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Induction from a Single Instance: Incomplete Frames

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Abstract In this paper we argue that an existing theory of concepts called *dynamic frame theory*, although not developed with that purpose in mind, allows for the precise formulation of a number of problems associated with induction from a single instance. A key role is played by the distinction we introduce between complete and incomplete dynamic frames, for incomplete frames seem to be very elegant candidates for the format of the background knowledge used in induction from a single instance. Furthermore, we show how dynamic frame theory provides the terminology to discuss the justification and the fallibility of incomplete frames. In the Appendix, we give a formal account of incomplete frames and the way these lead to induction from a single instance.

Keywords Concepts · Dynamic frames · Single instance induction · Induction

1 Induction from a Single Instance

The word 'induction' can refer to a whole variety of reasoning methods.[1] This paper however, is mostly concerned with a specific type of inference. Take the following two examples, motivated by Norton (2003: 649) and Steel (2008: 88), respectively:

$$\frac{\text{One sample of bismuth melts at 271 °C.}}{\text{All samples of bismuth melt at 271 °C.}}$$

$$\frac{\text{Bob's 2005 VW Beetle has its wheel-drive in the front.}}{\text{All 2005 VW Beetles have their wheel-drive in the front.}}$$

[1] Vickers (2010) gives a nice survey of those.

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These are inductions where from the fact that one element of a given class of objects has a certain predicate, a generalization is inferred which attributes that predicate to all elements of this class. We will call this inference pattern 'induction from a single instance', and henceforth abbreviate it as ISI. The general schema for ISI is:

$$\textbf{(ISI)}\quad \frac{\text{One } C \text{ has } P.}{\text{All } C\text{'s have } P.}$$

where C is a predicate determining a class and P is another predicate.

Now, it is obvious that this schema doesn't hold for all classes C and all predicates P. Although examples of apparently reliable ISIs are numerous, it is just as easy to give a counterexample to this type of inference. If we replace bismuth by wax in the first example, the induction would be unwarranted. It would be equally unreliable if the predicate in the second example was 'is blue' instead of 'has its wheel-drive in the front'.[2]

What is it that makes ISI rational in some cases, but not in others? There have been many approaches to this question. Notwithstanding the difference in terminology, most authors agree on the necessity of some specific background knowledge that accounts for the selective reliability of ISI. Mill (1973: 308–311) calls it a 'hidden major premise' and pursues the idea that every induction is actually a syllogism. Goodman (1978: 110) speaks of a 'positive overhypothesis', Thagard and Nisbett (1982: 380) refer to 'knowledge of variability within kinds', Davies (1988: 233) points at knowledge of 'determination relations', and more recently Norton (2003: 650) refers to a certain kind of 'material facts'.

This background knowledge (BK) has to be strong enough to make a single instance sufficient for the inductive conclusion. However, BK shouldn't by itself entail this generalization, for this would make the single instance redundant. It is BK *together with* the instance that lets one derive the desired generalization.[3]

Take our second example. On the face of it, the BK comes down to this: 'Either all 2005 VW Beetles have their wheel-drive in the front, they all have it in the rear, or they have a 4×4 wheel-drive'. With this assumption, knowing that at least one car of this year and making has its engine in the front enables one to infer that all 2005 VW Beetles have their engine in the front.

In cases such as the car example, the BK implies a *disjunction* of generalizations. Each of these generalizations is about the same class C of objects, assigning a certain predicate P to all its members. Also, we believe that if one of these generalizations is true, then all others are false—they are mutually exclusive. However, we don't know which one is true. The instance, itself a member of C, falsifies all but one of these generalizations, and hence 'picks out' that generalization as the inductive conclusion. This turns the induction into a classical inference, which explains its great strength compared to other cases of ISIs which

[2] The specific reasoning pattern that we call ISI is well-known in cognitive sciences and was shown to be a robust phenomenon in experimental settings. See e.g. Thagard and Nisbett (1982: 380) for an example concerning the physical behavior of a kind of metal, floridium. Moreover, the impact of the projectability of certain predicates on inductive inferences about these predicates is a well-established fact in cognitive psychology (see Heit 2000).

