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NEWTON'S TRAINING IN THE ARISTOTELIAN TEXTBOOK TRADITION: FROM EFFECTS TO CAUSES AND BACK

Steffen Ducheyne
Ghent University

1. INTRODUCTION

In this essay I will examine what I think is a neglected contribution to Newton's conception of experimental philosophy. I shall try to argue that certain works of the seventeenth-century 'Aristotelian'[1] textbook-tradition[2] (which were inspired by Aristotle's ideas on logic and science) are relevant to bringing some of Newton's ideas on natural philosophy into perspective. My aim is to establish that the tradition epitomized by these textbooks is important for some 'general' features of Newton's conception of science, although obviously they cannot explain all of them.

First of all, let me explain what I understand by 'general'. General features are features on a meta-level: the way natural philosophy is understood and conceptualized (i.e., the conception of natural philosophy as articulated by a *virtuoso*). They pertain to the most general and abstract level of science. Specific features refer to knowledge about how natural philosophy is actually done (i.e., the actual scientific praxis that is followed to gain concrete knowledge about nature). I will argue that these Aristotelian textbooks explain the following *general* features of Newton's methodological ideas:

(1) In the order of things ('ordo naturae') causes come first and effects follow from them; in the order of knowing we notice effects first and from them try to infer the causes.

(2) Hence, one of the most important modes (i.e., 'regressus demonstrativus') in natural philosophy is reasoning backwards from the effects to the causes; next we reason from the causes to the effects.

(3) There is a distinction between proximate causes (i.e., causes that produce their effect directly) and remote causes (i.e., causes that produce their effect by means of some intermediary). On the highest level we have remote causes, next proximate causes, and finally the observed effects.

Newton's causal outlook on scientific reasoning (especially in the *Principia*) has striking parallels with this textbook-tradition. I will argue that, although Newton reinterpreted the notion of 'cause', he formulated his views on natural philosophy on a more abstract level in Aristotelian terminology. He recast his new notion of 'cause' in a familiar language; and it turns out that Newton veiled his innovative new style of natural philosophy in Aristotelian terminology. This causal tradition has wrongly been neglected. If my claim is correct, then this article offers a more balanced view

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of the contribution of Aristotelianism on Newton. Newton drew from and was trained in a set of common texts and techniques, which were still the most important sources for university-trained natural philosophers at the mid-seventeenth century.

In the seventeenth century, Aristotle's inheritance was passed on by these textbooks on logic. Little or no attention has been paid to these books. This comes as no surprise since our knowledge of the Aristotelian tradition(s) between roughly 1400 and 1650 is rather poor compared to our knowledge of neo-Platonism.[3] Maurizio Mamiani however recently argued that the tract on logic, *Logicae artis compendium* (1615), written by Robert Sanderson, played an important role in the genesis of Isaac Newton's *regulae philosophandi*.[4] Sanderson's laws (such as the law of brevity) are, as far as logic and rhetoric is concerned, "the primary source of Newton's rules".[5] In this paper I will deal with such textbooks on logic from a different perspective: their ideas on *regressus* and causality. The relevant textbooks are: (1) *Logica sive Ars cogitandi* (originally in French, 1662) by Antoine Arnauld and Pierre Nicole; (2) *Aditus ad Logicam* (1613) by Samuel Smith; and (3) *Institutio logicae* (1687) by John Wallis.

I have chosen these books by applying two criteria: (1) the books were part of Newton's private library[6] (or they were written by natural philosophers whom Newton knew and who were contemporaries of his), and (2) the books treated scientific method and *regressus*. However, Newton did not refer to these books (or to the Aristotelian tradition). The more serious argument for studying these works is that they provide a *representative sample of Aristotelian textbooks available to Newton and contemporaries*. My choice is somewhat subjective, but it points at the importance of this tradition. As Cees Leijenhorst pointed out in his recent study on Hobbes: "Although our choice of Aristotelian authors thus remains somewhat arbitrary, nonetheless a fairly representative sample of the different kinds of Aristotelianism *en vogue* in Hobbes' days is possible."[7] The same holds for Newton. The ideas on *regressus* are essential, not the authors as such. Only Smith's work explicitly discusses all three of the relevant ideas I have mentioned.

Most historians consider the Greek geometers (such as Euclid and Pappus) as primary sources for Newton's conception of analysis and synthesis.[8] Alistair C. Crombie claims more generally that the seventeenth-century natural philosophers' model for scientific discovery and demonstration was the ancient mathematical method of analysis and synthesis.[9] I wholeheartedly agree that this mathematical tradition was important to Newton.[10] Guicciardini points out that René Descartes's *Geometria* (first Latin translation by Frans van Schooten in 1649) and John Wallis's *Arithmetica infinitorum* (1655) "made the strongest impact on Newton's mathematical mind".[11] Newton had a deep admiration for the geometrical writings of the ancients (especially for Pappus), which led him to criticize the modern symbolical mathematics.[12] Newton developed an analytical as well as a synthetical method of fluxions.[13] He abandoned his earlier analytical style in favour of the synthetical style of the ancients.[14] The mathematical tradition was a necessary ingredient, but not a sufficient one.

That the *Principia* proceeds as a highly mathematical exposition (especially Book

I) has probably biased the view of Newton's conception of natural philosophy. Let me explain why the mathematical tradition alone does not suffice. In the analysis one starts from what is sought — as if it had been achieved — and by working backwards one arrives at what is known; in the synthesis one works in the other direction: one starts with what is known and arrives at what is sought.[15] This is the general idea of the mathematical tradition. What seems to be neglected here is that Newton's conception of natural philosophy is explicitly causal (a feature that is obviously lacking in the mathematical tradition of analysis–synthesis, since the relation between what is sought and what is known in either direction is purely inferential) and that he correspondingly favoured abductive reasoning steps.[16] I am not in any way challenging the importance of the mathematical tradition of analysis and synthesis for Newton's natural philosophy.[17] My claim is that we need to add and incorporate causal aspects that are absent in this mathematical tradition: the mathematical tradition is not sufficient to explain all aspects of Newton's natural philosophy, for a crucial element of Newton's conception of natural philosophy is left unexplained. These textbooks fill in this gap.

