Condensed Version

Do We Need Paraconsistency in Commonsense Reasoning?*

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May 5, 2003

1 Introduction

In recent years, several attempts were made to apply paraconsistent logics or, more generally, inconsistency-tolerant logics, to the study of commonsense reasoning. Inconsistency-adaptive logics, for instance, were used to reconstruct circumscription ([3]) as well as default logic ([12]). Other examples can be found in [1], where Belnap's four-valued logic is applied to default reasoning, and in [19], where a many-valued paraconsistent logic is used as the basis for a query answering system that can handle rules with exceptions.

The aim of the present paper is twofold. First, I want to argue that, both from a technical and a philosophical point of view, paraconsistent logics are not the proper tools to study commonsense reasoning. What we need are *ampliative logics*, not inconsistency-tolerant logics. Next, I want to present an ampliative logic for a specific form of commonsense reasoning, namely default reasoning.

The logic will be called $\mathbf{R}^r$ and is intended to reconstruct Reiter's original version of default logic. When applied to default theories that have different extensions, the logic $\mathbf{R}^r$ will deliver those consequences that belong to the intersection of the different extensions (it thus captures the 'sceptical' form of default reasoning). One of the advantages of the present approach, however, is that it can easily be adjusted to other forms of default reasoning (the 'credulous' form, for instance).

To make its semantics and its proof theory as transparent as possible, $\mathbf{R}^r$ will be defined in an indirect way. More specifically, I shall present a *modal* translation of default theories and develop the logic $\mathbf{T}^r$ (based on the modal system $\mathbf{T}$) to make inferences from this translation. The logic $\mathbf{R}^r$ is obtained by stipulating that A is an $\mathbf{R}^r$-consequence of a default theory T iff $\Box A$ is a $\mathbf{T}^r$-consequence of the modal translation of T.

The system $\mathbf{T}^r$ is an adaptive logic. The first adaptive logic was designed by Diderik Batens around 1980 and was meant to handle inconsistent theories (see [2]). Later, the notion of an adaptive logic was generalized to include other types of logical abnormalities (negation-incompleteness, for instance) and several non-monotonic consequence relations were reconstructed in terms of adaptive logics (see, for instance, [4] and [11]). A more recent development is to use the framework of adaptive logics to study ampliative forms of reasoning (see [15]). At this moment, several ampliative adaptive logics are available—examples include logics for compatibility ([9] and [17]), enumerative induction ([7] and [8]), metaphorical reasoning ([13]), diagnostic reasoning ([10]) and causal reasoning ([14]).

The logic $\mathbf{T}^r$ has several nice properties. One is that it provides us with a *proof theory* for default reasoning. The proof theory is *dynamical*, but warrants that justified conclusions are obtained, even for undecidable theories.[1] It moreover warrants that 'in the end' different dynamical proofs lead to the same set of results, even for default theories that have different extensions (the derivability relation defined by $\mathbf{T}^r$ is proof invariant). Another important property is that it nicely captures the typical properties of default reasoning (for instance, that neither Contraposition nor *Modus Tollens* are valid in it).

I shall proceed as follows. After arguing why commonsense reasoning should not be studied within a paraconsistent framework (Section 2), I discuss the modal translation of default theories and the version of $\mathbf{T}$ that I shall rely on (Sections 3 and 4). Next, I present the semantics and the proof theory of $\mathbf{T}^r$ (Sections 5 and 6), and show how $\mathbf{R}^r$ is defined from $\mathbf{T}^r$ (Section 7). In the same section, I also present some examples that illustrate the intuitive adequacy of $\mathbf{R}^r$. I end with some conclusions and open problems (Section 8).

In view of the available results on adaptive logics (see especially [5] and [6]), the meta-theory of $\mathbf{T}^r$ (such as the Soundness and Completeness proofs) is rather straightforward. For reasons of space, it is not included in this condensed version.

*Research for this paper was supported by the Fund for Scientific Research – Flanders (Belgium), and indirectly by the Flemish Minister responsible for Science and Technology (contract BIL01/80).

