# The Methodological Relevance of the History of Mathematics for Mathematics Education

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## Abstract

Mathematics education benefits from an integration of the history of mathematics within the mathematics curriculum. We provide three basic arguments for such integration. The first is epistemological and addresses a contextual view on mathematical knowledge. The second concerns the phylogenic aspects of the development of mathematics. Conceptual difficulties with teaching children mathematics often correspond with historical periods of conceptual crisis in mathematics. A third, historical argument, draws on the vast repository of experience in mathematics education. We provide examples for each of these arguments from the history of algebra.

## Introduction

> [Most] problem situations occur in growing mathematical theories, where growing concepts are the vehicles of progress, where the most exciting developments come from exploring the boundary regions of concepts, from stretching them, and from differentiating formerly undifferentiated concepts. In these growing theories intuition is inexperienced, it stumbles and errs. There is no theory which has not passed through such a period of growth; moreover, this period is the most exciting from the historical point of view and should be the most important from the teaching point of view.
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> Imre Lakatos in *Proofs and Refutations*, (1976) p. 140.

In this paper we argue for the integration of the history of mathematics in mathematics education. Our motive for the study of the emergence of symbolic algebra is mainly epistemological. How are concepts formed in mathematics? Which factors influence or change the meaning of concepts? Is there an internal logic and order in the development of mathematical concepts? What is the role of symbolism in mathematical knowledge? How did the exposition of algebraic knowledge evolve in textbooks? The answers to these questions have an impact on mathematics education. As we considered the history of mathematics as an empirical basis for epistemology so it is also a relevant for the teaching of mathematics.

On first sight, arguing for the use of the history of mathematics may seem to be a redundant task. Official education plan for secondary education often define the role of the history of mathematics explicitly. One example from Belgium:[2]

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> Mathematics education is necessarily connected with other disciplines. Mathematics itself has developed through centuries in close connection with prevailing opinions and problems. Today, certain historical contexts still provide useful starting points to approach specific mathematical concepts and educational topics. The historical context shall therefore be integrated in our education plan.

However, when looking for concrete guidelines of how to integrate the history of mathematics, the examples offered by the plan are disappointing. We only find generalities such as "an approach with examples from architecture and painting can illustrate the role of mathematics in the development of certain art forms" (ibid. p. 11) and "assignments can be given to research historical facts, such as an internet search for mathematicians, important mathematical theorems, mathematical illustrations and applications" (ibid. p. 28). The history of mathematics is forced into an illustrative role. History delivers the pictures for lighting up dreary textbooks, to force a connection with other disciplines, and to keep students busy between other assignments. An *integrated* view on mathematics education in which the history of mathematics has a methodological relevance is absent.

We provide three basic arguments for the integration of the history of mathematics in mathematics education. The first is philosophical and addresses the epistemological status of mathematics. Mathematics is often wrongly considered as the outstanding example of a discipline providing absolute knowledge. The history of mathematics offers an excellent opportunity to impart a contextual approach to knowledge. The second we call the phylogenetic argument. Our conceptual study gave us an insight in the way algebraic concepts emerged and evolved through history. We believe this to be relevant for education. Periods of conceptual difficulties in history point to intrinsic epistemological difficulties with certain concepts. Such concepts require special attention in education. But possibly also the way concepts were established in history has its relevance for the method and order in which these concepts are taught to children. The third is a historical argument. The history of mathematics is at the same time a history of mathematics education. People have taught and learned mathematics since over 3000 years. A pluralism of historical frameworks and methods enriches the quality of education.