In Section 2, I will look at Newton's early training in Aristotelian philosophy, which will include a discussion of the notes from various Aristotelian authors (Joannes Magirus, Daniel Stahl and Gerardus Vossius) in Newton's Trinity notebook, and the aforementioned textbooks on logic. In Section 3, I will treat the parallels with Newton's *Principia*. I will not only show that several of Newton's assertions on natural philosophy are consistent with the Aristotelian tradition, but also that Newton's actual reasoning patterns in the *Principia* conform to some of the basic tenets of this tradition. Newton's conceptualization was heterogeneous and used different elements from these authors. In Section 4, I offer my conclusions, and raise questions that might guide future research.

2. NEWTON'S EARLY TRAINING

In this section I will discuss Newton's notes in his Trinity notebook (2.1), give some general remarks concerning the *regressus*-tradition (2.2), and finally present the relevant ideas of the aforementioned textbooks on logic (2.3).

2.1. *Notes from Aristotelian Works in Add. 3996 CUL*

When Newton was a student at Cambridge, study began with a heavy dose of Aristotle's logic, ethics, rhetoric and natural philosophy.[18] Aristotelianism was still the central system of thought in the educational system. In 1661 Newton as a student began to keep notes pertaining to his studies. His Trinity notebook (Add. 3996 CUL) contained copious notes from Aristotelian authors such as Joannes Magirus, Daniel Stahl and Gerardus Vossius.[19] I will briefly discuss what these works have to say on causal explanation. It will turn out that only Joannes Magirus and Daniel Stahl's books contained a thorough discussion of the Aristotelian doctrine of the causes. The two authors introduced Newton to natural philosophy.[20] Stahl wrote nothing explicit on *regressus*, but there are some statements that are compatible with the works I will present in Section 3. Let us look at the books from which Newton took notes.

The first is Gerardus Vossius's *Rheticores contractae, sive partionum oratorium, libri V* (1631).[21] This material dealt with demonstration, deliberation, conjectural reasoning, and the various states of the mind.[22] The book focused on rhetoric and does not engage in a discussion of causal explanation or *regressus* in natural philosophy. Joannes Magirus's scholastic compendium on physiology, *Physiologiae peripateticae: Libri sex cum commentariis* (1619) does contain a presentation of the Aristotelian doctrine of the causes, but has no discussion of *regressus* in natural philosophy.[23] Newton took various notes on such topics as: motion,[24] rest, infinity, place, vacuum, internal and external affections,[25] Aristotelian cosmology, and specific natural phenomena (such as eclipses, putrefaction, thunder, and rainbows).[26] Magirus defines physics as "the science of natural bodies (*corporum naturalium scientia*)": "**After** all, physics seeks the causes, principles and the proper affections of these natural bodies; and demonstrates the affections [of natural bodies] by their causes."[27]

Magirus does not elaborate further on this, but this passage entails a twofold directionality in scientific research: finding affections, principles and causes of bodies, and demonstrating the affections by means of the (previously discovered) causes. The authors I will present in Section 3 are more explicit on this matter. Daniel Stahl's *Axiomata philosophica, sub titulis XX* (1645), a more advanced Aristotelian compendium, was of considerable interest to Newton.[28] Stahl's work is more philosophical in orientation than Magirus's. Newton took notes on: the nature of essence, actuality and potentiality, the theory of the causes, the appetites, the will, agency and patient, matter, form, the theory of predication, the theory of genus, species, and difference, the idea of definition, the distinction between subject and accident, and the problem of truth and falsity.[29] "Titulus III" contains 21 rules concerning the doctrine of causes.[30] According to Stahl, effects are the "first part of experience (*prior pars patet experientia*)"; only later do we know the causes of things.[31] Effects occur first in our experiences, the causes only latently (*cf.* "incurrunt enim effectus in sensus, causis nos latentibus").[32] This is part of the human condition.

This brief discussion of these textbooks shows **that they would have ensured** Newton's familiarity with the Aristotelian tradition. In the following section, I will add some books that were — as I will argue — of even greater importance with respect to the three general features mentioned in the introduction.

2.2. *A Brief Note on* Regressus

I begin this section by clarifying what is meant by *regressus*. The Paduan Aristotelians — exemplified by Jacopo Zabarella — are usually credited with elaborating Aristotle's logic of demonstration into a scientific method of demonstration in which one first reasons from the effects to the causes (resolution), and then from the causes to the effects (composition). Nicolas Jardine states that demonstrative *regressus* is a procedure that combines an inference from an observed effect to its proximate cause with an inference from the proximate cause to the observed effect.[33] I will adopt

Jardine's terminology. Peter Dear notes:

> By no means wholly original with Zabarella but closely associated with his name throughout Europe in the later sixteenth and seventeenth centuries, the technique had developed from a commentary tradition that focused on Aristotle's *Posterior Analytics*, and in particular on Aristotle's distinction between two forms of demonstration: *apodeixis tou dioti* and *apodeixis tou hoti*, usually latinized as demonstration *propter quid* and demonstration *quia*.[34]

Aristotle's views on scientific inquiry which were embodied in his *Posterior analytics* were revived when they were rediscovered in the twelfth century. Jardine notes that *regressus* was often conflated with other methods and procedures. Aristotle's distinction between two procedures, one procedure going from the particular to the universal, the other from the universal to the particular, was identified with the first and second movement of demonstrative *regressus*, and likewise for his analysis–synthesis distinction.[35]

2.3. *The Aristotelian Textbooks on Logic*

We will now take a closer look at the Aristotelian textbooks on logic. Wilbur Samuel Howell noted that even until the beginning of the eighteenth century,

> it looked as if English logicians had permanently turned their backs upon the possibility of reforming logical theory in the direction of the revolutionary teachings of Bacon and Descartes, and had instead decided to do all that they could to restore the logic of Aristotle to its former pre-eminence in education and learning.[36]

As stated earlier, I shall not treat these books exhaustively, but will focus only on the relevant features: what these authors wrote on scientific method and *regressus*.[37] Recall that I do not wish to argue that these books individually influenced Newton, but rather that they provide a representative sample of Newton's Aristotelian training.