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[1] This is especially important in view of the fact that, for undecidable theories, default reasoning lacks a positive test.

2 Commonsense Reasoning

In [18], Nicholas Rescher discusses some important differences between scientific knowledge and commonsense knowledge. On his analysis, a central requirement for scientific knowledge is that it is as precise as possible. Accordingly, falsifying instances are regarded as problematic and as an incentive to alter the claims at issue. Commonsense beliefs, writes Rescher, are treated in an entirely different way: we safeguard them from falsification by keeping them vague and imprecise. Thus, the commonsense belief that peaches are delicious is not abandoned when we discover that our next-door neighbour does not like them. The reason is that, in everyday life, "peaches are delicious" is not intended to mean that all people at all times like all kinds of peaches, but, at best, that *most* people like peaches. In that sense, Rescher claims, commonsense knowledge is more *secure* than scientific knowledge.

It seems to me that this is indeed a crucial feature of commonsense knowledge. There is, however, an important price to be paid for the kind of security that Rescher describes: the more imprecise our commonsense beliefs are, the more fallible our commonsense *reasoning* becomes. By keeping our commonsense beliefs vague, we make them more secure (there is little that can force us to reject or alter them). At the same time, however, the *conclusions* we draw from them become more *insecure*. For instance, when we use the commonsense belief "peaches are delicious" to make predictions about the people around us, we will occasionally arrive at conclusions that have to be withdrawn.

This at once explains why commonsense reasoning is highly *non-monotonic*. In commonsense reasoning, unlike in scientific reasoning, we do not respond to falsified conclusions by questioning our premises. Instead, we keep the premises, but reject the conclusions.

This fact, namely that conclusions are withdrawn when they are contradicted by other findings, led many to believe that commonsense reasoning should be studied within a paraconsistent setting. There are, however, several problems with such an approach.

A first one is that, by using a paraconsistent logic, we make the wrong kind of assumptions about the conceptual structures that underlie commonsense reasoning. When commonsense reasoning seemingly leads to inconsistencies, this is *not* because the underlying conceptual structure is inconsistent. It is because inferences are made on the basis of incomplete information. Let me explain this somewhat further.

In commonsense reasoning, imprecise beliefs are used as *rules of thumb*—we know that they have exceptions, but apply them, even when we do not know whether or not we are dealing with one of the exceptions. Hence, when we learn more about the domain, some of our conclusions may turn out to be false. This, however, should not be taken as an indication that our conceptual framework precludes a consistent description of the domain. To give just one example, our commonsense knowledge about flying animals does not commit us to the belief that some animals fly and do not fly, even if the commonsense beliefs that birds fly and mammals do not will sometimes lead to false conclusions. However, when we use a paraconsistent logic to reconstruct commonsense reasoning, this is precisely the kind of assumption that we are making.[2]

This brings us to a second problem. Commonsense reasoning is *ampliative*: the conclusions go beyond the information implicitly contained in the premises. Most paraconsistent logics, however, are purely deductive systems. Hence, they are inadequate to capture the fact that, in commonsense reasoning, conclusions do not follow with logical necessity, but only with some degree of likelihood. For the same reason, they are inadequate to explicate why commonsense conclusions are defeasible.

A final problem is that reconstructions in terms of paraconsistent logics tend to be far too complex and very different from the way in which laypersons actually make commonsense inferences. A typical procedure is first to use a paraconsistent logic to define the consequence set and next to apply some selection mechanism to 'filter out' the inconsistencies (see, for instance, [3]). This, however, is not how we proceed in everyday life. When we discover that some bird, which was previously assumed to be flying, is a penguin, we do not need an extra-logical selection mechanism to decide that he does not fly. The mere fact that our original hypothesis is contradicted by a stronger inference is sufficient to withdraw it.

The logic presented in this paper suffers from none of these shortcomings. The ontological presuppositions it makes are simply those of Classical Logic. It moreover accounts for the ampliative character of commonsense inferences and allows for a reconstruction that is very close to actual reasoning (the withdrawal of conclusions is taken care of by the logic itself and not by some external selection mechanism).