## The philosophical argument [3]

### A contextual approach to rationality

The adolescent's notion of rationality often encompasses the epistemological view of mathematics as knowledge which offers absolute certainty. He probably has heard of a geometry in which the parallel postulate does not hold, but most likely believes that Euclidian geometry is the "real one". We can assume that he is not familiar with Gödel's theorems and undecidability. It is further unlikely that he has been taught about the existence of inconsistent arithmetic that performs finite calculations as correct as traditional arithmetic. These findings provide strong arguments against the

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[2] Cited from the education plan of the first two years of secondary school used by Catholic schools in Belgium(2002), p. 29. However, the education plans of other schools and other countries employ very similar formulations: "Wiskundevorming staat niet los van die van de andere vakken. Wiskunde zelf is doorheen eeuwen ontwikkeld precies in samenhang met de opvattingen en de problemen van die tijd. Een aantal historische contexten bieden ook vandaag nog een zinvolle instap om bepaalde wiskundeproblemen en leeronderdelen aan te pakken. Daarom zal die historische context geïntegreerd worden in de aanpak". Other representative descriptions are Calinger, 1996 and Fauvel and van Maanen, 2000

[3] For a more extensive argumentation, see Heeffer (2006b).

view that mathematics offers absolute truth. The static and unalterable mode of presentation of concepts in the mathematics curriculum, rather than lack of knowledge, contributes to this misconception. Mathematical concepts, even the most elementary ones, have changed completely and repeatedly over time. Major contributions to the development of mathematics have been possible only because of significant revisions and expansions of the scope and contents of the objects of mathematics. Yet, we do not find this reflected in class room teaching. While the room for integrating philosophy in mathematics education is very limited, an emphasis on the understanding of mathematical concepts is a necessary condition for a philosophical discourse about mathematics. The conceptual history of mathematics provides ample material for such focus and leads to a better understanding of mathematics and our knowledge of mathematics. We will argue for the integration of the history of mathematics within the mathematics curriculum, as a way to teach students about the evolution and context-dependency of human knowledge. Such a view agrees with the contextual approach to rationality as proposed by Batens (2004). We will draw some examples from the history of algebra. In line with Lakatos (1976) and Kitcher (1984) such example is motivated by the epistemological relevance of the history of mathematics.

## Absolute certainty in mathematics?

> Gentleman, that $e^i + 1 = 0$ is surely true, but it is absolutely paradoxical; we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth.

This well-known quote by Benjamin Peirce, after proving Euler's identity in a lecture, reflects the predominant view of mathematicians before 1930, when mathematical truth equaled provability.[4]
When Gödel proved that there are true statements in any consistent formal system that cannot be proved within that system, truth became peremptory decoupled of provability.
However, Peirce seems to imply something stronger: proving things in mathematics leads us to *the truth*. This goes beyond an epistemological view point and is a metaphysical statement about existence of mathematical objects and their truth, independent of human knowledge. The great mathematician Hardy formulates it more strongly (Hardy 1929):

> It seems to me that no philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner or another, the immutable and unconditional validity of mathematical truth. Mathematical theorems are true or false; their truth or falsity is absolute and independent of our knowledge of them. In some sense, mathematical truth is part of objective reality.

Such statements are more than innocent metaphysical reflections open for discussion. They hide implicit values about the way mathematics develops and have important consequences for the education and research of mathematics. An objective reality implies the fixed and timeless nature of mathematical concepts. The history of mathematics provides evidence of the contrary. Mathematical concepts, even the most elementary ones, such as the concept of number, continuously change over time. The objects signified by the ancient Greek concept of *arithmos* differ from that of 'number' by Renaissance mathematicians, which in turn differs from our current view. One could object that not mathematics but our understanding of mathematical reality changes. However, Jacob Klein's landmark study (1934-6) precisely focuses on the *ontological* shift in the number concept. In Greek arithmetic 'one' was not a number, later it was. After that, the root of two

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[4] Quoted in Kasner and Newman, 1940.

was accepted as a number and by the end of the sixteenth century the root of minus fifteen became a number.

Another implicit value hidden in the predominant view is the superiority of modern ideas over past ones, and possibly of Western concepts over non-Western ones. Again, the history of mathematics shows that mathematics always adapted to the needs of society. Mathematics was born in the fertile crescent, extending to the belt from North Africa to Asia, where wild seeds were large enough and mammals capable of employable domestication.[5] Modern algebra fertilized in the mercantile context of merchants and craftsman of Renaissance Italy. Several important figures in the development of symbolic algebra wrote also on bookkeeping, as well as on algebra often in one and the same volume.[6] If we accept that double-entry bookkeeping emerged in the fifteenth century as a result of the expanding commercial structures of sedentary merchant in Renaissance Italy, why not considering symbolic algebra within the same context? Ideas should be interpreted within the historical context in which they emerged and perhaps their superiority is dependent on the degree in which they were adapted to the needs of society.