[3] Davies (1988: 231) in particular stresses this point, and calls it "The non-redundancy problem".

lack such additional assumptions.[4] In the Appendix, we show how this case and similar ones can be formalized in a first order language.[5]

We agree that every ISI depends on some domain-specific background knowledge. However, once we accept this, the focus is immediately moved to more detailed questions: (i) is there any uniform relation between the classes about which we generalize and the predicates that we generalize about? (ii) how can we justify the BK for an ISI? (iii) how do we revise the BK once it becomes falsified by new evidence? (iv) can we give any formal account of (ii) and (iii)?

Our claim is that the BK that licenses ISI can be very conveniently studied using *dynamic conceptual frames*. We will start with a brief explanation of what these are in Sect. 2. Section 3 introduces the distinction between complete and incomplete dynamic frames. In Sect. 4 we discuss the role this distinction plays in a fairly uniform account of ISIs. Section 5 is devoted to an explanation of how incomplete dynamic frames come to be accepted. In Sect. 6 we describe certain ways the frames may be revised when anomalies are encountered in the context of ISI. In the remaining section, we will discuss some arguments in favor of our approach, and remark on some loose ends and prospects for further research.

2 Dynamic Frames

On the *classical theory of concepts*, to each concept there corresponds a set of necessary and sufficient conditions for falling under that concept, a set of conditions that can be discovered by conceptual analysis. Arguably, the classical view is not an adequate picture of how concepts work in human cognition (see e.g. Quine 1951; Fodor et al. 1999; Rosch 1973a, 1975a,b, 1978, 1983; Wittgenstein 1953). One of the major and most recent accounts of concepts put forward as an alternative to the classical theory, inspired by the work of Rosch (1973b, 1983), employs the notion of a *dynamic conceptual frame*. We will henceforth speak of *the theory of dynamic conceptual frames*, or more briefly, frame theory.[6] Some of the most well-known formulations of frame theory have been provided in Barsalou (1987), Barsalou and Hale (1993), Barsalou (1993), Barsalou and Yeh (2006). Motivated by the work of Kuhn (esp. Kuhn 1974), certain applications to the history of science have been put forward and it has been argued that dynamic frames are a useful tool to account for scientific revolutions and conceptual frame incommensurability (Andersen et al. 2006).

A frame developed for a single concept only is called a *partial frame*. In this paper we will be interested in various kinds of partial frames and certain relations between them. We will only explain the main ideas behind these and refer to the Appendix for their formal representation. A partial frame (for a concept R) is composed of two layers: *attributes* and *values*. Every object that falls under R is supposed to have all the attributes. Objects having a certain attribute are divided according to what values of those attributes they instantiate. For any object that falls under R and for any attribute in the frame for R, this object has to instantiate exactly one value for that attribute.[7] Take a simple example from Andersen et al.

[4] This doesn't mean that the assumptions used in this inference haven't been obtained by some sort of inductive reasoning. We'll discuss this possibility in Sect. 5.

[5] For the bismuth example, one might want to consider an infinite range of predicates, where the ISI allows us to single exactly one of them out. To deal with such cases, one has to turn to second-order classical logic—see also the Appendix.

[6] The psychological evidence for the adequacy of frame theory is surveyed by Andersen et al. (2006: 47–52).

[7] "…all of the attribute nodes are activated for every subordinate concept. However, value nodes appear in mutually exclusive clusters." (Andersen et al. 2006: 44)

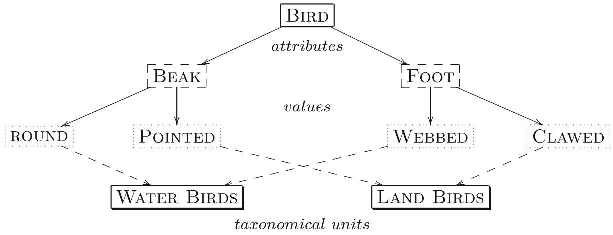

Fig. 1 A partial dynamic frame for the concept BIRD

(2006: 43). The concept BIRD can be considered in a frame where it has only two subordinate attributes: BEAK and FOOT, each having two values: ROUND, POINTED, and WEBBED, CLAWED respectively.[8]

Some combinations of values (each belonging to a distinct attribute) are taken to constitute a separate *taxonomical unit*. Taxonomical units are taken to be a division of the domain of objects that fall under the root concept: no object should belong to two taxonomical units (see Andersen et al. 2006: 56) and every object should belong to a taxonomical unit (Andersen et al. 2006: 27). For instance, in the exemplary frame there are only two such combinations: {POINTED, CLAWED} and {ROUND, WEBBED}, giving raise to the taxonomical units LAND BIRD and WATER BIRD respectively.[9] In this sense, a frame specifies a taxonomy of the concept under consideration. In the example, the concept BIRD is divided into two separate taxonomical units.