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[2] This should by no means be taken as a global rejection of paraconsistency. In [16], for instance, I argue that the study of scientific reasoning, unlike that of commonsense reasoning, often requires a paraconsistent logic.

3 Translating Default Theories in Modal Terms

Let $\mathcal{L}$ be the standard predicative language of Classical Logic (henceforth **CL**) and let $\mathcal{W}$ be its set of wffs (closed formulas). I shall assume that a default theory is a couple $\langle \mathcal{A}, \mathcal{D} \rangle$, in which $\mathcal{A} \subset \mathcal{W}$ is a set of (non-logical) axioms and $\mathcal{D}$ is a set of default rules of the form

(1) $\dfrac{A : C}{B}$

or the predicative generalization of this

(2) $\dfrac{A(x) : C(x)}{B(x)}$

in which case only the instantiations are applied. I shall further assume that a rule of the form (1) is interpreted as:

(3) If $\mathcal{A} \nvdash C$ and $\langle \mathcal{A}, \mathcal{D} \rangle \Vdash A$, then $\langle \mathcal{A}, D \rangle \Vdash B$.

in which $\vdash$ is monotonic (the consequence relation of **CL**) and $\Vdash$ non-monotonic.

In order to characterize default theories like this in modal terms, I shall rely on a predicative version of **T** (as described in the next section). The translation itself is simple. Let $\mathcal{A}^*$ stand for $\{\Box\Box A \mid A \in \mathcal{A}\}$, and let $\mathcal{D}^*$ be the (smallest) set that includes, for every $D \in \mathcal{D}$, the formula

(4) $\Box\Box\neg C \vee (\Box A \supset \Box B)$

if D is of the form (1), or

(5) $(\forall x)(\Box\Box\neg C(x) \vee (\Box A(x) \supset \Box B(x)))$

if D is of the form (2). In view of these stipulations, a default theory $\langle \mathcal{A}, \mathcal{D} \rangle$ is translated as $\mathcal{A}^* \cup \mathcal{D}^*$. Below, I shall use $\mathcal{T}$ to refer to the union $\mathcal{A}^* \cup \mathcal{D}^*$.

Note that, in **T**, $\Box\Box A$ does not follow from $\Box A$. In view of this, the proposed translation warrants that, in the proofs, a distinction can be made between members of $\mathcal{A}^*$ and formulas added on the basis of default rules. (Consider (3) again to see why the distinction is important.) As we shall below, the translation also warrants that Contraposition and *Modus Tollens* are invalidated.

Intuitively, a formula like (4) can be read as: "either C is not compatible with $\mathcal{A}^*$ or $\Box B$ may be derived from $\Box A$". The idea behind $\mathbf{T}^r$ will be to assume that C *is* compatible with $\mathcal{A}^*$, unless and until proven otherwise.

4 The Logic T

Let $\mathcal{L}^M$ be the standard modal language with $\mathcal{S}$, $\mathcal{P}^r$, $\mathcal{C}$, and $\mathcal{W}^M$ its sets of sentential letters, predicative letters of rank r, constants, and wffs. To simplify the clauses for the quantifiers, I introduce a (non-denumerable) set of pseudo-constants $\mathcal{O}$, *requiring that any element of the domain D is named by at least one member of* $\mathcal{C} \cup \mathcal{O}$. Let $\mathcal{W}^{M+}$ denote the set of wffs of $\mathcal{L}^{M+}$ (in which $\mathcal{C} \cup \mathcal{O}$ plays the role played by $\mathcal{C}$ in $\mathcal{L}^M$).

