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PARACONSISTENCY AND ITS RELATION TO WORLDVIEWS

ABSTRACT. The paper highlights the import of the paraconsistent movement, list some motivations for its origin, and distinguishes some stands with respect to para-consistency. It then discusses some sources of inconsistency that are specific for worldviews, and the import of the paraconsistent turn for the worldviews enterprise.

KEY WORDS: inconsistent knowledge, paraconsistency, worldviews

1. AIM OF THIS PAPER

Some readers will be puzzled by the first and last substantives in the title. While the first may be unfamiliar, the second may sound unconnected to science. So, let me start with some explanation relating to the relevance of the present essay.

During this century, isolated individuals and, later, small groups developed logics that are now called *paraconsistent*. Phrased quite generally, such logics enable one to reason sensibly in the presence of inconsistencies (logical contradictions and sets of statements from which contradictions are derivable). Nicolas Vasil'év, a forerunner living in pre-revolutionary Russia, was largely unaware of modern logic. Stanislaw Jaśkowski, working in the excellent Polish logic tradition, presented the first paraconsistent logic in a 1948 lecture. Key figures were, in the sixties, the Brazilian mathematician Newton da Costa and the North-American philosopher Nicholas Rescher, and, in the seventies in Australia, Richard Routley (later Sylvan) and Robert K. Meyer who came from the relevant logic school. The movement spread rapidly in the eighties and now counts important groups of scholars in all parts of the world where logic is

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alive.[1] In section 2, I characterize paraconsistent logics, argue for their need, and list some considerations that led people to construct and study them. In section 3, I outline my philosophical stand connected with paraconsistency, which is pretty acceptable from common and traditional views, and confront it with the most popular alternative. A special brand of paraconsistent logics is described in section 4. These inconsistency-adaptive logics appear to display several attractive and novel features.

One of my central claims will be that, today, any sensible person should recognize that paraconsistent logics are, at the very least, necessary instruments for reasoning and for understanding other people's reasoning, and hence that their advent was an important advance in the history of logic. Whatever one's view on reality, life, knowledge, or, more narrowly, science, one cannot escape the conclusion that one sometimes (if not always) has to face inconsistent knowledge and that one has to reason *from* such knowledge.

Now to the second substantive in the title. While, at least from Mach on, the importance of worldviews has been almost constantly affirmed in the philosophy of science, almost no serious study has been devoted to them. A small movement started in Belgium, with the late Leo Apostel as its main motor, and quickly found international allies. For Apostel, a worldview should provide us with the basic guidelines for experiencing the world, understanding it, and acting in it. It was typical for the depth of his thought that he saw worldviews as intrinsically connected to the sciences, but that he did not want to restrict them to what is traditionally considered the realm of the sciences.

In order to be acceptable, worldviews should be compatible with our best established and most advanced scientific theories; and they should incorporate the most fundamental ontological principles of those theories.[2] However, worldviews should provide the basic guidelines for *all* our experiencing of the world, for all our under-

[1] The early history of paraconsistency is well-documented in Arruda (1980) and (1989).

[2] Two common confusions should be denounced. First, worldviews need not incorporate full blown theories. Next, worldviews need not incorporate even the ontological principles of *all* respectable theories in some domain; the situation in some domain may entail that any coherent worldview has to make a choice between rival theories.

standing of it, and for all acting in it. Hence, they should play a central role with respect to our scientific activities, for example in hypothesis generation, but they should also play a central role in the organization of our lives. In this sense they have a direct moral and ethical function, and, more fundamentally, a direct function with respect to the meaning of life and the meaning of the world.

Thus conceived, worldviews have an important function and their justification presents inherent difficulties. As a consequence, worldviews should not be left implicit; they should be constructed. For the same reasons, they should be treated as hypotheses or 'theories', a diversity of them should be developed, and they should compete with each other in pretty much the same way as scientific theories do.

Allow me to introduce a methodological point here. The need for methodological requirements (such as systematicity, coherence, empirical adequacy, etc.), the need for considering 'conceptual problems' (Laudan's term), and the need for competition and intellectual fight, arise at the level of theories, not at the level of observations. Although the latter are theory-laden, this theory-ladenness cannot be corrected or adjusted at the level of the observations themselves. Just as observations are theory-laden, observations as well as scientific theories are worldview-laden. Only by explicitly stimulating the competition between worldviews – and this competition will partly take place at a level that is quite remote from experience – shall we be able to correct or adjust the worldview-ladenness of observations and of scientific theories. As the present volume contains an expository essay on worldviews, I leave the matter here.

Given the importance of worldviews, it is vital that one realizes that some sources of inconsistencies are specific for worldview construction. I discuss these in section 5. Those sources of inconsistencies form the reason for writing the present essay. I shall argue that both handling and (where possible) resolving these inconsistencies require inconsistency-adaptive logics. In section 6, I briefly comment on the import of the paraconsistent turn for the worldviews enterprise.

2. PARACONSISTENT LOGIC AND THE MOTIVES FOR ITS DEVELOPMENT

A logic is paraconsistent if and only if it does not validate[3] $A, \sim A \vdash B$, the so-called *Ex Falso Quodlibet* (*EFQ*, sometimes called *Ex Contradictione Quodlibet* or *Explosion*). *EFQ* turns any inconsistent set of premises into a trivial[4] one, and hence any inconsistent theory into the[5] trivial theory (the theory that asserts all statements to be true).