Logica sive Ars cogitandi was the first printing in England of the celebrated *Port-Royal logic*, which remained for well over a century a standard textbook of philosophy.[38] This work was originally printed in French in 1662 (*La logique ou L'art de penser*). It is generally known as a Cartesian work, but the insistence of the authors on syllogistic reasoning for instance gives away the Aristotelian concerns. The first Latin edition was printed in 1674, the first English edition in 1685. Newton owned a Latin version of 1687.[39] In chap. II of the fourth book, entitled "De duabis Methodus, Analysi & Synthesi", the authors Antoine Arnauld and Pierre Nicole discuss the nature of (scientific) method ('methodus'). The aim of this method is to guide the mind from a state of oblivion to a state of knowledge of truth. This method is twofold:

> the one to discover Truth, which is called *Analysis*, or *Method of Unfolding*, and which may also be called *Method of Invention*: And the other to make it understood by others when it is found out, which is called *Synthesis*, of the *Method of Composition*, and may also be called the *Method of Doctrine*.[40]

The most important difference between analysis and synthesis is this: analysis consists in beginning from particular cases in order to arrive at general rules; synthesis consists in positing the most general propositions in order to explain particular cases.[41] Every scientific investigation is carried out analytically: it attempts to resolve some question.[42] Let us look at the first two kinds of 'quaestio rei'.

> The first, when we seek for the causes by the effects.[43]

The authors give some examples. From the various effects of magnets we try to infer the causes. When we notice that *horror vacui* effects occur in nature, we try to find the cause that can produce such effects. From the flux and reflux of the sea we try to establish the true cause. The second kind concerns the reverse:

> The second is, when we seek to find out the Effect by the Causes.[44]

We find that wind and water have a great force to move bodies. For practical purposes (e.g. technological ones), we try to manipulate this force in order to obtain desirable effects. The difference from the first strategy is:

> So that it may be said, the first sort of Questions, whereby we seek the Causes by the Effects, include the speculative part of Physics, and the second part that seeks for the Effects by the Causes, contains the Practical part.[45]

In chap. XI, Arnauld and Nicole also formulate rules that are of use in science. These mainly concern the need for clarity of definitions and axioms.[46] Two features are important here: (1) analysis proceeds from particulars to universal (and conversely for synthesis), and (2) in (theoretical) physics we infer causes from effects.

In 1613 Samuel Smith published his *Aditus ad Logicam*,[47] of which Newton owned the 1649 version. I will discuss some relevant fragments from Liber III. Smith notes that science involves knowing things: to know something is to know the cause of it.[48] This entails knowing what the proximate cause is ('causa proxima'). In Caput IV and V Smith deals with 'demonstratio propter quid' and 'demonstratio quod'. The first kind of demonstration involves preceding from conclusions about the proximate causes, i.e. "a prioribus secundum naturam, & causis proximis". The second kind proceeds from the effects (to the proximate cause) or from the remote cause ('causa remota') to the effect.[49] So the last kind of demonstration can be done in two directions: "From a remote cause to the effect and from the effect to the [proximate] cause."[50]

Concerning this last direction type Smith writes:

> The demonstration from the effect to the cause takes place when the effect is more known and the cause is unknown: then after all we show that some cause is present (and therefore the effect is present).[51]

We therefore have three different reasoning steps:
(1) proximate causes → effects (*demonstratio propter quid*);
(2) effects → proximate causes (*demonstratio quod* type 1); and
(3) remote causes → effects (*demonstratio quod* type 2).

The *regressus*-strategy is treated explicitly:

> Regressus is that method of demonstration in which we collect a prior unknown cause from a known effect. Thereafter we truly demonstrate the same effect from the same cause.[52]

Smith's usage of *regressus* is, as we have seen, not entirely identical to Jardine's description of it (see 2.2). According to Jardine, *regressus* is a procedure that combines an inference from an observed effect to its proximate cause with an inference from the proximate cause to the observed effect. Smith's account is different in that it also incorporates a distinction between proximate and remote causes. But Smith's account agrees with the more general idea that science essentially involves reasoning from effects to causes and back. Although we first know the effect (and only later the cause), in nature ("ordo naturae") the cause takes place first.[53]

In 1687, the year in which the first edition of Newton's *Principia* appeared, John Wallis published his *Institutio logicae*.[54] Newton did not own a private copy of it. The work was, however, dedicated to the officers and Fellows of the Royal Society and what he said on method was intended primarily for young natural philosophers.[55] Wallis is known primarily as a mathematician, his *Arithmetica infinitorum* (1655) being his best-known work. In "Caput XV De Inductione & Exemplo" of the third part, Wallis discusses induction as a type of imperfect syllogism where one proceeds from particulars to a universal. Experimental philosophy ("Philosophia Experimentalis") proceeds from effects to causes:

> For although in the order of nature the progression is from causes to effects, yet in the order of knowing the progression is from observed effects to the investigation of causes. And indeed in the magnetic effects which I have already observed before (and that originally in a concrete case, I believe), that a magnet attracts iron and points to the north, no one would know or indeed suspect such a thing from the nature of the magnet. And so equally in many other things.[56]

Again we encounter demonstrative *regressus*. In the speculative sciences the method is

> to begin with the cause (or what is first in regard to way of operating) and from there to proceed to the effect; or alternatively to begin with the subject (the name, nature, and species which are first investigated) and thence to proceed to accidents, adjuncts, properties, and relations, along with the principles and causes of these last....[57]

The speculative sciences consist of two methods: one used for investigation (analysis) and one for teaching or education (synthesis). The first proceeds from individuals to universals ("a Particularibus ad Universalia procedit"); the second proceeds in the opposite direction.[58] Finally, Wallis discusses the use of method in mathematics where one begins with laying down definitions, citing axioms and postulates, and proving propositions from them.[59]

3. THE EFFECTS OF THE TEXTBOOKS ON NEWTON

In this section I will indicate the parallels between these textbooks and Newton's ideas on natural philosophy, focusing on the *Principia*. In 3.1 I will briefly analyse Newton's writings (notably the *Lucasian lectures*, *Opticks* and the *Principia*) and compare them with the ideas presented in the previous section. It will turn out that Newton's natural philosophy, even in its early form, always had a strong causal interpretation. In 3.2 I will examine Newton's explanations in the *Principia* in more detail. It will be shown that Newton conceived centripetal forces as proximate causes. First he inferred them (analysis), then he used them to explain effects (synthesis).