A **T**-model M is a quintuple $\langle W, w_0, R, D, v \rangle$ in which W is a set of worlds, $w_0 \in W$ the real world, R a binary relation on W, D a non-empty set and v an assignment function. The accessability relation R is *reflexive*. The assignment function v is defined by:

C1.1 $v : \mathcal{S} \times W \longrightarrow \{0, 1\}$
C1.2 $v : \mathcal{C} \cup \mathcal{O} \times W \longrightarrow D$ (where, for all $w \in W$, $\{v(\alpha, w) \mid \alpha \in \mathcal{C} \cup \mathcal{O}\} = D$)
C1.3 $v : \mathcal{P}^r \times W \longrightarrow \wp(D^r)$ (the power set of the r-th Cartesian product of D)

The valuation function $v_M : \mathcal{W}^{M+} \times W \longrightarrow \{0, 1\}$, determined by the model M is defined by:

C2.1 where $A \in \mathcal{S}$, $v_M(A, w) = v(A, w)$
C2.2 $v_M(\pi^r \alpha_1 \ldots \alpha_r, w) = 1$ iff $\langle v(\alpha_1, w), \ldots, v(\alpha_r, w) \rangle \in v(\pi^r, w)$
C2.3 $v_M(\alpha = \beta, w) = 1$ iff $v(\alpha, w) = v(\beta, w)$
C2.4 $v_M(\neg A, w) = 1$ iff $v_M(A, w) = 0$
C2.5 $v_M(A \vee B, w) = 1$ iff $v_M(A, w) = 1$ or $v_M(B, w) = 1$
C2.6 $v_M((\exists \alpha) A(\alpha), w) = 1$ iff $v_M(A(\beta), w) = 1$ for at least one $\beta \in \mathcal{C} \cup \mathcal{O}$
C2.7 $v_M(\Diamond A, w) = 1$ iff $v_M(A, w') = 1$ for at least one w' such that Rww'.

The other logical constants are defined as usual. A model M verifies $A \in \mathcal{W}^M$ iff $v_M(A, w_0) = 1$. A is valid iff it is verified by all models. Where Γ is a subset of $\mathcal{W}^M$, M is a **T**-model of Γ iff M verifies all $A \in \Gamma$. $\Gamma \models_{\mathbf{T}} A$ will denote that all **T**-models of Γ verify A.

I shall write $\Gamma \vdash_{\mathbf{T}} A$ to refer to the proof theory. The latter is obtained by extending the predicative version of **CL** with the usual axioms for **T** (including the Barcan formula).

5 Semantics of $\mathbf{T}^r$

In view of the intended application context, the logic $\mathbf{T}^r$ will only be defined for sets of premises of the form $\mathcal{T} = \mathcal{A}^* \cup \mathcal{D}^*$ (where $\mathcal{A}^*$ and $\mathcal{D}^*$ are as in Section 3).

As is the case for all adaptive logics, the semantics of $\mathbf{T}^r$ is obtained by selecting, for each set of premises, a *subset* of the models of some monotonic logic (in this case $\mathbf{T}$). More specifically, those $\mathbf{T}$-models are selected that verify a formula of the form $\Box\Box A$ only if it is 'unavoidable' in view of $\mathcal{A}$. This is in line with the plot behind $\mathbf{T}^r$, namely to assume that some formula A is compatible with $\mathcal{A}^*$, unless proven otherwise (that is, unless $\Box\Box\neg A$ is $\mathbf{T}$-derivable from $\mathcal{A}^*$).

If a formula of the form $\Box\Box A$ is $\mathbf{T}$-derivable from $\mathcal{A}^*$, it will be said to behave 'abnormally' with respect to $\mathcal{A}^*$ (it violates the assumption that $\neg A$ is compatible with $\mathcal{A}^*$). For some theories $\mathcal{T}$, a disjunction of abnormalities is derivable from $\mathcal{A}^*$, but none of its disjuncts is (we shall see an example in Section 7). In cases like this, $\mathbf{T}^r$ considers all disjuncts as 'unreliable' (none of them is assumed to be compatible with $\mathcal{A}^*$).[3]

Let Ω be $\{\Box\Box A \mid A \in \mathcal{W}\}$ and let $Dab(\Delta)$ refer to the disjunction $\bigvee(\Delta)$ in which $\Delta \subset \Omega$. Intuitively, $Dab(\Delta)$ stands for a disjunction of abnormalities. A formula of the form $Dab(\Delta)$ will be called a *minimal Dab-consequence of* $\mathcal{T}$ iff $\mathcal{T} \vDash_{\mathbf{T}} Dab(\Delta)$, and $\mathcal{T} \nvDash_{\mathbf{T}} Dab(\Delta')$, for any $\Delta' \subset \Delta$.