So one of the main constituents of a dynamic frame is a *tree-like structure*. The idea seems fairly simple. Any object that falls under the root concept is supposed to have one of the values for each of the attributes. Attributes are just aspects in which objects that fall under the root concept are classified and values are various features that an object can have with respect to those aspects. This two-layered structure provides frame theory with rich means of expression, and we will rely on this feature in subsequent sections.

Another important constituent of a dynamic frame are *activation patterns*. These decide which combinations of values for the attributes in the frame actually occur together and they put additional restrictions one the domain of objects falling under the root concept. For instance, in the frame from Fig. 1, the combination {ROUND BEAK,WEBBED FOOT} and the combination {POINTED BEAK,CLAWED FOOT} constitute the two activation patterns in the frame. On the other hand, the above frame does not admit an activation pattern where an object has a pointed beak but webbed feet, or a round beak and clawed feet.[10]

3 Complete and Incomplete Frames

Now that the notion of a dynamic frame has been introduced, we will elaborate on a particular aspect of the theory, in order to explain the relation between dynamic frames and ISI.

[8] We will refer to partial dynamic frames as *frames*, and a distinction soon will be made between complete and incomplete frames. Completeness will be opposed to incompleteness, not to being partial.

[9] The distinction between these two groups might be introduced for instance because there are certain useful generalizations that apply to all land birds but not to all water birds, and so on.

[10] This might be the case for instance because we have a causal story that tells us why birds with webbed feet are less likely to survive if they have a pointed beak, or because we simply have no evidence for there actually being birds instantiating this combination.

For some frames, each taxonomical unit has a fixed value for every attribute of the frame, i.e.:

[Strong Relevance Requirement] For any taxonomical unit and for any attribute in that frame, there exists exactly one value for that attribute such that all objects in that taxonomical unit have that value.[11]

Note that **SRR** implies that every taxomical unit in the frame has to correspond to an activation pattern, as in the example from Sect. 2. Also, it is important to observe that **SRR** is different from the claim that for any object that falls under the root concept, for any attribute in the frame, there exists exactly one value for that attribute possessed by that object. For even if the latter condition holds, it still might be the case that a taxonomical unit contains objects that disagree on a certain attribute.

The basic intuition in support of **SRR** is that attributes should be, in a fairly strong sense, relevant for our taxonomy. **SRR** is quite strong, because we also have another candidate for capturing the idea that attributes should be relevant for our classification:

[Weak Relevance Requirement] For any taxonomical unit and for any attribute in the frame, there exists at least one value for that attribute such that no object in that taxonomical unit has that value.

We will focus on frames that obey **SRR**, arguing that these provide the most straightforward justification of ISI. However, in Sect. 7 we will briefly show that ISI can take place in the context of other frames as well. In other words, the rather strict and abstract model we are presenting can be easily loosened such that real-life examples are within reach.

Now, even if **SRR** holds, a distinction should be made between two cases: (i) we are able to associate each taxonomical unit with a particular activation pattern, and (ii) we believe that every taxonomical unit is associated with an activation pattern, but are sometimes unable to tell which taxonomical unit is associated with which pattern. (i) and (ii) are epistemically different situations, and—as the examples we will give indicate—both are quite common.[12] If (i) is the case, we speak of a *complete frame*, whereas in the case of (ii), we speak of an *incomplete frame*. In the Appendix we show how complete and incomplete frames can be distinguished in more formal terms.

Since they are less specific, incomplete frames are less informative than their complete counterparts. In this sense, when we "fill in" the details in an incomplete frame, associating taxonomical units with specific values, epistemically speaking, we are making progress. In what follows, we describe ISI as the process of rendering an incomplete frame complete.

Take for example the concept BROADLEAF TREE. Suppose the attributes of the frame for this concept are LEAF- SHAPE, FRUIT and MODE OF REPRODUCTION.[13] Say we know three kinds of broadleaf trees: chestnut trees, elms and cherry trees. These are our taxonomical

[11] See the Appendix for a formal representation of **SRR**. We may refer to Andersen in this context: "Conventionally, all of the attribute nodes are activated for every subordinate concept. However, value nodes appear in mutually exclusive clusters. Only one value for any given attribute may be activated, but different activation patterns, or different choices of value, generate many different subordinate concepts, within the limits allowed by the attribute and value constraints already described. Each pattern of selection constitutes a subordinate concept; for example, a waterfowl is a bird whose values for BEAK and FOOT are restricted to ROUND and WEBBED." (Andersen et al. 2006: 44) We use the term "taxonomical unit" where Anderson et al. speak of subordinate concepts.