Finally, the idea of an objective reality of mathematical concepts evades the reality of conceptual problems in mathematics. Time and again there have been serious crises in the conceptual foundations of mathematics.[7] There have been inconsistent theories, such as the early use of analysis, and set theory which have existed for several decades. It is precisely in times of crisis and conceptual difficulties that new ideas emerge and breakthroughs are made. According to Lakatos (1967, 140) such periods are "the most exciting from the historical point of view and should be the most important from the teaching point of view". This brings us to our second argument.

### The phylogenetic argument

The philosophical argument addresses the mode of presentation in of mathematics education as the absolute and eternal truth. Mathematics education is all too much focused on the presentation of the achievements, the established theories. But, as we have argued, new theories or new topics in mathematics are often preceded by a period of conceptual crisis, even inconsistencies. Sixteenth-century algebra, seventeenth-century calculus and eighteenth-century function theory are some examples. Piaget and Garcia (1983) argue that the psychogenesis of concept development in children and adolescents may learn us about the development of concepts in the history of the mathematical sciences. We have critically assessed their approach in chapter 2. The phylogenetic argument goes in the reverse direction. The emergence and development of concepts in mathematics may provide us with insights into the way mathematical concepts are acquired by individuals in education. The phylogenetic argument advocates the use of history of mathematics in mathematics education from periods of conceptual disarray. History of mathematics has a double function in this respect. One addresses the teacher, the other, the student. One acts on a meta level, the other on the object level. We will discuss these two functions separately.

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[5] For an eye-opening study on the relation between these coincidental factors and the development of culture and thus mathematics see the excellent work of Jared Diamond (1996).

[6] Between 1494 and 1586: Luca Pacioli, Grammateus, Valentin Mennher, Elcius Mellema, Nicolas Petri and Simon Stevin.

[7] An important case study on crisis in mathematics is Carl Boyer (1959), *The History of the Calculus and Its Conceptual Development*. As the title suggests Boyer concentrates on the conceptual difficulties in developing the modern ideas of the calculus.

## On the meta level

The conceptual history of mathematics allows teachers to understand why certain concepts are difficult for students to understand. It has taken the geometers and natural philosophers centuries to get a grasp on the concept of continuity in mathematics which suggests that the concept is challenging for adolescents as well. The study of the conceptual development of mathematical concepts will serve the teacher to relate the students' difficulties in understanding with conceptual difficulties in the history of mathematics. Placing the students' recurring errors and mistakes in mathematical practice within the broader context of conceptual development raises teaching to the epistemological level. Mistakes can be expected when there are intrinsic conceptual difficulties with the methods for solving the problem.

Let us look at an example related to the history of symbolic algebra: negative solutions of algebraic problems. We have elsewhere argued against the interpretation of negative solutions of two accounts on the history of subject by Sesiano (1985) and Gericke (1996) (Heeffer 2006b). Algebraic practice of solving linear problems has lead repeatedly to situations in which one arrives at "a negative value". Before the sixteenth century, such solutions were consistently called 'absurd' or 'impossible'. The abacus master, convinced of the correctness of his algebraic derivations, could interpret the negative value in some contexts as a debt. This does not imply that he accepted the solution as a negative value. At the contrary, by interpreting the solution as a debt, he removed the negative. Only from the beginning of the sixteenth century do we see the first step towards negative values, in the form of algebraic terms affected by a negative sign. The fact that negative solutions were considered absurd for several centuries of algebraic practice is of significance to the teaching of algebra. Being aware that negative solutions formed a conceptual barrier for the Renaissance habit of mind, prepares teachers for potential difficulties in the teaching of the concept.