In order to define the selection criterion, I first define the *abnormal part* of a $\mathbf{T}$-model M and the set of formulas that are *unreliable* with respect to $\mathcal{T}$:[4]

Definition 1 $Ab(M) = \{A \mid A \in \mathcal{W};\ M \textit{ verifies } \Box\Box A\}$.

Definition 2 $U(\mathcal{T}) = \bigcup\{\Delta \mid Dab(\Delta) \textit{ is a minimal Dab-consequence of } \mathcal{T}\}$.

The selected models are those that verify a formula of the form $\Box\Box A$ only if it is unreliable with respect to $\mathcal{T}$:

Definition 3 *A* $\mathbf{T}$*-model* M *is a* $\mathbf{T}^r$*-model of* $\mathcal{T}$ *iff (i)* M *verifies all* $A \in \mathcal{T}$*, and (ii)* $Ab(M) \subseteq U(\mathcal{T})$*.*

As may be expected, the semantic consequence relation is defined with respect to the selected models:

Definition 4 $\mathcal{T} \vDash_{\mathbf{T}^r} A$ *iff* A *is true in all* $\mathbf{T}^r$*-models of* $\mathcal{T}$*.*

To see what the $\mathbf{T}^r$-semantics comes to, consider $\mathcal{T} = \{\Box\Box p,\ \Box\Box\neg q \vee (\Box p \supset \Box r)\}$. In that case, some $\mathbf{T}$-models of $\mathcal{T}$ verify $\Box\Box\neg q$, others verify $\neg\Box\Box\neg q$. However, as $U(\mathcal{T}) = \{A \in \Omega \mid \Box\Box p \vDash_{\mathbf{T}} A\}$, all $\mathbf{T}^r$-models of $\mathcal{T}$ *falsify* $\Box\Box\neg q$, and hence, verify $\Box r$.

6 Proof Theory of $\mathbf{T}^r$

As is usual for adaptive logics, I shall present the proof theory of $\mathbf{T}^r$ in a generic format. In addition to a premise rule PREM, there is an unconditional rule RU and a conditional rule RC. The deductive inferences are governed by RU, the ampliative ones by RC. Both RU and RC are formulated in terms of $\mathbf{T}$.

The proofs themselves look like those of other logics, except that every line has a *condition* attached to it. Thus, lines in a $\mathbf{T}^r$-proof have five elements: (i) a line number, (ii) the formula A that is derived, (iii) the line numbers of the formulas from which A is derived, (iv) the rule by which A is derived, and (v) the condition.

Intuitively, the rule RC allows one to introduce the hypothesis that a formula is compatible with the premises *on the condition* that the opposite has not been proven (see also the examples in the next section). A *marking definition* determines when a line in the proof should be marked (a line is marked if the condition that is attached to it is no longer fulfilled). A formula is considered to be derived at a certain stage in a proof iff it occurs on a line that is not marked.

Here are the generic rules that govern dynamic proofs from $\mathcal{T} = \mathcal{A}^* \cup \mathcal{D}^*$:

PREM If $A \in \mathcal{T}$, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) "–", (iv) "PREM", and (v) $\emptyset$.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A$ and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the line numbers of the B_i, (iv) "RU", and (v) $\Delta_1 \cup \ldots \cup \Delta_m$.

RC If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A \vee Dab(\Delta)$, and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the line numbers of the B_i, (iv) "RC", and (v) $\Delta \cup \Delta_1 \cup \ldots \cup \Delta_m$.

[3] The terminology is the usual one for adaptive logics—see the full version or [5] for more explanation.