3.1. *Newton on Scientific Explanation*

There are many places in which Newton stresses the importance of proceeding from effects to causes. Let me give a brief overview. At the beginning of the *Principia* Newton declared:

> For the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces.[60]

The true forces of nature were unknown. In the *scholium* to the definitions, Newton wrote:

> But in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and conversely, of how to determine from motions, whether true or apparent, their causes and effects.[61]

Newton spoke of gravity as a causal entity.[62] In *Opticks* (first edition: 1704) he wrote that the main business of natural philosophy is "to argue from Phaenomena without feigning Hypotheses, and to deduce[63] Causes from Effects".[64] In Query 31 he wrote:

> As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For hypotheses are not to be regarded in experimental Philosophy.... By this way of Analysis we may proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general, from Effects to their Causes, and from particular Causes to more general ones, till the Argument end in the most general. This is the Method of Synthesis: And the method of Synthesis consists in assuming the Causes discover'd and establish'd as Principles, and by them explaining the phaenomena proceeding from them, and proving the Explanations.[65]

Natural philosophy consists in two kinds of demonstrations: first from effects to causes (the analysis) and then from causes to effects (the synthesis). The parallels with the authors of the textbooks on logic should be clear by now. Another point may be added. Let us look at the following famous quotation:

> Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity. Indeed, this force arises from some cause that penetrates as far as the centers of the sun and planets without diminution of its power to act, and that acts not in proportion to the quantity of the surfaces of the particles on which it acts (as mechanical causes are wont to do) but in proportion to the quantity of solid matter, and whose action is extended everywhere to immense distances, always decreasing as the squares of the distances.... And it is enough that gravity really exists and acts according to certain laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies.[66]

Newton took this to mean that he had proved gravity as a primary or proximate cause for the heavenly and terrestrial motions, but that he had not succeeded in discovering a further secondary or remote cause for gravity. This is confirmed by an unpublished draft intended for the *Scholium Generale*. Newton wrote as follows:

> First, the phenomena should be observed, then their proximate causes — and afterward the causes of the causes — should be investigated, and finally it will be possible to come down from the causes of the causes (established by phenomena) to their effects, by arguing a priori.[67]

Newton, like Smith, is thinking in terms of proximate and remote causes, along the lines of *demonstratio quod* types 1 and 2. He endorsed the difference between immediate causes and the highest level.[68] His thinking about nature always was in causal terms. But in his physical work, as in his optical work, he succeeded only in demonstrating the primary causes: in physics the force of gravitation, in optics the heterogeneity of white light. Many authors have taken Newton's explanations as anti-causal phenomenalism. But this is to neglect the causal language Newton used on several occasions. I have already pointed out several causal fragments from the *Principia* — which we will discuss in the following section — and *Opticks*. Newton's early optical work also testified to a causal outlook on scientific explanation. In his 1672 paper on light and colour Newton conceived of the heterogeneity of light as a cause,[69] and in his Lucasian lectures on optics (1670–72), he was even more explicit on his causal outlook:

> But had they sufficiently examined the cause of these colors, the inequal refrangibility of different rays would have immediately become known, and hence it would have been established that the defects of telescopes do not derive from the unsuitability of a spherical shape to perform refractions properly.[70]

The heterogeneity of white light is also considered to be a cause. Likewise for the explanation of other phenomena:

> Besides the phenomena of the colors that we have treated, there are still not a few others (especially the colors of very thin transparent plates, such as the aqueous globes of bubbles, air compressed between two plates, and very thin skins of many bodies) whose causes and measures can scarcely be accurately determined without mathematical reasoning.[71]

After Newton had discovered the cause of colours, he explained other phenomena (such as the rainbow) not originally among those used in establishing the heterogeneity of white light by means of this cause. The following fragment seems to be an early adumbration of the analysis–synthesis part in *Opticks*. First we discover causes from some effects, then assuming these causes we explain other phenomena:

> Thus far I have erected the foundations whereby the phenomena of colors produced in any way can be explained, but now I will describe individually the particular and immediate causes of the effects that I have not previously treated, not for the sake of the geometers (to whom, it will appear unnecessary) but for others.[72]

This indicates that Newton's conception of scientific explanation was causal from his earliest scientific works on.

Many scholars have neglected Newton's causal parlance. According to Ernan McMullin, in the *Principia* Newton's innovative approach "appeared to allow him to dispense with the troubling hypothetical element that the search for causal explanation had led his predecessors to admit into physics".[73] McMullin argues that the *Principia* offers "dynamical explanation" and that Newton's talk of attraction and forces is merely dispositional; but the present writer's opinion is that Newton's language is essentially phenomenal.

Alan Shapiro also stresses the phenomenal character of Newton's optical work.[74] Both McMullin and Shapiro claim that Bernard Cohen's "Newtonian Style" supports their views. McMullin explicitly claims that Cohen's account precludes abductive reasoning.[75] However, that this claim is unwarranted can be seen from the following: Cohen states that, in commenting on Book I, Propositions 1–3, Newton had demonstrated that a mathematically descriptive law of motion was shown by mathematics to be equivalent to a set of *causal* conditions of forces and motions.[76] Given that the laws of motion are valid, Newton is able to deduce that the area law is caused by its necessary and sufficient causal condition: a centripetal force.[77] This would allow reasoning from effects to causes. These centripetal forces are real entities and hence the real causes of the celestial motions we observe. However, Newton remains agnostic about the further explanation and cause of these centripetal forces. What physically causes gravity, is postponed to a later moment. These phenomenalist accounts are at odds not only with Newton's frequent usage of causal talk, they are also incompatible with Newton's practice in the *Principia*, as we will now see.