[12] As far as we know, the distinction has not been made in literature of the subject.

[13] For the sake of brevity and simplicity we use a greatly simplified example here, in the sense that we've reduced the number of attributes, values and taxonomical units to a minimum.

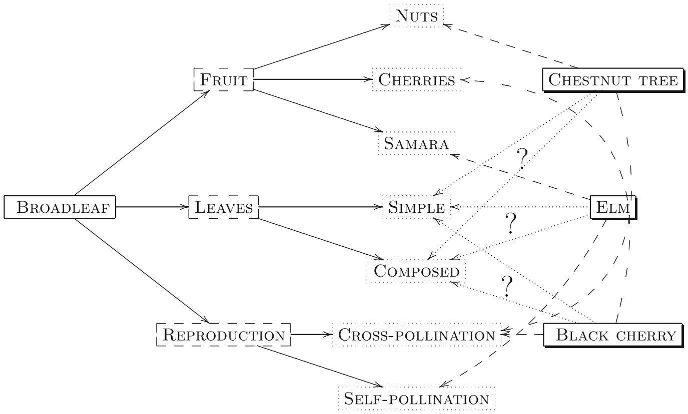

Fig. 2 An incomplete frame for BROADLEAF TREE

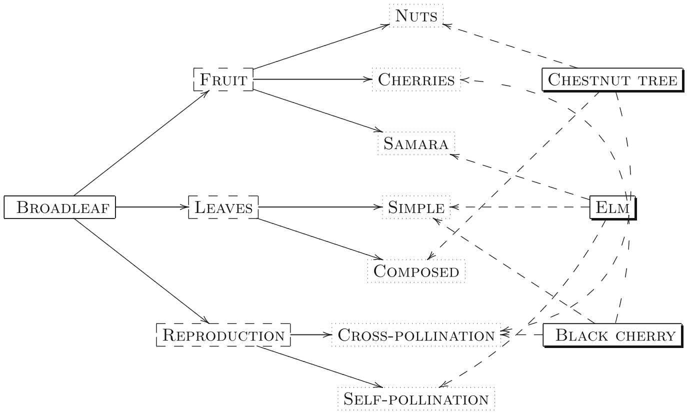

Fig. 3 A complete frame for BROADLEAF TREE

units. We may once have learned the shape of the leaves for each of these units. However, few of us still know the leaf-shape of e.g. elm trees. We know that all elms must have the same kind of leaves, but we can't figure out what exactly these look like, and we may not be able to choose the right item from a list of all possible leaf-shapes. Hence, our frame for broadleaf trees is incomplete (Fig. 2):

A tree expert, on the contrary, may have a complete frame for broadleaf trees (Fig. 3):

The incomplete frame presented in Fig. 2, even though less informative than the one pictured in Fig. 3, is still in an important sense more informative than a frame without LEAF-SHAPE as an attribute. In the latter case, we wouldn't even have the incomplete knowledge about taxonomical units being uniform with respect to this attribute. Moreover, given that it might be rather hard to memorize all the shapes of leaves of different tree species, and that we often can quite easily consult exemplars of trees in our environment and obtain this information, the incomplete frame may be seen as a "shorthand guide" to certain information.[14]

[14] One may compare this to information gathering in general: it is sometimes more useful and certainly easier to know where you can find the right information about a certain class of subjects when you need it, than to know as much as possible.

4 Incomplete Frames and ISI

What is the relation between incomplete frames and ISI, as we described it in the first section? The answer is fairly straightforward: incomplete frames provide the background knowledge for ISI. The predicate that the generalization assigns to certain classes of objects is a value of an attribute of such a frame, and the class of objects that the generalization is about is a taxonomical unit in that frame. Each ISI which draws a connection between a particular taxonomical unit and a certain value is a step towards a complete frame.[15] More formally, the incomplete frame entails a disjunction of generalizations, whereas the instance falsifies all but one of these—see the Appendix where this is explained by means of first order predicate logic. To get the intuitive picture, let's reconsider the examples from section one.