[4] I leave it to the reader to check that a formula of the form $\Box\Box A$ is $\mathbf{T}$-derivable from $\mathcal{A}^*$ iff it is $\mathbf{T}$-derivable from $\mathcal{T}$ and hence that it safe to define the set of unreliable formulas with respect to $\mathcal{T}$.

The marking definition is formulated in terms of the (disjunctions of) abnormalities that have been derived in the proof. A formula $Dab(\Delta)$ is called a *minimal Dab-formula at stage s of a proof* iff, at that stage, $Dab(\Delta)$ occurs in the proof on the empty condition and, for any $\Delta' \subset \Delta$, $Dab(\Delta')$ does not occur in the proof on the empty condition.

Definition 5 $U_s(\mathcal{T}) = \bigcup\{\Delta \mid Dab(\Delta)$ *is a minimal Dab-formula at stage s of the proof*$\}$.

Definition 6 *Line i is marked at stage s of a proof from $\mathcal{T}$ iff, where Δ is its condition, $\Delta \cap U_s(\mathcal{T}) \neq \emptyset$.*

A formula is said to be *derived at stage s* in a $\mathbf{T}^r$-proof from $\mathcal{T}$ iff A is the second element of a line that is not marked in the proof at that stage. In addition to this, a notion of *final derivability* is defined:

Definition 7 *A is finally derived on line i of a $\mathbf{T}^r$-proof from $\mathcal{T}$ iff (i) A is the second element of line i, (ii) line i is not marked in the proof, and (iii) any extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.*

It may been shown that, if A is finally derivable from $\mathcal{T}$, then *any* $\mathbf{T}^r$-proof from $\mathcal{T}$ may be extended in such a way that A is finally derived in it. The derivability relation is defined with respect to the finally derived conclusions:

Definition 8 $\Gamma \vdash_{\mathbf{T}^r} A$ *(A is finally $\mathbf{T}^r$-derivable from $\mathcal{T}$) iff A is finally derived on some line of a $\mathbf{T}^r$-proof from $\mathcal{T}$.*

7 The Logic $\mathbf{R}^r$

The modal approach presented in the previous section has the advantage that default rules can be expressed at the object-level and therefore integrated in the proofs. A logic for the non-modal version of default theories can be obtained by the following definition:

(6) $\langle \mathcal{A}, \mathcal{D} \rangle \vdash_{\mathbf{R}^r} A$ iff $\mathcal{A}^* \cup \mathcal{D}^* \vdash_{\mathbf{T}^r} \Box A$

In the rest of this section, I present two examples that illustrate the dynamic proof theory and the adequacy of the logic obtained by Definition 6. The first example shows that $\mathbf{R}^r$ allows for the application of *Modus Ponens* to default rules, but not for the application of *Modus Tollens*. The second one illustrates the dynamical character of the proofs as well as the way in which default theories with multiple extensions are handled.

Example 1. $T = \left\langle \left\{ Bird(Tweety), \neg Flies(Billy) \right\},\ \left\{ \frac{Bird(x):\, Flies(x)}{Flies(x)} \right\} \right\rangle$

Intuitively, the default rule of this theory expresses that, if one knows that some a is a bird, one may infer from this that a flies, provided that it is compatible with one's knowledge that a flies. Following the modal translation from Section 3, this gives us the following premise:

1	$(\forall x)(\Box\Box\neg Flies(x) \vee (\Box Bird(x) \supset \Box Flies(x)))$	–	PREM	$\emptyset$

Note that the premise is written down on the *empty* condition. This is as it should be: at no stage of the proof should the premises be withdrawn. The two other premises are translated as:

2	$\Box\Box Bird(Tweety)$	–	PREM	$\emptyset$
3	$\Box\Box\neg Flies(Billy)$	–	PREM	$\emptyset$

As is easily observed, the rule RU allows one to add any instantiation of the formula on line 1 to the proof. So, we may add, for instance:

4	$\Box\Box\neg Flies(Tweety) \vee (\Box Bird(Tweety) \supset \Box Flies(Tweety))$	1	RU	$\emptyset$