Triviality is disastrous – any sensible person agrees to that. If every statement that can be phrased in English is true, there is no point in me going on writing or you going on reading. There is no point in it because, if every statement is true, no statement provides any information. If all statements are true, but you do not know this, you may think to learn something from a person who tells (or convinces) you of this truth. But even that is questionable. Indeed, if all statements are true, so is the statement 'Not all statements are true.' and even the statement 'All statements are false.'[6]

EFQ is validated by Classical Logic and by many other logics (*e.g.*, Intuitionistic Logic). That is so because the people who devised these logics were convinced that inconsistencies *cannot* be true. John may be mistaken in asserting A. Mary may be mistaken in asserting $\sim A$. Who is mistaken depends on what the world looks like (whether A or $\sim A$ corresponds to the world). But if Chris asserts both A and $\sim A$, then, so those logicians thought, Chris is bound to be mistaken *independently of what the world looks like*. Chris is *always* mistaken; he is mistaken for *logical* reasons.

Those logicians were relying on a 'firm tradition': already Aristotle had taught so, and this view of his is well-entrenched in Western culture. (Aristotle wrote many books, and is not free of

[3] Read $\sim A$ as 'not A', and $A_1, \ldots, A_n \vdash B$ as 'B is derivable from $A_1, \ldots, A_n$'.

[4] A set of premises is trivial if and only if every sentence/formula can be derived from it.

[5] Indeed, if, for an arbitrary statement A, both A and $\sim A$ are affirmed and *EFQ* is considered as valid, then any statement B should be affirmed. Remark that, given a language, there is only one trivial theory (see the subsequent explanation in the text).

[6] The matter is not better if a theory that is formulated in some formal language turns out trivial, except that it is informative to state, in a different language, that the theory is trivial.

inconsistency, even in the claim that contradictions cannot be true, but *passons*.) The firm tradition raises two questions. The first *seems* simple to answer: Is it indeed correct that inconsistencies cannot be true? The second question is more complex: Even if inconsistencies cannot be true, is this a reason to proclaim *EFQ* a law of logic?

Presumably you answer the first question in the affirmative. Most likely, your reason for this answer is the following consideration. When you affirm $\sim A$, you *mean* that A is false, you mean to state that A does *not* correspond to the world, you mean to reject A, you mean to exclude that A is true. If this is the meaning of $\sim A$, A and $\sim A$ cannot both be true. Let me grant you this for the moment – but I shall have to return to it later in the present section – and move on the *second*, more complex, question. Even if inconsistencies cannot be true, does it follow that *EFQ* is justified? Two difficulties arise.

The first and simpler difficulty is that there is something fishy about *EFQ*. The (Aristotelian!) criterion for validating a schema of the form $A, B \vdash C$ is that C is true whenever A and B are true (for these specific forms A, B, and C). In model-theoretic terms: C is true in all models in which both A and B are true. Now consider *EFQ*: $A, \sim A \vdash B$. According to the criterion, B is true whenever both A and $\sim A$ are true (B is true in all models in which A and $\sim A$ are true). Is that so? Yes, according to Classical Logic (and many other logics), but for a rather unexpected reason: it is never the case that both A and $\sim A$ are true (there are no models in which both A and $\sim A$ are true). The least one should say is that this is slightly misleading. One would expect to go through the models in which both A and $\sim A$ are true, and find B true in all of them. But one can do so only in the limit-case sense: there are *no* models in which both A and $\sim A$ are true, and hence there is no such model in which B is false. The validity of *EFQ* resembles the truth, in **CL**-languages, of ‘All unicorns are blue.’ As there are no unicorns, you cannot find a unicorn that is not blue. This is fishy (as is known for a long time), but it might be the inescapable outcome of a definition. However, there is worse.

During many (if not all) past periods, the best model of the world that was available to humans turned out to be inconsistent. There were inconsistencies in (individual as well as culturally shared) views on values and norms, in so-called everyday knowledge, in ontologies, in methodologies, and in other 'philosophical' theories, and even, at times, in some (mathematical and empirical) scientific theories. With respect to individual convictions the matter is even worse.

Is our present-day knowledge inconsistent? Some inconsistent theories are around. Some people reject them precisely for this reason. But the inconsistent theories are around because either there is no alternative or the alternatives are problematic in other respects. Moreover, we have no warrant – that much is obvious after Gödel – that the theories which we think to be consistent are indeed consistent. In the past, inconsistencies surfaced at points where they were totally unexpected. Next to Gödel's first theorem, this constitutes a very good reason for modesty with respect to consistency claims. The other limitative theorems of Arithmetic and the undecidability of Classical (Predicate) Logic reinforce such reasons. We better conclude that our present-day best theories may very well be inconsistent.[7]

There is no good reason to consider this situation as a temporary one. The cumulative view on scientific progress has been given up in the early nineteenth century. We are not collecting chunks of knowledge that are reliable in an absolute way. To the contrary, we often had to give up theories that were taken to be established. After Kuhn, Lakatos and Laudan, the converging (self-correcting) view on scientific progress was also given up. There is no warrant that our knowledge converges towards the truth or towards correctness. More importantly, there is no reason to believe that we shall ever reach a stage at which our knowledge will be free of the *kinds* of flaws that affect it today. This, it seems to me, provides a ground as firm as any to believe that, at any point in the future, our best theories may very well be inconsistent.