Antoine Arnauld, who wrote an important philosophical work known as *The Logic of Port-Royal* (Arnauld, 1662), also published a *Geometry* (Arnauld, 1667). In the book he includes an example of symbolic rules that he considers to be against our basic intuitions on magnitudes and proportions. His reasoning goes as follows. Suppose we have two numbers, a larger and a smaller one. The proportion of the larger to the smaller one should evidently be larger than the proportion of the smaller to the larger one. But if we use 1 as the larger number and – 1 as the smaller one this would lead to

$$\frac{1}{-1} > \frac{-1}{1}$$

which is against the rules of algebra. Witnessing the multiple instances in which this discussion turns up during the seventeenth century, the clash between symbolic reasoning and classic proportion theory, taught within the *quadrivium*, was experienced as problematic. Also Leibniz found it important enough to write an article about (Leibniz, 1712, 167). He acknowledges the problem as a genuine one, but states that the division should be performed as a symbolic calculation, the same way as we do with imaginary numbers. Indeed, when blindly applying the rules of signs there is no problem at all. When dividing a positive number by a negative one, the result is negative, and dividing a negative number by a positive one, the result is also negative. Therefore

$$\frac{1}{-1} = \frac{-1}{1}$$

The discussion was not closed by Leibniz. Several eighteenth-century authors return to the question. E.g. Rolle (1690, 14-22), Newton (1707, 3), Maclaurin (1748, 6-7) and d'Alembert (1751-81).

As it was a source of controversy and discussion in the seventeenth and eighteenth century, it should come as no surprise that it raises questions and difficulties in the classroom. A conceptual history of mathematics can prepare teachers for such difficulties and show them that such questions must be taken seriously. It may show also that certain didactic approaches have their potential trap falls. The use of the number line in teaching negative numbers is in direct conflict with d'Alembert who argues that most of the difficulties with Arnauld's identity arise from viewing negative numbers as smaller than zero (Diderot and d'Alembert 1780, XXII, 289). Furthermore, we have elsewhere demonstrated that the negation sign was introduced within an algebraic context that functioned as a precondition for the acceptance of negative numbers (Heeffer 2006a). Current education plans in Belgium prescribe the introduction of negative numbers in the first year of secondary education, separate from algebra.[8] Knowing the historical context of the introduction of the negation sign, would teachers and decision makers in education account for the conceptual chronology?

### On the object level

Those involved with the daily teaching of mathematics can generally be convinced of the relevance of the history of mathematics on the epistemological level. However, it is difficult to persuade them of the importance of integrating the history of mathematics in mathematics education. Two arguments are commonly used against such a proposal. Firstly, the curriculum is overloaded and no subject matter can be added. Secondly, teachers are afraid of confusing students. If these historical discussions were perplexing for philosophers such as Arnauld and Leibniz, why trying to teach these to students? It is strenuous enough to teach students the proper meaning of mathematical concepts, why bother with the historical difficulties of arriving at these concepts. Let us address these two concerns. Firstly, we are not arguing to add the history of mathematics to the mathematics curriculum. Instead we advocate an integrated approach in which the history of mathematics is employed for explaining certain concepts. In as far as concepts are approached in mathematical school books, the explanation is limited to definitions, symbols and formulas. Secondly, an explanation of conceptual difficulties does not need to confuse students. In the same way that conceptual difficulties were overcome in history can the historical discussions clarify students' difficulties in classroom practice.

### Three thousand years of mathematics education

Every attempt to explain mathematical theorems or procedures contributes to mathematics education. Mathematics education is as old as mathematics itself. This long experience in teaching and explaining mathematics can be brought to use in twenty first-century mathematics education in two ways.

Every reflection on the epistemology of mathematics or the nature of mathematical methods has its direct consequences on mathematics education. Chapter nine of the fourth book of *La logique, ou,*

[8] Tellingly, the teaching of addition of negative numbers is no longer allowed in basic education in Belgium. Negative numbers can only be used in "concrete situations". The examples provided are the floors of a building and temperature.