As this is a purely deductive move, no condition is introduced. Suppose, however, that we add the following line to the proof (relying on the fact that $\Diamond\Diamond A \vee \Box\Box\neg A$ is a theorem of $\mathbf{T}$):

5	$\Diamond\Diamond Flies(Tweety)$	–	RC	$\{\Box\Box\neg Flies(Tweety)\}$

Intuitively, this line expresses that "$Flies(Tweety)$" is compatible with the members of $\mathcal{A}$. The condition warrants that the line will be marked as soon as it is discovered that "$Flies(Tweety)$" is *not* compatible with them. In view of line 5, we may now add:

6	$\Box Bird(Tweety) \supset \Box Flies(Tweety)$	4, 5	RU	$\{\Box\Box\neg Flies(Tweety)\}$
7	$\Box Flies(Tweety)$	2, 6	RU	$\{\Box\Box\neg Flies(Tweety)\}$

At this stage of the proof, the formula on line 7 is considered as derived in the proof (the line is not marked). I leave it to the reader to the check that the formula is also *finally derived*: $\Box\Box\neg Flies(Tweety)$ is not a disjunct of some minimal *Dab*-consequence of the premises. Hence, in view of Definition 6, $Flies(Tweety)$ is $\mathbf{R}^r$-derivable from T.

Let us now check whether also $\neg Bird(Billy)$ is derivable. Each of the following lines can be added:

8	$\Box\Box\neg Flies(Billy) \vee (\Box Bird(Billy) \supset \Box Flies(Billy))$	1	RU	$\emptyset$
9	$\Diamond\Diamond Flies(Billy)$	–	RC	$\{\Box\Box\neg Flies(Billy)\}$
10	$\Box Bird(Billy) \supset \Box Flies(Billy)$	8, 9	RU	$\{\Box\Box\neg Flies(Billy)\}$

However, $\Box\neg Bird(Billy)$ is neither $\mathbf{T}$-derivable from $\Box Bird(Billy) \supset \Box Flies(Billy)$ and $\Box\Box\neg Flies(Billy)$ nor from any other formulas that could be written down in the proof.

Example 2. $\left\langle \{Republican(Nixon),\ Quaker(Nixon)\},\ \left\{\frac{Republican(x)\colon \neg Pacifist(x)}{\neg Pacifist(x)},\ \frac{Quaker(x)\colon Pacifist(x)}{Pacifist(x)}\right\}\right\rangle$

Let us begin by writing down the information we have about Republicans:

1	$\Box\Box Republican(Nixon)$	–	PREM	$\emptyset$
2	$(\forall x)(\Box\Box Pacifist(x) \vee (\Box Republican(x) \supset \Box\neg Pacifist(x)))$	–	PREM	$\emptyset$

From these we may derive:

3	$\Diamond\Diamond Pacifist(Nixon)$	–	RC	$\{\Box\Box\neg Pacifist(Nixon)\}$
4	$\Box Republican(Nixon) \supset \Box\neg Pacifist(Nixon))$	2, 3	RU	$\{\Box\Box\neg Pacifist(Nixon)\}$
5	$\Box\neg Pacifist(Nixon))$	1, 4	RU	$\{\Box\Box\neg Pacifist(Nixon)\}$

At this stage of the proof, $\Box\neg Pacifist(Nixon))$ is considered as derived in the proof. In a similar way, we can also derive $\Box Pacifist(Nixon))$:

6	$\Box\Box Quaker(Nixon)$	–	PREM	$\emptyset$
7	$(\forall x)(\Box\Box\neg Pacifist(x) \vee (\Box Quaker(x) \supset \Box Pacifist(x)))$	–	PREM	$\emptyset$
8	$\Diamond\Diamond\neg Pacifist(Nixon)$	–	RC	$\{\Box\Box Pacifist(Nixon)\}$
9	$\Box Quaker(Nixon) \supset \Box Pacifist(Nixon))$	7, 8	RU	$\{\Box\Box Pacifist(Nixon)\}$
10	$\Box Pacifist(Nixon))$	6, 9	RU	$\{\Box\Box Pacifist(Nixon)\}$