Assume that we have an incomplete frame for PURE LIQUIDS, containing the attribute BOILING POINT. Since BISMUTH is a taxonomical unit in this frame, all samples of this fluid will have the same boiling point. Knowing the boiling point of one sample of bismuth thus suffices to associate the right value for the attribute BOILING POINT to the taxonomical unit. Wax, on the contrary, is not a taxonomical unit of this frame (or any other incomplete frame with BOILING POINT as the attribute, for that matter). Hence a similar ISI doesn't work for wax.[16]

As for the car example, say an incomplete frame CAR contains an attribute WHEEL- DRIVE (2-wheel-front, 2-wheel-rear, 4-wheel). The taxonomical units are version-specific makes of cars. Our incomplete frame expresses our knowledge that cars of the same version-specific makes have the same wheel-drive, and this licenses an ISI: once we observe that a certain car of a particular version has a certain kind of wheel-drive, we can legitimately infer that all cars of that version have the same kind of wheel-drive. On the other hand, COLOR is not a relevant attribute of this frame, since cars of the same type can have different colors.[17] The richness of frame theory, which for each concept introduces two levels of predicates (attributes and their values), makes it very suitable for an account of ISI—all the different constituents of a frame play their part in the explanation.

5 Justifying Incomplete Frames

Once we have an incomplete frame, it allows for sensible inductions from a single instance. Also, inductions from a single instance not based on an available incomplete frame are not legitimate, and in this sense our assembly of available incomplete frames allows us to draw the line between convincing and unconvincing ISIs. But how is the frame itself justified? Let us, for the sake of example, briefly discuss two ways this can be done.

One kind of justification is based on a causal story. Such a story informs us that every member of the same taxonomical unit has been involved in the same causal process, and for this reason has a fixed value for a given attribute. For instance, suppose we are developing a frame for our concept CAR, where the basic taxonomical units are version-specific car makes. We may then obtain information that all cars of the same version are the product of one and the same assembly line in a factory, and that this assembly line determines where the

[15] Whenever a series of ISIs eventually leads to a complete frame, one can speak of the "saturation" of an incomplete frame.

[16] Notice that the expression "has a fixed boiling point" can be captured within frame theory only given the distinction between complete and incomplete frames we made in the previous section.

[17] Note that, since no colors are excluded for any type of car, even a frame for CAR that only conforms to the Weak Relevance Rule cannot contain the attribute COLOR.

wheel-drive of a car is situated.This amounts to the conclusion that every version-specific car make is associated with a single value for the attribute WHEEL- DRIVE, even though we are not sure what the connections exactly are. At that point, we are justified in our acceptance of an incomplete frame for the concept CAR.

Another kind of justification is based on a specific type of second-order induction. Let $V_1, V_2, \ldots$ be values of a certain attribute that is new to the frame, and let $T_1, T_2, \ldots$ be taxonomical units of that frame. Suppose we come to believe that T_1 is associated with V_1, T_2 is associated with V_4, and in general we come to accept a certain number of generalizations of this sort. Then, prior to discovering those connections for all of the taxonomical units, we reason by induction *on the taxonomical units themselves* to the conclusion that *all* the taxonomical units available in that frame are associated with a single value. Hence we obtain an incomplete frame. This, of course, may motivate our search for a causal story explaining this connection, but that does not mean that the second-order induction itself cannot justify the incomplete frame.

For instance, suppose we are developing a frame for the concept *pure liquids*. Our attribute candidate is BOILING POINT. We learn that quite a few types of pure liquids have a fixed boiling point. We then might be justified to infer inductively that each taxonomical unit in our frame is associated with a fixed boiling point.

We tend to be pluralists about the ways an incomplete frame can be justified, and the above is not a complete list of the ways this can be done. Just like there are many ways one can be rational in accepting certain beliefs, there are many ways one can be rational to accept a certain structured conceptual frame. Rather our intention is to display at least certain ways an incomplete frame can be justified, to indicate that this justification can be clarified fairly easily, using the terminology of frame theory.

6 Revising Frames

While reasoning with a certain (complete or incomplete) frame in the background, reliable data might force one to revise the frame. It might turn out that the taxonomization provided by the frame is inadequate. For instance, an object falling under the root concept can be discovered which does not belong to any of the taxonomical units (or which should belong to two distinct taxonomical units). There are quite a few ways in which data may go against a given frame. According to our approach, revising the background knowledge that grounds an ISI is but a particular case of frame revision in general.