*L'art de penser* (Arnauld and Nicole, 1662; 1996, 306-12) deals with methods in the *Art of thinking*. Arnauld lists six basic mistakes made by geometers in explaining their discipline:

1. to be more concerned with certainty than with obviousness and more trying to convince that to enlighten the mind
2. proving things that do not need a proof
3. proving by the impossible (i.e. by contradiction)
4. demonstrations drawn by too elaborate ways
5. ignoring the true order of nature
6. failing to make use of divisions and partitions

With the exception of the last, each of these principles touches the foundations of the mathematical method. At the same time, these are the basic questions about the methodology of mathematics education. Let us look at the fifth only. Arnauld, having written a basic *Geometry* himself (Arnauld, 1667), stresses the importance of following a natural order of explanation. An exposition should start from the simplest and most general concepts moving to the more complex and particular ones. He criticizes Euclid's *Elements* for failing to follow such basic principle and provides several examples of concepts introduced in the wrong order.[9] Indeed, the order of explanation from the Euclidean axiomatic method may not be the best order to teach geometry to children. Also, different axiomatizations lead to different approaches in teaching geometry. van der Waerden, who received great recognition as a pedagogue of mathematics, wrote an elementary geometry accounting for historical developments as well as different axiomatizations (van der Waerden, 1937). He rejects the axiomatization of Hilbert (1899) and adopts one based on congruence axioms because of its didactic superiority. Undoubtedly, the order in which concepts are explained have important didactic consequences. Through its ontogenesis, history of mathematics itself proposes one order of explanation, but the history of mathematics education provides many alternatives.

Another argument for drawing material from the history of teaching mathematics is that of the plurality of methods. Mathematics in secondary education is taught at heterogeneous groups. They can broadly be divided into the strong, the weak and the average. The best students will usually be able to master the material taught by any method, so this group is neglected. Methods directed to students with serious difficulties are often not adequate for others, hence is neither this group the focus of attention. Mathematics education is therefore methodologically directed towards the average student. An approach to improve on such situation, which has found some recognition during the past years, is to employ a plurality of methods. A new concept, method or theorem, explained in multiple ways is more likely to reach a broader range of students. Evidently, there are more differentiations between students than in our threefold, somewhat cynical, characterization. Some students have difficulties with purely symbolic accounts of mathematics. Others are weak in spatial representations. Still others need numerical examples to be able to grasp abstract relations

[9] Arnauld and Nicole (1996, 306-12): "Les éléments d'Euclide sont tout pleins de ce défaut. Après avoir traité de l'étendue dans les quatre premiers livres, il traite généralement des proportions de toutes sortes de grandeurs dans le cinquième. Il reprend l'étendue dans le sixième, et traite des nombres dans les septième, huitième et neuvième, pour recommencer au dixième à parler de l'étendue. Voilà pour le désordre général; mais il est rempli d'une infinité d'autres particuliers. Il commence le premier livre par la construction d'un triangle équilatère; et vingt-deux propositions après, il donne le moyen général de faire tout triangle de trois lignes droites données, pourvu que les deux soient plus grandes qu'une seule; ce qui emporte la construction particulière d'un triangle équilatère sur une ligne donnée".

and functions. Teaching concepts by a plurality of methods levels out these difficulties. The history of mathematics provides a vast repository of alternative cases, representations and methods. Previous chapters contain many examples of alternative methods to the strictly symbolic approach of teaching algebra. For example, for the quadratic equation we discussed several alternative approaches: the Babylonian cut-and-paste method (Høyrup 2002), the Arabic proof of completing the square (Rosen 1831) and the Hindu methods. We could add the discussion of multiple roots to the quadratic equation and the Arabic system of five rules for quadratic equations as useful material for further elaboration. Similar comments can be made for the concept of equations and the role of multiple unknowns. Luis Radford (1995, 1996, 1997) has demonstrated how material from the abacus tradition can contribute to a better didactic understanding of the use of multiple unknowns. We further maintain that the inventiveness of the abacus masters in solving difficult arithmetical problems can successfully be applied in teaching and practicing algebraic problem solving. Allowing students to compare their own approach and problem solution methods with alternative ones from the abacus tradition can induce interesting results. The thousands of word problems to be found in the existing corpus of algebra textbooks and manuscripts provide ample opportunities to enrich and empower the teaching of algebra and illustrate the plurality of methods and the dynamics of concepts in mathematics.

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