However, neither $\Box\neg Pacifist(Nixon))$ nor $\Box Pacifist(Nixon))$ is finally derivable. This may be seen from the fact that the following formula is derivable:

11	$\Box\Box Pacifist(Nixon) \vee \Box\Box\neg Pacifist(Nixon))$	1, 2, 6, 7	RU	$\emptyset$

At this stage in the proof, the formula on line 11 is a minimal *Dab*-formula. Hence, all lines that have one of its disjuncts as a member of their condition are *marked*. This is how the result looks like:

1	$\Box\Box Republican(Nixon)$	–	PREM	$\emptyset$	
2	$(\forall x)(\Box\Box Pacifist(x) \vee (\Box Republican(x) \supset \Box\neg Pacifist(x)))$	–	PREM	$\emptyset$	
3	$\Diamond\Diamond Pacifist(Nixon)$	–	RC	$\{\Box\Box\neg Pacifist(Nixon)\}$	$\checkmark_{11}$
4	$\Box Republican(Nixon) \supset \Box\neg Pacifist(Nixon))$	2, 3	RU	$\{\Box\Box\neg Pacifist(Nixon)\}$	$\checkmark_{11}$
5	$\Box\neg Pacifist(Nixon))$	1, 4	RU	$\{\Box\Box\neg Pacifist(Nixon)\}$	$\checkmark_{11}$
6	$\Box\Box Quaker(Nixon)$	–	PREM	$\emptyset$	
7	$(\forall x)(\Box\Box\neg Pacifist(x) \vee (\Box Quaker(x) \supset \Box Pacifist(x)))$	–	PREM	$\emptyset$	
8	$\Diamond\Diamond\neg Pacifist(Nixon)$	–	RC	$\{\Box\Box Pacifist(Nixon)\}$	$\checkmark_{11}$
9	$\Box Quaker(Nixon) \supset \Box Pacifist(Nixon))$	7, 8	RU	$\{\Box\Box Pacifist(Nixon)\}$	$\checkmark_{11}$
10	$\Box Pacifist(Nixon))$	6, 9	RU	$\{\Box\Box Pacifist(Nixon)\}$	$\checkmark_{11}$
11	$\Box\Box Pacifist(Nixon) \vee \Box\Box\neg Pacifist(Nixon))$	1, 2, 6, 7	RU	$\emptyset$	

The formulas that are considered as derived in the proof at this stage are those that occur on non-marked lines. I leave it to the reader to check that the formula on line 11 is a minimal *Dab*-consequence of the premises. Hence, neither $Pacifist(Nixon)$ nor $\neg Pacifist(Nixon)$ is $\mathbf{R}^r$-derivable from T.

8 In Conclusion

As is clear from the last example, the logic $\mathbf{R}^r$ corresponds to the sceptical variant of default reasoning: when a default theory has several extensions, only the formulas that belong to their intersection are derivable. Importantly, however, it can easily be changed to a logic for other types of default reasoning. As readers familiar with adaptive logics will have noticed, the present logic is based on the *Reliability* strategy. This is the most cautious way to deal with 'abnormalities': when a disjunction of abnormalities is derivable, all disjuncts are considered as unreliable. By changing the strategy to one of the other available strategies (the Minimal Abnormality strategy, for instance), one obtains a logic for a different type of default logic.

A topic for further research is to elaborate these alternative versions and to study how they relate to the types of default reasoning that have been described in the literature on default logic. It would be interesting, for instance, to know which kind of adaptive strategy is needed to reconstruct the credulous form of default reasoning. It is my guess that such a study would not only lead to a successful reconstruction of available default logics, but also to the discovery of new types.

A problem of a different kind concerns the meta-theory of $\mathbf{R}^r$. This, however, is well in line with those on other adaptive logics and is presented in the full version of this paper.[5]

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[5] All unpublished papers in the reference section can be downloaded from `http://logica.rug.ac.be/centrum/writings`.