Suppose we have filled in the blanks in an incomplete frame, by drawing inductions from a single instance. Every taxonomical unit T_i is thus associated with a single value for the new attribute A, say V_j. We happily continue to use the newly obtained complete frame for a while.

Alas, at some point we realize that two objects that fall under the same taxonomical unit T_k have a different value for A. So we are faced with a contradiction between the newly obtained complete frame and the available data.In that case, we may continue reasoning for a while, relying on the non-problematic parts of our frame (see Urbaniak 2010) for a formal account of this. However, having inconsistent beliefs is usually not the ideal state that we are after, whence we need to revise our frame somehow and restore consistency.

Perhaps, even though we have reasons to reject our generalization about T_k, our claim that taxonomical units are associated with specific values is still strongly supported for all the other taxonomical units. In such a case, we would be rather inclined to keep the new attribute, and divide T_k into as many "real" taxonomical units as needed: if all the objects from T_k have one of the values V_i, V_n or V_u, we might simply move to a frame where instead of T_k we have three taxonomical units T_{k1}, T_{k2}, T_{k3}, all of them associated with the same values for all the attributes different from A, and each of them associated with a different value for A. For instance, in the case of wheel-drives, once we discover that something that we considered a specific version of a car comes in two sorts, e.g. those that have 2-wheel-front and those that have 2-wheel-rear drive, instead of deleting the attribute of wheel-drive, we might just decide that what we considered a version is not really a version and comprises at least two versions.

One may also decide to split up the original frame into two separate frames for two different root concepts: one containing the new attribute, and the other one without it. This implies that one also decides which elements of $T_1, T_2, \ldots, T_k$, belong to the first frame, and which to the second one. For instance, one may initially have an incomplete frame for SUBSTANCES, containing the attribute BOILING POINT. After discovering that wax doesn't have a fixed boiling point, one may distinguish between pure substances and mixtures. The frame for PURE SUBSTANCES still contains the attribute BOILING POINT and remains an incomplete frame, while the one for MIXTURES doesn't. A good reason to do this would be that one knows of other relevant differences between both subclasses of the initial root concept (as is the case for the example we just gave).

As in Sect. 5, the above picture is not exhaustive—there are many rational ways one can respond to inconsistencies that result from new data. The exact details of these reactions require further attention, and more formal accounts. Our goal was to indicate that frame theory can provide us with the theoretic tools to do this.

7 Summary and Further Research

In the previous sections, we explained that the problem of the appropriateness of single instance inductions can be formulated and studied in a fairly clear way in terms of an independently developed theory of *dynamic conceptual frames*. Only a few specifications and one distinction—that between complete and incomplete frames—suffice to phrase the necessary BK for ISI in terms of a partial dynamic frame. Furthermore, questions concerning the acceptance of this BK and the reaction when confronted with data that contradict it can be addressed nicely using this terminology.

The concept of an incomplete frame thus helps us to formulate the necessary background knowledge for ISI, bringing with it all the theoretic tools that are inherent in frame theory. Interestingly, frame theory was not constructed with ISI in mind, and its development by cognitive scientists is quite independent of questions concerning induction. In this sense, developing frame theory to account for ISI does not seem to be too ad hoc.

The most striking feature of frame theory, compared to preceding alternatives to the classical view of concepts, is that it allows for two levels of predicates: attributes and values. Hence it makes it possible to group values according to the respective attributes that they are values of. Whether the BK is attained through causal knowledge or through a kind of second order induction about predicates and classes of objects, there is always a reference to a set of predicates, namely all the values of a certain attribute. Most of the older alternatives—for instance the prototype theory (Rosch 1973b) or the exemplar theory (Medin and Schaffer 1978)—lack the division of predicates into levels which would allow for this sort of move.[18]

[18] See Barsalou and Hale (1993) for a lengthy discussion of this difference.

What may be slightly unusual about our approach is that we consider the appropriateness of ISIs to be a matter of concepts. However, we are not the first to state that a conceptual frame contains more than just analytic knowledge in the sense of possible combinations of values that fall under a root concept, but can contain factual and law-like knowledge as well.[19] The notion of complete and incomplete frames is simply an extension of this idea.

We promised to come back to the possibility of frames that only obey the weak relevance requirement. Even for such frames it is still possible that at least one attribute behaves according to **SRR** (even though the whole frame doesn't), and yet we don't know which taxonomical unit is associated with which value. The justification for the assumed behavior of such an attribute can of the kind we gave in Sect. 5, and ISI proceeds in the same manner as before.

