and his answer to it

In the last two subsections, I have explained why the proposed interpretation of conditionalization satisfies Interpretation, I have give reasons that have led me to believe that it does not fail to meet Admissibility and I have discarded as not diriment the difficulties that the interpretation specifically raises. As already stated, my reasons to believe that the proposed interpretation does not fail to meet Admissibility are no conclusive proof of the fact that it actually meets the criterion. In accordance with this, David Miller proposed an argument in favor of the interpretation failing to meet the criterion.[48]

Miller's argument focuses on the law of total probability, and aims at showing that the interpretation I proposed cannot account for this law being a true statement concerning conditional probabilities. In other words, the claim is that

[46] [12] 5.

[47] [12] 8.

[48] This argument and the answer to the objection it constitutes were proposed on the occasion of a workshop in Paris in June 2008. It is with David Miller's full agreement that I here give an account of his insightful comments on my proposal.

my proposal fails to account for Bayesian conditionalization making the law of total probability a true statement about conditional probabilities. More exactly, Miller shows that my proposal accounting for this would have implausible consequences.

Let us indeed consider the law of total probability:

Theorem 1 (Law of total probability) *For any probability function P, any A, and any E such that $P(E) \neq 0$ and $P(not - E) \neq 0$,*
$P(A) = P(A|E).P(E) + P(A|not - E).P(not - E)$.

Following my claim that probability functions given a propensity interpretation are determined by sets of physical conditions, consider the function Pr_{PC} that is determined by set $PC = (S, t)$. Theorem 1 implies that:

For any events A and E such that $Pr_{PC}(E) \neq 0$ and $Pr_{PC}(not - E) \neq 0$,
$Pr_{PC}(A) = Pr_{PC}(A|E).Pr_{PC}(E) + Pr_{PC}(A|not - E).Pr_{PC}(not - E)$.

This equality can also be written as:

$$Pr_{PC}(A) = Pr_{c_p(PC,E)}(A).Pr_{PC}(E) + Pr_{c_p(PC,not-E)}(A).Pr_{PC}(not - E).$$

As noticed by Miller, the first, third and fifth terms of this equality have values fixed by PC. Thus, the equality implies that, given PC, there is an algebraic relationship between the second and fourth terms – that is $Pr_{c_p(PC,E)}(A)$ and $Pr_{c_p(PC,not-E)}(A)$. Under the interpretation proposed in section 4, $c_p(PC, E)$ is (S^E, t) with S^E the system the most similar to S among those that give probability 1 to E at t, and $c_p(PC, not - E)$ is (S^{not-E}, t) with S^{not-E} the system the most similar to S among those that give probability 1 to $not - E$ at t. Hence, under the proposed interpretation of conditionalization, the law of total probability implies the following: given $PC = (S, t)$, at t the values of the probabilities in the system the most similar to S among those that give probability 1 to E give (and are given by) those of the probabilities in the system the most similar to S among those that give probability 0 to E. Miller claims that "such a pre-established harmony [...] would be remarkable" and, to be more explicit, that it is indeed very implausible.

Rather welcome is the "unsophisticated" example designed by Miller to illustrate his claim. Consider the system S constituted by a plane driven by an agent furnished with a parachute and the environment the plane evolves in. At t, the plane is in difficulty and the agent is dithering about jumping. More precisely, the propensity for him to find himself in the air out of the plane within the 5 minutes following t (an event that will be labeled E) has the same value as the propensity for him to stay inside the plane: $Pr_{(S,t)}(E) = Pr_{(S,t)}(not{-}E) = 1/2$. Moreover, the propensity for the agent surviving the next 10 minutes (event A) is also $1/2$; $Pr_{(S,t)}(A) = 1/2$. Therefore, following last paragraph, the law of total probability implies that

$$1/2 = Pr_{c_p((S,t),E)}(A).1/2 + Pr_{c_p((S,t),not-E)}(A).1/2,$$

which is equivalent to:

$$1 = Pr_{c_p((S,t),E)}(A) + Pr_{c_p((S,t),not-E)}(A).$$

In words, the propensity tending to produce A that is determined by $c_p((S, t), E)$

and the propensity tending to produce A that is determined by $c_p((S,t), not-E)$ have complementary values. Now, one can suppose with Miller:

- that S^E that is the most similar to S among those that give probability 1 to E at t consists of the same plane, driven by the same agent furnished with the same parachute, evolving in the same environment, but equipped with a device that pushes the agent out of the plane at a time shortly posterior to t,
- that S^{not-E} consists of the same plane, driven by the same agent furnished with the same parachute, evolving in the same environment, but equipped with a device designed to sharply knock the agent on the head at a time shortly posterior to t.