What are we to conclude from all this? According to Classical Logic, there is no point in reasoning from a set of statements once you derived an inconsistency from it. This, however, seems quite

[7] If they are, this very fact may make them problematic. But many of our best theories are problematic anyway in other respects. The aforementioned modesty, however, requires that we do not consider inconsistent theories as for that very reason worthless.

unjustified, as our knowledge often turns out to be inconsistent.[8] If the world is consistent, and one of our best theories is inconsistent (and hence false), we face two problems: (i) to live with the theory as long as we did not develop a consistent alternative that is at least as good in other respects, and (ii) to develop such an alternative.

Consider Cantor's set theory. It was know (by Cantor in the first place) to be inconsistent. Frege tried to get rid of the inconsistencies and presented an axiomatic system. Russell found an inconsistency in the latter. Russell thought he also found a way to avoid the inconsistency (and later, in collaboration with Whitehead, another way). In the intermediate periods, people lived with the inconsistent theories. Moreover, Frege *reasoned from* Cantor's set theory in trying to locate and eliminate the inconsistencies. And Russell reasoned from Frege's set theory in trying to locate and eliminate its inconsistencies. All this pleads against Classical Logic. According to the latter, Cantor's set theory and Frege's set theory are identical, viz. the trivial theory. As we saw earlier, there is no sensible way to reason from the trivial theory: once you realize it is trivial, you know that all statements are derivable from it and hence applying any rule of logic is just lost time: triviality warrants that *anything* is derivable. I do not mean to say that any statement *about sets* is derivable from it. '2 + 2 = 17' as well as 'The sun is made of blue cheese.' are just as derivable from it as 'For all P and Q, $P \in Q$.' (provided only that these sentences belong to the language).

It follows that Classical Logic is on the wrong track as far as *living with* our best theories is concerned, and that it is equally on the wrong track when it comes to removing inconsistencies from our knowledge. Indeed, we do not, and happily enough so, follow the policy to throw our knowledge overboard and start from scratch.[9] This point may be strengthened. Newton's infinitesimal calculus was inconsistent, and hence so was his mechanics (that contained the calculus). But no one would say (even in the present relativistic era) that Newton was *just all wrong*. Quite to the contrary, we all

[8] That you may refuse to call it knowledge for this very reason is rather immaterial. Moreover, it is not wise to do so, as, in view of historical facts, the presumable result of this convention is that knowledge is unavailable to humans.

[9] Some interesting case studies: Norton (1987) and (1993), Smith (1988), Nersessian (199+), Meheus (1993) and (199+c).

agree that he was definitely wrong to a lesser extent than Descartes or Kepler. And justly so. Hence, Classical Logic is mistaken in viewing Newton's mechanics as just a nonsense (and equivalent to Cantor's set theory).

So, even if the world is consistent – that is, even if all inconsistencies are false – and hence *EFQ* is justified with respect to true statements, this does not justify that we apply *EFQ* with respect to our fallible (and often false and inconsistent) knowledge.[10] We should eschew *EFQ* both in order to live with inconsistent theories and in order to remove the inconsistencies from them.

So much being established, let us now return to the first question: Is it correct that inconsistencies cannot be true? Many paragraphs ago, I granted you that someone asserting $\sim A$ usually *means* to exclude A by this. But the question remains: Can one exclude A? This question is deeper than it seems to be. Suppose for a moment that the world *is* inconsistent. Suppose that both A and $\sim A$ are true. Then, after establishing $\sim A$ by means however excellent, you may, in asserting $\sim A$, very well intend to exclude A, but you cannot exclude it. Your *intention* will not change anything to the truth of A. If, asserting $\sim A$, you mean to reject A, then your assertion is simply mistaken. My point is not, please beware, that you may assert $\sim A$ and be mistaken (because $\sim A$ is false). My point is that you may assert $\sim A$, and justly so because $\sim A$ is indeed true, but that nevertheless A may also be true. Of course, this argument relies on the supposition that the world is inconsistent. Most people take this supposition to be false. But how do we know that the world is consistent?

Graham Priest has been going around demonstrating that people's answers to this question are circular; they presuppose that the world is consistent. I think I have something to say in diagnosis of the problem. The statement that the world is consistent (or inconsistent) is confusing. The world consists of facts, events, and processes. It is hard to see in which way these could be either consistent or inconsistent. When we claim that the world is consistent, we mean to say that the true *description* of the world is

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[10] See Brown (1990) and de Costa and French (199+) on the truth of inconsistent theories with respect to a consistent world.

consistent.[11] A description presupposes a language and a correspondence relation that ties this language to the world. Whatever the world looks like, it is absolutely obvious that we may choose a language L and a correspondence relation R such that the true description of the world as determined by L and R is *inconsistent*. So, if one claims that the world is consistent, one can only intend to claim that, whatever the world looks like, *there is* a language L and a relation R such that the true description of the world as determined by L and R is consistent.

So, what about this? Remark, first, that the italicized 'is' in the last sentence of the previous paragraph signifies existence in the mathematical sense. For *any* subset Σ of the natural numbers, *there is* a function f that maps the natural numbers on the set $\{0, 1\}$ in such a way that $f(n) = 1$ if and only if n is a member of Σ. The number of such functions is uncountable, and most of them are undecidable. But they all exist (in that the statement asserting so is a mathematical truth). Let us now return to the question: Is it the case that, whatever the world looks like, *there is* a language L and a relation R such that the true description of the world as determined by L and R is consistent? The answer may be disappointing: no one demonstrated anything even remotely resembling an answer to this question. The reasons are that we have only a very tentative idea of what the world looks like, and none at all of what it *might* look like,[12] and that we have no idea of the languages that we are able to handle, let alone of those that might exist in the mathematical sense of the term. The situation gets even worse when we think about knowledge. Even if there is a language L and a relation R such that the true description of the world as determined by L and R is consistent, there is no warrant at all that humans will ever be able to handle[13] L or to sufficiently

[11] If you are a positivist, you mean something even weaker: the correct description of the phenomena is consistent, and the best theory (the one that saves the phenomena and fulfils some further criteria) is consistent.