Numerous issues still require consideration. Further research should include both a more formal account of the acceptance, use and rejection of incomplete frames, continuing the work initiated by Urbaniak (2010) and more psychological research pertaining to the empirical adequacy of frame theory. The relation between ISI and other kinds of induction within the context of frame theory also deserves some consideration.Seen from that perspective, the current paper is just the starting point of an investigation into the relation between frames and induction as a whole. Following Nelson Goodman's critique of a purely syntactical approach to induction,[20] frame theory can provide a semantic complement to logics of inductive generalization[21] and thus viewed, may result in a more general theory of inductive generalizations. The general idea behind this is that our inductive reasoning can only reach stable and reliable universal propositions, once it limits its scope to a certain set of predicates—those that are values of attributes in a frame. Such a model of induction would be able to account for both ISI and more complex cases of induction.

Appendix

In this Appendix, we will briefly illustrate how cases like the car example and the tree example can be captured in a first order predicative language. Although our formal explication is very basic, it can easily be refined and enriched in several ways.[22] Also, it shows the road to representations in second order logic, which can account for more complex issues such as the second-order induction mentioned in Sect. 5, or ISIs about attributes that have an infinite range of values.

In the remainder, we assume that a frame is rooted and consists of only finitely many attributes and values (see Urbaniak 2010: pp. 436–438) for a justification of these restrictions. We will consider complete frames as a starting point, and gradually extend these to obtain incomplete frames. This is done by taking the complete frame and stepwise adding new attributes and their corresponding values to it. Intuitively, this corresponds to adding new branches to the tree-like structure. However, we do not assume that complete frames are more primitive or basic from a psychological point of view.

[19] This is stressed in particular by the followers of the so-called Theory-Theory view. See Andersen et al. (2006: 60–64) where this view is commented on. More generally, activation patterns and structural invariants already go beyond what we would be inclined to treat as "analytical knowledge".

[20] See Goodman (1978), where the author presents his famous "Grue paradox".

[21] See Batens and Haesaert (2003) for examples of a specific class of logics of induction that we have in mind, adaptive logics of induction. These are being developed by the Ghent Group.

[22] For instance, many other approaches in the dynamic frames-literature represent attributes in terms of binary predicates.

Information encoded by complete frames. Suppose we have a frame with the tree-like structure, where R is the root predicate, $A_1, A_2, \ldots, A_i$ are its attributes, and each attribute A_k has n_k values falling under it: $V_1^k, V_2^k, \ldots, V_{n_k}^k$.

To express the tree structure we first have to say that all objects that fall under the root concept possess all its attributes and that any object that has an attribute A_k instantiates at least one value for that attribute. That is, for $1 \leq k \leq i$ and i is the number of all attributes in the frame we need:

$$\forall x(R(x) \rightarrow A_k(x)) \tag{1}$$

$$\forall x[R(x) \rightarrow (A_k(x) \rightarrow \bigvee_{n_k}^{k} V_{n_k}^k(x))] \tag{2}$$

where $\bigvee_i^n P(x)$ abbreviates $P_1^n(x) \vee P_2^n(x) \vee \cdots \vee P_i^n(x)$ for an optional superscript n, a predicate P and a individual variable x. Let's call the set of formulas falling under (1) $\mathbb{R}$ and the set of all needed instances of (2) $\mathbb{A}$. We also need to say that values falling under each of the attributes are exclusive. Consider the values $V_1^k, V_2^k, \ldots, V_{n_k}^k$. For each such k, we extend our set of formulas that describe the frame by all formulas of the form:

$$\neg\exists x(R(x) \wedge A_k(x) \wedge V_m^k(x) \wedge V_l^k(x)) \tag{3}$$

where $m \neq l$, $1 \leq m \leq n_k$, $1 \leq l \leq n_k$. Call the set of formulas falling under this schema $\mathbb{V}$.

A set of predicates Π is a **choice set** of the tree if and only if:

1. Every predicate in Π is a V_m^l, that is, Π is a set of value predicates.
2. For every k there is an m such that V_m^k is in Π, that is, Π contains at least one value for each of the attributes.
3. For no k there are $m, l, m \neq l$ such that V_m^k and V_l^k are both in Π, that is, Π contains no two different values for one and the same attribute.

The set of all choice sets of the tree will be called γ.