3.2. *Explanation in the* Principia

I will argue that Newton not only favoured causal explanation (as is shown in 3.1), but actually offered it. It is not my aim here to treat Newton's use of *regressus* exhaustively. I will limit myself to his use of it in the beginnings of Book III (*Systema mundi*) of the *Principia*. Newton introduced a new causal entity, "Universal Gravitation", which was completely different from the Aristotelian notion of cause. By means of abstract mathematics he was able to demonstrate that the force of gravity is the cause of the motion of the celestial and terrestrial bodies. Let us focus on Newton's abductive reasoning steps in Book III. I start with Propositions 1 and 2, where Newton deals with the dynamical implications of Kepler's second law:

> Book I, Proposition 1: The areas which bodies made to move in orbits describe by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times.[78]

The proof proceeds as follows (see Figure 1). Let a body by its inherent force, i.e. by its *vis inertiae*, describe the straight line *AB*. After *B* the body would normally (by

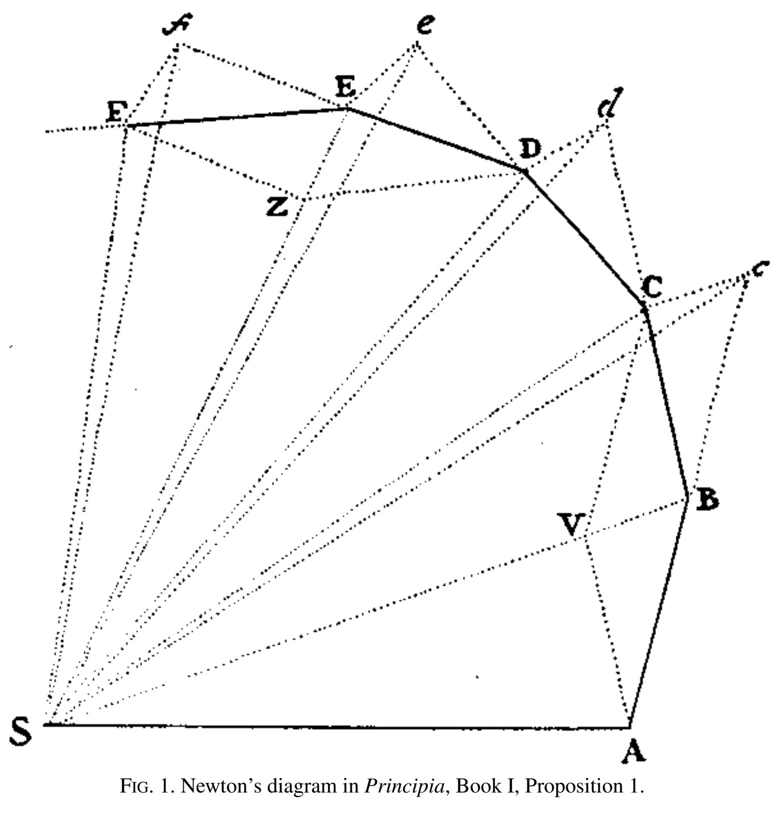

FIG. 1. Newton's diagram in *Principia*, Book I, Proposition 1.

Law 1) go straight to *c*. Let *cC* be parallel to *BS* and meet *BC* at *C*. A second force acts on *B* and makes it deviate from *Bc* and follow *BC* (by Corollary 1 of the laws of motion). When the second part of the time has been completed — the time is divided in equal intervals and the force acts instantaneously after each such interval — the body will be found at *C* in the same plane as triangle *ASB*. Since *SB* and *Cc* are parallel, triangle *SBC* will be equal to *SBc* and thus to *SAB*. *SBC* and *SBc* are equal since both their heights and their bases are equal. That their bases *SB* are equal is evident. The height is the same in each because both triangles are between the parallels *SB* and *Cc*. Hence they describe an equal area in an equal amount of time.

By a similar argument this can be extended to all triangles. If the number of triangles is increased infinitely, one can still apply the same reasoning: they will describe equal areas in equal times. Proposition 1 is essentially based on: the first law of motion, Corollary 1 of the laws, and a limiting procedure.[79] Several interesting suppositions are made: the inertial component is visualized as *Bc*, *Cd*, *De*, *Ef*, ...; the centripetal force as *cC*, *dD*, *eE*, *fF*, ...; the resulting force of these forces as *BC*, *CD*, *DE*, *EF*, ... This model is a one-body system, i.e. a system with one point mass drawn towards an empty point.

Let us now turn to Proposition 2.

> Book I, Proposition 2: Every body that moves in some curved line described in a plane and, by a radius drawn to a point, either unmoving or moving uniformly forward with a rectilinear motion, describes areas around that point proportional to the times, is urged by a centripetal force tending toward that same point.[80]

Proposition 2 amounts to saying that Kepler's second law (which supposes a constant areal velocity and equality of the plane of motion) presupposes a centripetal force (thus: it demonstrates the converse of Proposition 1). Newton shows that a centripetal force is a necessary (Proposition 1) and sufficient (Proposition 2) cause for Kepler's area law: Kepler's area law is valid if and only if there is a centripetal force.[81] The proof proceeds as follows (again see Figure 1). By Newton's first law we know that a body that moves in a curved line is deflected from a rectilinear course by some force acting on it. The force that deflects the body and makes it describe in equal times around an immoveable point *S* the equal minimally small triangles *SAB*, *SBC*, *SCD*, ... acts along a line parallel to *cC*, i.e. along the line *BS*; at point *C*, **it acts parallel to** the line *dD*, i.e. along *CS*; **and so on.** Therefore it always acts along lines tending towards *S*. Similar suppositions are made as in the previous proposition.

We now proceed to Proposition 4.

> Book I, Proposition 4, Corollary 6: If the periodic times are as 3/2 powers of the radii, and therefore the velocities are inversely as the square roots of the radii, the centripetal forces will be inversely as the squares of the radii; and conversely.[82]

The reasoning proceeds as follows. In Proposition 4 Newton had already proved that the magnitude of this force is proportional to v^2/r. Corollary 6 shows that this fact combined with Kepler's harmonic law delivers an inverse square law and vice versa. In modern terminology this can be demonstrated as follows (Newton does not give any further comment to this corollary). Huygens published the result that a body travelling in a circle with constant angular speed needs a force proportional to v^2/r to keep it in orbit: $F = kv^2/r$. Since v equals $2\pi r/t$, $F = k4\pi^2 r^2/t^2 r$. Multiplying by r/r, we have $F = k4\pi^2 r^3/t^2 r^2$. Since r^3/t^2 is a constant according to Kepler's third law, we can write: $F = (\text{constant})/r^2$. Thus, Kepler's third law implies an inverse square law, and conversely.