[12] My point concerns the *structure* of the world. If this is (possibly a refinement of) the structure of a standard model of Classical Logic, then there obviously is a language L and a relation R such that the true description of the world as determined by L and R is consistent.

[13] It is generally accepted that humans are unable to handle uncountable languages – remember the Löwenheim-Skolem Theorem.

get a grasp[14] on R in such a way that our knowledge of the world will be consistent.[15]

Let me summarize. Even if the world is consistent, our knowledge often requires that we apply a paraconsistent logic rather than Classical Logic. Moreover, no one proved (in a non-circular way) that 'the world is consistent'. As we saw, to find such a proof is wildly beyond human capacities – whence the Aristotelian consistency tradition seems to reduce to sheer prejudice (but see the next section).

To conclude this section, I make two historical remarks. The first concerns the multiplicity of motivations for developing paraconsistent logics, which I list in a somewhat random manner. The set theoretical paradoxes, especially the Russell paradox,[16] are standardly avoided by introducing restrictions that seem *ad hoc*. This fact made it attractive to study paradoxical set theories; the aim is to avoid *ad hoc* restrictions at the expanse of making the underlying logic paraconsistent. A similar motivation derives from the semantic paradoxes, first and foremost Tarski's:[17] they are only avoided by introducing restrictions that seem *ad hoc*. The occurrence of inconsistencies in mathematical and empirical theories, and in knowledge systems in general, forms a further partly independent motivation. A motivation of a different nature derives from the relevance tradition – see especially Anderson and Belnap (1975), Anderson et al. (1992), Routley (1982), and Read (1988). This tradition aimed at circumventing the 'paradoxes'[18] of Classical Logic. As *EFQ* is an

[14] This presupposes that our *criteria* to determine the truth (or acceptability) of a sentence are such that they select the true ones.

[15] I am fighting the traditional view here. Let me add that no one demonstrated that a specific possible structure of the world *cannot* be consistently described with respect to some L and R, and that no one demonstrated that our future knowledge is necessarily inconsistent.

[16] The axioms of Frege's set theory are generally recognized to be extremely natural. From them, Russell derived that $R = \{x \mid x \notin x\}$, the set of all sets that are not a member of themselves, is and is not a member of itself ($R \in R$ and $R \notin R$ are theorems).

[17] The simplest one: given the principle that 'A' is true if and only if A, and given that to be false means not to be true, 'This sentence is false.' is easily shown to be both true and false.

[18] Classical logic is consistent (if A is a theorem, $\sim A$ is not). However, it contains some oddities (usually called paradoxes) that agree neither with everyday

obvious paradox of Classical Logic, relevant logics do not validate it. Hence they are paraconsistent (even if no founding father of the relevance tradition seems to have questioned the consistency of the world). Finally, possibly attractive aspects of the so-called dialectical tradition (as re-founded by Hegel), with its emphasis on movement and change, provided some further motivation of a still different nature – see for example Apostel (1979). The recent development of artificial intelligence and the actual inconsistency of many massive databases aroused interest in those circles as well.

The second remark concerns the reaction to this development. The consistency of the world has been so deeply entrenched in our culture that many people are shocked when they hear that paraconsistent logics are *possible*. In the seventies and early eighties, some logicians still reacted in an overtly hostile way to colleagues working on paraconsistent logics.

3. A PHILOSOPHICAL STAND

In the previous section I defended a positive answer to the second question: we need paraconsistent logics even if the world is consistent. My treatment of the first question will have disappointed some (especially Continental) readers. I argued that the consistency of the world should be phrased in terms of possible languages and correspondence relations. And I noted that no one has offered a demonstration that settles the answer to the thus reformulated question. But what do I think the answer to be? And what about related questions? In the present section, I briefly outline my position and oppose it to a different position (that I shall not be able to fully do justice to).

My answer to the second question of the previous section may have been unexpected for some readers. Still, neither the answer nor the arguments for it entail that the world is inconsistent. We nevertheless need paraconsistent logics because logic is to be applied to our theories, not to the world. Those who believe that there is One True Logic, will conclude that it is paraconsistent. Those who,

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reasoning practice nor with our intuitions. Some examples: where A and B are arbitrary sentences, according to Classical Logic, $A \supset B$ (A implies B) is true whenever B is true; and $B \vee \sim B$ (B or not B) is derivable 'from' A.

like me, see logics as instruments or as theories about fragments of languages – theories determining the meaning of such non-referring words as ‘not’, ‘and’, ‘all’, etc. – will conclude that paraconsistent logics should be applied whenever inconsistent knowledge is involved.[19]

Is the world consistent? The most notorious argument in favour of its inconsistency may be found in Priest (1987). The position defended there, relying on the semantic paradoxes, is called dialetheic: some sentences are both true and false. Those sentences are both true and false for logical reasons, viz. because of the meaning of ‘true’ and ‘false’. As the linguistic realm definitely belongs to the world, it follows that the world is inconsistent. Given this, and given Priest’s view on logic, it follows that the True Logic is paraconsistent.