If a set of predicates Π is non-empty and finite, and $P_1, P_2, \ldots, P_k$ are all the members of Π, by $\bigwedge \Pi(x)$ ($\bigvee \Pi(x)$) we abbreviate $P_1(x) \wedge P_2(x) \wedge \cdots \wedge P_k(x)$ ($P_1(x) \vee P_2(x) \vee \cdots \vee P_k(x)$). We allow the degenerate case when $k = 1$. In this case both $\bigwedge \Pi$ and $\bigvee \Pi$ are the same.

The set of **activation patterns** is a subset α of γ. Let $\overline{\alpha} = \gamma - \alpha$. Suppose $\Pi_1', \Pi_2', \ldots, \Pi_n'$ are all the members of $\overline{\alpha}$. We need to say that no object that falls under the root concept falls under one of the Π_i''s. That is, we need n formulas of the form:

$$\neg\exists x[R(x) \wedge \bigwedge \Pi_i'(x)] \tag{4}$$

The set of all needed formulas falling under (4) will be called $\mathbb{P}$.

Recall that in a frame that obeys **SRR**, taxonomical units are identified with activation patterns. A complete frame moreover contains the explicit association of each taxonomical unit with a particular activation pattern. Given that α has m members: $\Pi_1, \ldots, \Pi_m$, we need m letters $T_1, \ldots, T_m$ in our language to represent taxonomical units. We introduce m definitions of the form:

$$\forall x[R(x) \rightarrow (T_i(x) \equiv \bigwedge \Pi_i(x))] \tag{5}$$

for $1 \leq i \leq m$. Call the set of all formulas of this form $\mathbb{T}$. This ends our description of the frame. The set of formulas $\mathbb{F}$ that expresses the frame is now defined by:

$$\mathbb{F} = \mathbb{R} \cup \mathbb{A} \cup \mathbb{V} \cup \mathbb{C} \cup \mathbb{P} \cup \mathbb{T} \tag{6}$$

Information encoded by incomplete frames. Consider the case where an otherwise complete frame is extended to an incomplete frame by adding a new attribute A_{k+1} and its corresponding values $V_1^{k+1}, \ldots, V_{n_{k+1}}^{k+1}$. How do we extend the set describing the initial complete frame to capture this new information? There are a few moves that have to be made:

- We add the $\mathbb{R}$-formula for A_{k+1}.
- We add all $\mathbb{A}$-formulas for A_{k+1}.
- We add all $\mathbb{V}$-formulas for $V_1^{k+1}, \ldots, V_{n_{k+1}}^{k+1}$.

Now, we have to encode the information that each taxonomical unit from the original frame is homogenous with respect to the values of A_l. This is done by adding, for every $1 \leq h \leq m$:

$$\forall x[R(x) \rightarrow (T_h(x) \rightarrow V_1^{k+1}(x))] \vee \ldots \vee \forall x[R(x) \rightarrow (T_h(x) \rightarrow V_{n_{k+1}}^{k+1}(x))] \qquad (7)$$

Note that formulas of the form of (7) capture **SRR**, described on page 6.

ISI inferences with incomplete frames. With these assumptions, it is not too difficult to see how ISI can be modeled. Say we learn that an object a falls under the root concept, belongs to a taxonomical unit T_h and has the value V_z^l of attribute A^l:

$$R(a) \wedge T_h(a) \wedge V_z^l(a)$$

The last conjunct together with the $\mathbb{V}$-formulas for A^l entail $\neg V_g^l$ for any $g \neq z$. This means that we can use Ra, T_h and $\neg V_g^l$ to derive the negations of all formulas:

$$\forall x(R(x) \rightarrow (T_h(x) \rightarrow V_g^l)$$

whenever $g \neq z$. Finally, we can use negations thus obtained and disjunction elimination applied to (7) in order to obtain the desired generalization. [23]

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[23] One might worry that since the approach is based on classical monotonic predicate logic, it fails to model prototypical effects and framework revision. However, once we are able to express the information embodied in an (in)complete frame in a first-order language, many ways to account for such processes are available in the literature. As for non-monotonic and defeasible aspects of conceptual frames, these can be handled separately by adding a layer of adaptive logic(s) on top of the sets we use to represent frames (see Urbaniak 2010) for details. As for typicality and similarity degrees, this is handled by another machinery which already assumes first-order representation. All those issues lie beyond the scope of this paper, though.

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