Then, that the propensity tending to produce A that is determined by $c_p((S,t), E)$ and the propensity tending to produce A determined by $c_p((S,t), not-E)$ have complementary values exactly means the following: at t, the more probable it is that the agent survives the next 10 minutes if he is pushed out of the plane shortly after t, the less likely it is that he survives the next 10 minutes if he is sharply knocked on the head shortly after t. According to Miller, and I follow him in this respect, there is no reason why this should be so. Quite the opposite, it is plausible that, both relative to (S^E, t) and relative to (S^{not-E}, t), the propensity for the agent surviving the next 10 minutes has a value smaller than the one it had relative to (S, t). More generally, the proposed interpretation of Bayesian conditionalization implies that there exists a certain relationship between (values of) propensities determined by different physical systems, such a relationship has a metaphysical content, but there does not seem to be a metaphysical reason for it.

If Miller is right – which I think he is – the propensity interpretation of Bayesian conditionalization that is built up in section 4 fails to satisfy Admissibility and, therefore, cannot be accepted. However, Miller had the good taste to come not only with a strong objection to my proposal, but also with an adaptation of this proposal that overcomes the objection. The adaptation suggested by Miller is as follows: define S^E not as the system the most similar to S among those that give E probability 1 at t, but as the system the most similar to S among those which, at t, give any A the probability $Pr_{(S,t)}(AE)/Pr_{(S,t)}(E)$. As noticed by Miller, it is evident that for any E such that $Pr_{(S,t)} \neq 0$, $Pr_{(S,t)}(E|E) = 1$. In other words, Property 1 is satisfied by the proposal as adapted by Miller. Moreover, the interpretation thus adapted indeed overcomes the difficulty identified by Miller. Indeed, by definition of this interpretation,

$$Pr_{PC}(A) = Pr_{c_p(PC,E)}(A).Pr_{PC}(E) + Pr_{c_p(PC,not-E)}(A).Pr_{PC}(not-E)$$

is equivalent to:

$$Pr_{PC}(A) = Pr_{PC}(A.E) + Pr_{PC}(A.not-E):$$

the consequence of the law of total probability that was shown to be problematic under my proposal now reduces to "an innocuous consequence of the addition law". More generally, the interpretation suggested by Miller is constructed so as to account for all the properties of conditional probabilities standardly defined as ratios of absolute probabilities: Admissibility is satisfied by construction. This may not be satisfactory: one may expect an interpretation not to be *constructed to be* admissible, but rather to be formulated in terms different from the ones appearing in the definition of the notion to be interpreted and that turn out to guarantee Admissibility. Still, I currently cannot see any more satisfactory way to avoid the difficulty highlighted by Miller. Moreover, the interpretation suggested by Miller belongs to the same vein as the one that was formulated in section 4. It shares its advantages, in particular the ones it has over the conditional-event interpretation supported by Milne. All in all, my contention is that, for the time being, the interpretation defined by Miller is our best proposal for a propensity interpretation of Bayesian conditionalization.

6 Conclusion

I have given an update on Humphreys' paradox and on the possibility to give a propensity interpretation of conditional probabilities. More precisely, I have shown that, among existing proposals, only Milne's one may be accepted. Yet this proposal suffers from its resorting to the hardly intelligible notion of a conditional event. Therefore I have tried to construct a propensity interpretation of conditional probabilities that does not resort to this notion and, correlatively, does not suffer from the same intelligibility problem as Milne's proposal. This was done after I came to the view that an interpretation of conditionalization is a function describing the way conditioning events redefine determinants of probability functions.

The interpretation I formulated was criticized by David Miller. I think his criticism is right and I subscribe to his suggestion for subsequently adapting the interpretation I formulated. The adapted interpretation may still fail to be satisfactory. On the one hand, it may be considered problematic that it satisfies Admissibility *by construction*. On the other hand, it faces the same specific difficulties as the proposal I formulated in section 4, and some of the arguments that were given in subsection 5.2 in order to show that these difficulties are not diriment are acknowledgedly rather weak. However, I do not see the content of the proposed interpretation as the most interesting aspect of the present paper, and not even as the most important aspect of its second, constructive part. Positively, the core claim of this constructive part is that one who renounces Milne's conditional events is left with the task of interpreting conditionalization. In other words, *contra* the widespread idea that an interpretation of conditional probabilities is analytically contained in the propensity interpretation of absolute probabilities, I have shown that conditionalization requires its own *interpretation* and I have given an analysis of what such an interpretation formally is. Specifying the interpretation, then, is only the last job. The way I have carried it out turned out to be unsatisfactory. Maybe the way Miller suggested to carry it out can be improved. In any case, I have shown that there can be a propensity interpretation of conditional probabilities that does not resort to Milne's conditional events and I have indicated the lines along which such an interpretation should be constructed.

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