This is not the place to quarrel about Priest’s position (some explicit objections may be found in Batens (1990) and some (mainly) implicit ones in Batens (199+c)). I shall just spell out and somewhat defend my (quite different) position on the matter. A comparison is useful here. In his (1986), John Earman argued convincingly that the question whether the world is deterministic should not be settled on the basis of our present best theories; determinism is a *methodological requirement*: whenever some theory is not deterministic, this is seen as a problem and research is continued in an attempt to make it deterministic. Consistency is pretty much like determinism in this respect. Whenever one of our best (mathematical, empirical, or evaluative) theories is inconsistent, this should be seen as a problem and we should strive for a consistent replacement of the theory.

How may this position be justified? The justification of methodological determinism is straightforward. Only the search for deterministic theories may provide us with theories that catch the deterministic mechanisms present in the world. This is why determinism is a sensible general methodological requirement. The justification for the consistency requirement runs along the same lines.

[19] These two views on logic are connected to distinct conceptions of natural languages – see Batens (199+c).

There is, however, a further question. To search for deterministic theories seems the only sensible way to systematically increase our chances of locating deterministic mechanisms that act in the world. This clearly advances our knowledge. Knowledge, and more specifically lawlike knowledge, presupposes determinism. But does knowledge presuppose consistency? As is apparent from the previous section, the answer to this question is ‘No’. Nevertheless, the consistency requirement is justified because consistent knowledge is, *ceteris paribus*, preferable over inconsistent knowledge. Let me argue briefly for this important point.

Let P be a unary predicate of the language of an inconsistent theory, and let some paraconsistent logic **PL** be the underlying logic of the theory. Let us suppose that **PL** is not paracomplete (hence, for any sentence A, either A or $\sim A$ is true). P divides the objects into three[20] subsets: those that are P only, those that are $\sim P$ only, and those that are both P and $\sim P$. The sentence $Pa \,\&\, \sim Pa$ unequivocally locates a among the objects that are inconsistent with respect to P. There is no way, however, to locate a in the union of the first and third set, not in the second only.[21] Compare this situation to the one in which P belongs to a consistent theory (of which the underlying logic validates *EFQ*). Here P introduces two sets only; Pa unequivocally locates a in the first set, $\sim Pa$ unequivocally locates a in the second one. If there is a need for three sets, then one introduces a family of predicates (Carnap’s term), say P_1, P_2, and P_3. The predicates of a family are exhaustive and mutually exclusive. So, they divide the objects in three sets, $P_1 a$ unequivocally locates a in the first, $P_2 a$ in the second, and $P_3 a$ in the third. Whether you need

[20] For the sake of the example, I suppose that $\sim(A \,\&\, \sim A)$ – not both A and $\sim A$ – is **PL**-equivalent to $A \vee \sim A$. I also suppose that the latter is a theorem, and I suppose some more stuff that reduces the sets to three.

[21] If the logic would contain a means to locate a in the third set only, say in that $*Pa$ excludes that a belongs to the second set, then $\sim Pa \,\&\, *Pa$ might be used to define a negation of Pa, say $\neg\, Pa$, that might be weaker than that of Classical Logic but still validates *EFQ* (intuitionistic negation is a good example). In this case, the logic still contains a paraconsistent negation, but it also enables one to state that some sentence does not behave inconsistently.

two or three sets (this depends on ‘the world’), the consistent theory is more precise.[22]

Let me summarize my position. Our present-day best knowledge may very well be inconsistent. At any future point in time, our best knowledge may still be inconsistent. Yet, consistency is a sound methodological requirement, justified by the advantages of precise knowledge.

Two further remarks are in place. Some people might conclude from the next to last paragraph that nothing is simpler than turning an inconsistent theory into a consistent one. They are mistaken. The point is easily illustrated by means of the Russell paradox. Frege’s axioms lead to the existence of a set, call it R, such that $R \in R$ if and only if $R \notin R$ – whence $R \in R$ and $R \notin R$ are both theorems. It is indeed easy to replace the membership relation by a family of three relations: to be in the only-a-member-of-relation, to be in the only-a-non-member-of relation, and to be in the mixed relation. However, as any reader familiar with the stuff will easily find out, any ‘translation’ of the abstraction axiom into the new terminology will provoke another paradox (see Batens (199+c)). So, although we have a way to ‘resolve’ the inconsistency at the linguistic level, we still have no consistent set theory expressed within the new language. Of course, we might translate into the new language the sentences derivable from Frege’s theory in the old language. But as Frege’s theory is trivial, its translation will be fully uninteresting (all sets are in the mixed relation to each other). The moral is that an algorithmic means to eliminate an inconsistency might, if it succeeds at all, lead to a result that is more problematic than its inconsistent predecessor (provided the latter is handled paraconsistently).