The real world is not the subject of Book I; Book I lacks physical content.[83] As such, Book I is written with the purpose of demonstrating what will be the case if certain physical conditions hold (neglecting at that moment the real conditions in the actual world). Put differently: it is an investigation of what follows from the laws of motion given certain force functions. It is an enterprise different from that undertaken by Newton in Book III, the *Systema mundi*. Book I is an investigation of the mathematical consequences that follow from certain physical conditions, i.e. an "investigation of those quantities of forces and their proportions that follow from any conditions that may be supposed". Then "coming down to physics", i.e. in the process of explaining real-world motions, one starts by looking at which mathematical properties are present in nature and infers from this which mechanisms are active (*cf.* "these proportions must be compared with the phenomena, so that it may be found out which conditions (or laws) of forces apply to each kind of attracting bodies").[84] In other words: from the mathematical properties present in nature one infers the physical agents.[85] Bernard Cohen states, in commenting on Propositions 1–3 of Book I, that Newton was able to demonstrate that a mathematically descriptive law of motion was shown by mathematics to be equivalent to a set of causal conditions of forces and motions.[86] Newton is arguing from effects (Kepler's laws, or Kepler's rules as they were called at the time) to causes (centripetal forces). Newton's method is far more complex than characterized here.[87] It incorporates many features of Smith's account (e.g., the idea of systematic dependencies), but is explicitly "model-based". Newton starts with a set of physical conditions, P_x, and next he deduces the mathematical properties, M_x. Newton then observes some empirical regularities which can be expressed mathematically (M_x) and finally concludes to P_x. This amounts to abduction:

(1) $P_x \to M_x$
(2) M_x
(3) P_x

The astonishing feature of Newton's method is that he allows for *quam proxime* "if–then" propositions.[88] For instance in Book I, Proposition 65, of the *Principia*, he is able to infer that in a many-body system in which several smaller bodies orbit around one greater body by an inverse-square law (i.e., P_x), the area law will hold *very nearly* (i.e. M_x (*quam proxime*)).[89] We obtain propositions of the kind:

(1) $P_x \to M_x$ (*quam proxime*)
(2) M_x (*quam proxime*)
(3) P_x

These reasoning steps can then be reconstructed as:

(1a) If a centripetal force acts on a body, then Kepler's second law holds very nearly.
(1b) If the centripetal force varies inversely as the square of the distances, Kepler's third law holds very nearly.
(2) Both Kepler's second and third laws hold very nearly for the primary and the secondary planets.
(3a) The motions of the primary and secondary planets are caused by centripetal forces.
(3b) These centripetal forces vary inversely as the square of the distances.

Book I, Proposition 2, allows one to conclude that Kepler's second law entails a centripetal force. So we could say that one can "deduce" a centripetal force from Kepler's second law. However, in the natural order it is the centripetal force that produces or causes motion that conforms to Kepler's second law. That is why it amounts to abduction. This agrees with Newton remark on "arguing from Effects to Causes". The essential thing is that the conditional sentences in (1) are deduced from the laws of motion. We establish that if P_x (a centripetal force) is active then M_x (e.g. the area-law) holds. Then we observe that M_x holds very nearly. From this we can abductively infer that P_x is the cause of the observed mathematical regularities.

This is a striking feature of Newton's methodology. This interpretation also enables one to alleviate the apparent tension between the fact that these models do not exhaustively describe the actual world and Newton's realistic-causal stance. The awareness of this incongruence can be found in various statements of Book III.[90] Although these propositions do not exactly coincide with nature, they can teach us something about the causal agents in the world. This was of the greatest importance to Newton.

Newton surely must have thought that he provided causal explanations in the *Principia*. This was not in the strong causal-agent sense nor in the traditional Aristotelian sense, since Newton admits that he did not assign the cause of the gravity that "penetrates" the cosmos.[91] But how then are we to understand *this sense*? In my opinion there is only one option: to take Newton's causal parlance at face value. He conceptualized gravity as a proximate cause. The cause of this proximate cause is left unspecified. Newton's notion of cause was broader than a purely mechanical notion. Newton is inferring instances of centripetal forces. Gravity provides explanations as a proximate cause. Gerd Buchdahl makes clear that we should make the distinction between "the logical status of gravity itself, as a 'primary' cause, and the modus operandi, if any, of a secondary explanatory mechanism for gravity".[92] What Newton was doing here, was not — at least considered from a *general* methodological perspective — new: he was proceeding from effects to (proximate) causes. However, Newton proposed a radically new kind of causal entity: *a non-mechanical entity inferred by means of abstract mathematics*.[93] Most contemporaries, such as Leibniz and Huygens, criticized Newton for not giving a physical, i.e. mechanical, explanation of gravity. They viewed Newton's force of universal gravitation as an occult quality.

On of the more careful observers, the Jesuit Castel, criticized Newton because there was nothing "physical" in Newton's abstract demonstrations.[94] They were purely mathematical. It is far from obvious how they — in virtue of being purely mathematical — can explain physical behaviour. The novelty involved in Newton's method baffled most of his contemporaries.[95] Nevertheless, gravity was a causal entity for Newton and he thought that mathematics could unravel the underlying causes in nature. After some unification steps, where Newton identifies the various centripetal forces in our solar system, he arrives at universal gravitation.[96] It is here that the synthetical moment of science enters the scene. The synthesis starts after Proposition 8 and extends to the very end of Book III. Newton shows that the motion of the planets, the motion of the Moon, the tides, and the motion of comets can all be deduced from the causes proposed by the theory of universal gravitation. So in the synthesis we assume "the Causes discover'd and establish'd as Principles, and by them explaining the phaenomena proceeding from them, and proving the Explanations". This is identical to Smith's *demonstratio propter quid*: proceeding from proximate causes to effects. At its most general, abstract form, Newton was using the strategy of *regressus*, reasoning to causes and back. He radically modified the content of the notion 'cause'. This Aristotelian textbook tradition perceived science as a dual process of proceeding from effects to causes and then from causes to effects. Newton's explicit usage of analysis–synthesis in a causal way emerged as he introduced a new causal entity: universal gravitation. Newton recast his innovating scientific demonstrations in an Aristotelian terminology.

4. CONCLUSION

The impact of these textbooks is limited to what I have termed 'general' features of Newton's view on natural philosophy. These textbooks conceive of natural philosophy as a dual process: its first moment is regressive, the unravelling of proximate causes (and if possible the causes of these causes, i.e. the remote causes) of the effects we observe; the second is progressive, the postulating of proximate causes (and if possible, of remote causes) in order to demonstrate their effects. Newton, although he was reforming the very notion of cause, took over the Aristotelian terminology of explaining phenomena by arguing from effects to causes and conversely. They are representative of certain aspects of the intellectual climate of the seventeenth century. When we juxtapose Newton's work to these textbooks, Newton's causal outlook comes as no surprise.

My analysis also helps to explain why Newton did not endorse a probabilistic account of natural philosophy. Newton's stress on causes and certain knowledge was at odds with the general climate at the Royal Society.[97] Newton favoured certain knowledge, and disliked hypotheses.[98] Certain knowledge includes knowing the causes of things. Contrary to the spirit of Bacon, most natural philosophers of the Royal Society endorsed a probabilistic view on human knowledge, which included hypotheses.[99] In their willingness to accept probability they "moved away not only from thinkers like Hobbes, but even from Bacon (and for that matter, from Aristotle)".[100] People could arrive only at morally certain knowledge. They could not know nature "in their true immediate, necessary causes".[101] Newton thought otherwise.

This case-study indicates the need for futher research into such questions.[102] We are not claiming that Newton was an Aristotelian concerning all his ideas on science, or that these textbooks are the only relevant sources for his conception of science. Newton's abstract, mathematical way of dealing with force was surely unprecedented. It is obvious that the abductive mathematical propositions in Book I are mathematized in a way that was not carried through to a conclusion. There certainly remain questions. These textbooks are only the tip of the iceberg. What lies beneath, i.e. the relations between these and other books, remains unclear. Were the works I mentioned important to other natural philosophers? Are there further relevant textbooks? How exactly was this textbook tradition introduced and assimilated in seventeenth-century England?[103] Was there a relation or mutual connection between Hobbes and Bacon? What was the role of these textbooks in this? What was the role of these books in the milieu of the Royal Society? What is the link between Newton and Hobbes? These are questions that merit investigation.

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1. I am somewhat suspicious of using the term 'Aristotelian' here. Any tradition is modified as different authors interpret it. However, these authors "take the principal concepts and divisions of their natural philosophy from Aristotle's books". See Patricia Reif, "The textbook tradition in natural philosophy", *Journal of the history of ideas*, xxx (1969), 17–32, pp. 19–20. The Aristotelian tradition was not a monolithic whole. For a recent account of the tradition and its variety from Antiquity to the seventeenth century see Cees Leijenhorst, Chrostoph Lüthy and Johannes Thijssen (eds), *The dynamics of Aristotelian natural philosophy from Antiquity to the seventeenth century* (Leiden, Boston and Cologne, 2002).
2. Near the end of the seventeenth century, the textbook mode of exposition became more and more popular. Direct reading of the Aristotelian text correspondingly began to decline. It was during the seventeenth century that the philosophical textbooks really began to dominate the teaching of the subject in most formal courses in institutions of higher learning. See Charles B. Schmitt, "The rise of the philosophical textbook", in Charles B. Schmitt (ed.), *The Cambridge history of Renaissance philosophy* (Cambridge, 1988), 792–804, p. 801.
3. See Charles B. Schmitt, *Aristotle and the Renaissance* (Cambridge, MA, and London, 1983), 3, 108.
4. See Maurizio Mamiani, "To twist the meaning: Newton's *regulae philosophandi* revisited", in Jed Z. Buchwald and I. Bernard Cohen (eds), *Isaac Newton's natural philosophy* (Cambridge, MA, and London, 2001), 3–14.
5. Mamiani, *op. cit.* (ref. 4), 4.
6. I admit that the brute fact that these books were present in Newton's library does not by itself constitute

evidence that these books contributed to Newton's conception of natural philosophy.