My second remark concerns the persistence of methodological requirements. In the history of the sciences, many methodological requirements were first considered as absolute and later given up or relativised (in that they have to be combined with other requirements). But this does not affect my view. I do not believe in any methodological requirements that are absolute in the here intended sense: as a rule, the choice between several theories depends on multiple criteria. Just like partly indeterministic theories, partly

[22] Those unfamiliar with the advantages of precision should read Popper (1935).

inconsistent theories may very well be the best amongst the theories available at a particular point in time. Given this, we should distinguish between methodological requirements that are ontologically justified and those that are methodologically justified. Galileo and Kepler (and others in the seventeenth century) were convinced that the mathematics of their days correspond to 'the creator's frame of thought'. Even Newton shared this conviction, provided mathematics was upgraded to his infinitesimal calculus. Simplicity in *this* sense ('if it is not simple with respect to these mathematical theories, there is a problem') is an ontologically justified methodological requirement. We meanwhile gave it up because our ontology changed. Indeed, the latter is not independent of the history of the sciences. The above justification of determinism and consistency is different, viz. methodological.[23] I concede that the distinction between ontological and methodological justifications is not an absolute one.[24] There may come a time when determinism or consistency are justly given up as methodological requirements (or are unmasked as relying on ontological presuppositions). For the time being, however, nothing points in that direction. So, even if the truth theory for natural languages defended in Priest (1987) is the best available one today – I promised not to discuss the point – this in itself is not a sufficient reason to give up the consistency requirement.

4. ADAPTIVE LOGICS

Paraconsistent logics prevent an inconsistent theory from being turned into triviality, but they do not presuppose that all, or even some, true theories are inconsistent. They allow for inconsistencies, but do not require them. In model-theoretic terms: where **PL** is a paraconsistent logic, some **PL**-models are inconsistent, while others

[23] So is the justification of simplicity in a different sense: pick the simplest hypothesis that fits the phenomena. This maxim is still relied upon today in curve fitting and other hypothesis generating methods. (Its use is restricted in view of the absence of a general criterion for simplicity.)

[24] Our methodological justifications are in general more stable than our ontology (because they are more indirectly justified), but they are by no means perpetual.

are consistent. Actually, all models of Classical Logic are models of most paraconsistent logics. Paraconsistent logics extend the set of models; they introduce inconsistent models next to the consistent ones.

As paraconsistent logics allow for inconsistencies, they do not validate several inference forms that are correct according to Classical Logic. Consider Disjunctive Syllogism: from $A \vee B$ and $\sim A$ to derive B. Think about this in model-theoretic terms. $A \vee B$ is true in a model if and only if A is true in it or B is true in it. If all models are consistent, then B is true in all models in which both $A \vee B$ and $\sim A$ are true. The picture changes if one allows for inconsistent models. In some of them A and $\sim A$ are true, and B is false. As A is true in them, so is $A \vee B$. Hence, B is false in some models in which both $A \vee B$ and $\sim A$ are true. It follows that Disjunctive Syllogism is invalid in paraconsistent logics.[25]

Consider a theory, such as Frege's set theory, that the author supposed to be consistent, but later turned out to be inconsistent. As I argued in section 2, we need a paraconsistent logic in order to reason sensibly about such theory. However, replacing Classical Logic by a paraconsistent logic is a rather drastic move. We discovered one inconsistency in our theory, say A and $\sim A$. Obviously, we do not want to apply Disjunctive Syllogism to, say, $A \vee B$ and $\sim A$.[26] But if also $\sim C$ and $C \vee D$ are derivable from the theory, and C does not behave inconsistently on the theory, then why should we refrain from applying Disjunctive Syllogism to *these* formulas. In other words, why should we not derive D? It seems then that replacing Classical Logic by a paraconsistent logic is too drastic a move in the present circumstances.

If the world is consistent, or if consistency is a justified methodological requirement, the same reasoning applies whenever our knowledge about the world is inconsistent. We want to safeguard our knowledge against triviality, but we want to apply all rules of

25 There are a few exceptions that combine a non-standard negation with a non-standard disjunction. A fascinating example is presented in Meheus (199+a).

26 The reason is also obvious from a proof theoretic point of view. A and $\sim A$ have been derived from the theory. From A follows $A \vee B$ (by Addition) for any B. From $A \vee B$ and $\sim A$ follows B (by Disjunctive Syllogism). So, Disjunctive Syllogism and Addition make *EFQ* a derivable rule.

Classical Logic whenever the risk for triviality is absent. Precisely this effect is realized by inconsistency-adaptive logics.

This special brand of paraconsistent logics, discovered[27] around 1980, allow for inconsistencies but presuppose the consistency of all sentences 'unless and until proven otherwise'. Interpreting a theory 'as consistently as possible', they *adapt* to the *specific* inconsistencies that occur in it. Unlike what is the case for Classical Logic, the consistency requirement is not presupposed generally. But unlike what is the case for (usual) paraconsistent logics, consistency is presupposed whenever the premises do not require inconsistency.[28]

Inconsistency-adaptive logics have a number of fascinating properties that cannot be described here. The following list is intended as merely suggestive. These logics have a dynamic proof theory: as we proceed in deriving consequences from the premises or theory, we locate the inconsistencies and hence may have to review our previous judgement on what is derivable from the theory. Still, the 'final consequence set' is stable and proof independent: in the long run, all proofs from a set of premises result in the same set of finally derived consequences. Both the dynamic proof procedure, and the stable final-consequence set is characterized by an adequate semantics. Inconsistency-adaptive logics are non-monotonic (adding premises may render some consequences underivable). This is as expected: if $A \vee B$ and $\sim A$ are derivable from a theory, but A is not, we want B to be derivable; if, however, A is added to the theory, we want B not to be derivable any more. If applied to a consistent set of premises, adaptive logics deliver exactly the same consequences as Classical Logic, as desired. If applied to an inconsistent set, then, except for some limit cases, adaptive logics deliver *more* consequences than the corresponding monotonic paraconsistent logics, but less than Classical Logic (which delivers triviality). It is instructive to offer an

[27] See Batens (1989), which was written earlier than the subsequent papers, and Batens (1985), (1986a), and (1986b). For a study of the predicative version see Batens (1998). A survey of the domain is presented in Batens (199+b). For an informal description and the relation with argumentation, see Batens (1996).

[28] A special case is where $(A \,\&\, \sim A) \vee (B \,\&\, \sim B)$ is derivable from the premises, while neither disjunct is. Such cases – there may be more disjuncts, and they may be existentially quantified – lead to the first diversification in strategies to interpret the premises 'as consistently as possible'. See Batens (1989) and (199+b).

idea of the semantics of inconsistency-adaptive logics. For one particular brand, it simply comes to this: the final consequences of the premises are the formulas true in all minimally inconsistent models of the premises.[29] All this is as desired, but that formal logics (in which no extra-logical preferences are invoked) have such properties is rather unexpected.

I have suggested that inconsistency-adaptive logics are adequate in situations in which our knowledge happens to be inconsistent. This holds even if the world is consistent (or if consistency is justified as a methodological requirement). This is why those logics should be acceptable even to people that have rather traditional views on logic and consistency.[30] However, inconsistency-adaptive logics are also acceptable to dialetheists. While they claim that there are true inconsistencies, they by no means take all statements to be inconsistent; rather, they agree that consistency is the 'normal' case, that *A* should be taken to behave consistently unless and until we have a reason to draw the opposite conclusion. This means that they are eager to upgrade their preferred paraconsistent logic to an adaptive logic – see Priest (1991).

So, inconsistency-adaptive logics seem to have turned into a centre of unity with respect to inconsistencies. Our best knowledge may be inconsistent. When it is, we have to reason from this knowledge (for example in order to find a consistent improvement). Whichever one's view on the consistency of the world, inconsistency-adaptive logics seem the right tool to do so.

5. SOURCES OF INCONSISTENCIES IN WORLDVIEWS

An inconsistency in a worldview may provide from the best (empirical or mathematical) theory that is available at the time. As I

[29] A model *M* is less inconsistent than a model M' if and only if M' verifies all inconsistencies verified by *M*, but not *vice versa*.

[30] Such people often argue that inconsistencies should be handled by the mechanism of Rescher (1964) and Rescher and Manor (1970): divide the premises in consistent chunks and see what follows from some of them, from all, or from the preferred ones. As may be seen from Batens (199+b) and (199+d) this mechanism is indeed a special inconsistency-adaptive logic, but often other inconsistency-adaptive logics are more adequate.

remarked in footnote 2, a worldview need not incorporate full blown theories. Hence, only some inconsistencies that occur in theories will have an effect on worldviews. A further source of inconsistencies may reside in our observational criteria (or in the way we handle instruments, etc.). If these were the only sources of inconsistencies in worldviews, the specific topic of the present paper would hardly be interesting; one should simply consider the means by which inconsistencies in theories or in observational criteria are approached. The same remark applies to cases were the solution of a scientific problem involves inconsistent constraints – see especially Nickles (1980). However, worldview construction may lead also to inconsistencies of a different origin; some inconsistencies surface only in worldview construction.

The most obvious case is where theories from different domains contradict each other in aspects that are relevant to worldviews. Some useful examples are mentioned in chapter 2 of Laudan (1977). In such cases, there may be no problem whatsoever if the involved disciplines are considered in isolation. The inconsistency ensues when the ontologies underlying the best theories in the domains are combined into a worldview.[31]

An even more earnest problem may arise (for worldview construction) when conflicting theories in some domain happen to be of equal merit – in some periods corpuscular and wave theories of light were equally justified (and equally incomplete in explanatory power). Such cases may force one to integrate the ontology of one of the theories in a worldview, which will not lead to inconsistency, but to conflicting worldviews. However, each of the conflicting theories may be so obviously incomplete that any decent worldview should incorporate elements from both. In this case, the worldviews are likely to be inconsistent.

[31] In this sense, worldviews have a problem generating function with respect to the sciences. Of course, this function is just as well served by Mach's much weaker idea of a unified science. Similarly, the hypothesis generating function of worldviews is served by Mach's interpretations (he usually calls them 'hypotheses') – see Meheus (199+b). The worldviews enterprise provides a justification for both unification and the generative role of interpretations. Mach would have objected to Apostel's worldviews, but only for reasons that are now generally rejected.

In view of the function of worldviews – see section 1 – the agnostic alternative may be ruled out. If one chooses for one of the conflicting theories, the worldview will be affected by a serious anomaly (determined by the merits of the alternative theory). For this reason, it may be wiser to take both theories into account, even if this causes the worldview to be inconsistent. The inconsistency will constitute a problem, but will also enable one to find out which other parts of the worldview go along with each 'half' of the inconsistency. Moreover, searching for a solution of the problem will generally be preferable to ignoring one of the conflicting theories. For one thing, a conceptual reorganization may result from the search for a solution of the inconsistency in the worldview. If this is the case, the solution at the worldview level may prepare for a solution at the level of the scientific theories.[32] To the extent that the worldview is better integrated, it will provide a richer set of constraints. For example, several constraints will derive from theories that are only linked to the problem through the worldview.[33] Even if the problem with the scientific theories is solved independently, it is likely that the inconsistent worldview will be adapted in a rather smooth way to this solution. Compare this to the case where one of the conflicting theories is ignored. If that theory turns out to prevail, one will be forced to reorganize one's worldview in a much more drastic way – the full bet was on the wrong alternative. All this suggests that, in the case under discussion, inconsistent worldviews are superior to consistent ones. In worldviews as in scientific practice, it is usually preferable to face an inconsistency rather than to neglect one half of it.[34]

An equally critical problem arises when some scientific development results in the prevalence of a theory that conflicts with an earlier established worldview. In some cases, one simply has to

[32] Needless to say, the solution at the worldview level will not be final unless there is a solution at the level of the theories.

[33] During the problem-solving process, some constraints may be eliminated while others are transformed. Nevertheless, to start from a 'richer set of ideas' will in general further the solution of the problem.

[34] Worldviews are the right place to face certain inter-theory inconsistencies. It should be stressed, however, that to do so makes only sense in specific cases, in which the incompleteness of the conflicting theories makes it plausible that no sound solution will be reached without their integration.

rework the worldview (as suggested in the previous paragraph). In other cases, especially if the worldview is well integrated and is the result of careful critical examination, these features of the worldview may justify that one opposes the new theory. I again refer to Laudan (1977), viz. to pp. 57–61 and especially to pp. 61–64 (with as title 'Worldview difficulties'). If both the worldview and the new theory are highly valuable, and their combination results in a serious inconsistency, the most sensible (and effective) reaction may be to integrate the ontology of the new theory into the worldview and to face the inconsistency. The advantage of this approach is as discussed in the previous paragraph.

As I remarked in section 1 (the methodological point), the construction of worldviews is especially important with respect to disagreements that cannot be settled in terms of sensory experience, empirical generalization, etc. This holds especially for a host of 'theories' that are rather central to worldviews: normative and evaluative theories (on moral, aesthetic and methodological matters); stands on the meaning of life and the meaning of the world; nominalism versus platonism ('realism'); all sorts of realism versus corresponding forms of idealism, positivism, and pragmatism; absolute versus relative justification, and similar epistemological stands; and so on. Disagreements in such matters can only be settled within worldviews – piecemeal arguments may be relevant, but may always be rebuffed by referring to the coherence of a more embracing 'theory'. In connection with the intended tenets, someone's implicit worldview will unavoidably play a decisive role in the construction of an explicit worldview. And precisely because an implicit worldview is usually gathered from sundry sources, it likely will result in a multiplicity of inconsistencies in the constructed worldview.

Although this list may be prolonged, it seems sufficient to show that worldview construction tends to lead to inconsistencies that do not arise at the level of observation or at the level of scientific theories (or scientific practice).

By which logical means an inconsistent worldview should be approached depends in part on the worldview itself. People who share my view that consistency is a methodological requirement, will regard all inconsistencies as problematic. Dialetheists will consider some inconsistencies as established and hence as unproblematic. In general, however, dialetheists will consider inconsistencies as problematic unless they are established by an argument that they consider as convincing. From both points of view, the right logical approach is an adaptive logic. The specific logic that will be chosen may vary. Methodological consistency will lead to an adaptive logic that 'oscillates' between Classical Logic[35] and a paraconsistent logic 'derived' from it (by giving up the model-theoretic consistency requirement). Dialetheism will lead to an adaptive logic that oscillates between Classical Logic and the worldview's preferred paraconsistent logic (the True Logic according to the worldview).

6. IN CONCLUSION

I hope to have shown that the paraconsistent turn constitutes an extremely important development in logic. The central reason for its importance is not that it liberates Western thinking from a deeply rooted prejudice on logic, but rather that it provides us with reasoning instruments for important and apparently unavoidable situations that cannot be handled by Classical Logic (and other logics validating *EFQ*). I also hope to have shown that the application of paraconsistent logics and of inconsistency-adaptive logics is compatible with traditional logical views. I did not offer any direct objections to the dialetheist position, and leave that matter unsettled here. The discussion on the justified application of the afore-mentioned logics seems sufficiently thought provoking for now.

An important lesson to be drawn from the preceding sections is that an inconsistent set of statements may be (i) less problematic than any consistent alternative available at the time (static evaluation), and (ii) an important instrument to arrive at a consistent improvement (dynamic evaluation).

All this is extremely important for the worldviews enterprise. Given the results on paraconsistency, I was able to follow the line of argument of section 5, and defend the inclusion of inconsistencies in worldviews as the most justified move in some circumstances

[35] Or, depending on the worldview, an extension of it, or another logic validating *EFQ*, for example intuitionistic logic.

– a move that may prove effective for resolving intra-scientific inconsistencies. If we had not been liberated from the absolute and all-embracing consistency requirement (and hence from *EFQ* as a logical closure condition), such line of argument would have been impossible. Let me phrase this in a different way. Notwithstanding the strong arguments for worldview construction, this enterprise would appear as extremely problematic in that, if taken serious, it requires inconsistent worldviews in the situations described in section 5. If consistency is an absolute requirement, worldview construction seems impossible in most historical situations. By the insights gained from the paraconsistent turn, this restriction is overcome. And fortunately so, for worldviews are important, even for the sciences.

Let me summarize. The paraconsistent turn enables us to *face* and *handle* inconsistencies as problems, rather than as irrevocable verdicts that force us to start again from scratch. Without this change, worldview construction would not be a sensible enterprise.

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