7. See Cees Leijenhorst, *The mechanisation of Aristotelianism: The late setting of Thomas Hobbes' natural philosophy* (Leiden, Boston and Cologne, 2003), 8.
8. See Jaakko Hintikka and Unto Remes, *The method of analysis, its geometrical origin and its general significance* (Dordrecht and Boston, 1974), 107–8; Richard Westfall, *Never at rest: A biography of Isaac Newton* (Cambridge, 1980), 377–81; and Peter Dear, *Discipline and experience: The mathematical way in the Scientific Revolution* (Chicago and London, 1995), 240. McGuire and Tamny have a rather different explanation: they claim that "Newton probably became familiar with the method of analysis and synthesis in a philosophical context" from Thomas Hobbes's *Elements of philosophy* (1656). See James E. McGuire and Martin Tamny (eds), "Commentary", in *Certain philosophical questions: Newton's Trinity notebook* (Cambridge, 1983), 24. Hobbes indeed discusses the synthetical and analytical method as respectively preceding from and ascending to causes (especially in chap. 6 of *Of method*). The conjecture of McGuire and Tamny can be consistent with my findings. See Thomas Hobbes, *Elements of philosophy* (London, 1656), and Leijenhorst, *op. cit.* (ref. 7).
9. See Alistair C. Crombie, *Styles of scientific thinking in the European tradition: The history of argument and explanation especially in the mathematical and biomedical sciences and arts* (London, 1994), 283. In an accompanying footnote Crombie refers to the *Opticks* but not to the *Principia*. See *ibid*., 716n.
10. For a recent account and further references see Niccolò Guicciardini, "Analysis and synthesis in Newton's mathematical work", in I. Bernard Cohen and George E. Smith (eds), *The Cambridge companion to Newton* (Cambridge, 2002), 308–38.
11. Guicciardini, *op. cit.* (ref. 10), 311.
12. Guicciardini, *op. cit.* (ref. 10), 317.
13. Guicciardini, *op. cit.* (ref. 10), 319. See especially Niccolò Guicciardini, *Reading the Principia: The debate on Newton's mathematical methods for natural philosophy from 1687 to 1736* (Cambridge, 1999), 17–38.
14. Guicciardini, *op. cit.* (ref. 13), 38.
15. From a purely logical point of view this mathematical account of analysis–synthesis is incompatible with Newton's conception of analysis as discovering causes, and synthesis as assuming these causes to explain phenomena (see 3.1). In the mathematical tradition analysis means reasoning from what is sought to what is known and conversely for synthesis. In Newton's view analysis is reasoning from what is known, the effect, to what is sought, the cause, and conversely for synthesis.
16. Few authors have explicitly labelled Newton's reasoning abductive. See e.g. George E. Smith, "The methodology of the *Principia*", in Cohen and Smith (eds), *op. cit.* (ref. 10), 31–70.
17. Newton's *Principia* for instance contained the synthetical method of ultimate ratios as well as the analytical method of fluxions. See Guicciardini, *op. cit.* (ref. 10), 320–4, and especially Guicciardini, *op. cit.* (ref. 13), 39–98.
18. See Westfall, *op. cit.* (ref. 8), 81–82. Students read Aristotle's *Physica*, *De caelo*, and *De anima*. See especially Phyllis Allen, "Scientific studies in the English universities of the seventeenth century," *Journal of the history of ideas*, x (1949), 219–53, p. 220. For a general survey see William T. Costello, *The scholastic curriculum at early seventeenth-century Cambridge* (Cambridge, MA, 1958).
19. McGuire and Tamny (eds), *op. cit*. (ref. 8).
20. Westfall, *op. cit.* (ref. 9), 84.
21. Gerardus Ioannis Vossius, *Rhetorices contractae, sive partitionum oratorium, Libri V* (Oxford, 1631).
22. McGuire and Tamny (eds), *op. cit*. (ref. 8), 19.

23. Johannes Magirus, *Physiologiae peripetaticae, libri sex cum commentariis* (Canterbury, 1619; quotation from 1642 edn). Magirus's treatment of the doctrine of the causes can be found in "Librum I: De natura deque naturalium principiis, affectionibus & accidentibus", *ibid*., 1–56. For Magirus's discussion on efficient and final causes, see *ibid*., 21–25.
24. *Ibid*., 26–56.
25. *Ibid*., 9, 21.
26. McGuire and Tamny (eds), *op. cit*. (ref. 8), 15–17.
27. Magirus, *op. cit.* (ref. 23), 1.
28. Daniel Stahl, *Axiomata philosophica, sub titulis XX* (Cambridge, 1645).
29. McGuire and Tamny (eds), *op. cit*. (ref. 8), 18.
30. Stahl, *op. cit.* (ref. 28), 60ff.
31. *Ibid*., 69.
32. *Ibid*., 76.
33. See Nicolas Jardine, "Epistemology of the sciences", in Schmitt (ed.), *op. cit.* (ref. 2), 685–711, p. 686.
34. Dear, *op. cit.* (ref. 8), 27. Also see John H. Randall, "The development of scientific method in the school of Padua", *Journal of the history of ideas*, i (1940), 177–206; William Wallace, *Galileo's logic of discovery and proof: The background, content, and use of his appropriated treatises on Aristotle's Posterior Analytics* (Dordrecht, Boston and London, 1992), 166–7.
35. See Jardine, *op. cit.* (ref. 2), 687–8.
36. Samuel W. Howell, *Eighteenth-century British logic and rhetoric* (Princeton, 1971), 13.
37. I will focus on the similarities in these works. This inevitably harms the content of these works. However, my aim is not to present these books in their full complexity, but to point to some representative features common to the Aristotelian tradition in which Newton was trained. We have to bear in mind that "beneath the umbrella of 'Aristotelianism' are a very large number of thinkers of very diverse orientation". See Charles B. Schmitt, *John Case and Aristotelianism in Renaissance England* (Kingston and Montreal, 1983), 218. Those interested in a broader perspective I refer to Samuel Wilbur Howell, *Logic and rhetoric in England, 1500–1700* (New York, 1961).
38. For a presentation of this work see Howell, *op. cit.* (ref. 36), 350–63.
39. See John Harrison, *The library of Isaac Newton* (Cambridge, 1978), 182.
40. Antoine Arnauld and Pierre Nicole, *Logica sive ars cogitandi* (London, 1687), 386.
41. *Ibid*., 375.
42. *Ibid*., 375–6. Arnauld and Nicole point out that the greatest part discussed here concerning questions was taken from "a manuscript of the Deceas'd *Descartes*".
43. *Ibid*., 375.
44. *Ibid*., 369.
45. *Ibid*., 369.
46. *Ibid*., 416. I will not insist on this point since these features also are present in the mathematical tradition.
47. For a presentation see Howell, *op. cit*. (ref. 36), 292–8. It is bound together with Edward Brerewood's *Elementa logicae* (edition of 1649). See Edward Brerewood, *Elementa logicae* (London, 1649), and Harrison, *op. cit.* (ref. 39), 110, 240. Since Brerewood does not add anything fundamental for my present purposes I will omit a presentation of it here.
48. Samuel Smith, *Aditus ad Logicam* (London, 1613), 97.
49. *Ibid*., 111.
50. *Ibid*., 112. These translations are mine. I have opted for a free translation (without doing harm to

the original content).

51. *Ibid*., 113.
52. *Ibid*., 116.
53. *Ibid*., 154.
54. Howell, *op. cit.* (ref. 36), 29.
55. *Ibid*., 39.
56. Quoted from *ibid*., 36.
57. Quoted from *ibid*., 39.
58. John Wallis, *Institutio logicae* (Oxford, 1687), 212–13.
59. See Howell, *op. cit*. (ref. 36), 40–41; and Wallis, *op. cit.* (ref. 58), 215–17. As we have seen, Arnauld and Nicole also stressed the importance of clear and evident axioms and principles.
60. Isaac Newton, *The Principia: Mathematical principles of natural philosophy*, transl, by I. Bernard Cohen and Anne Whitman (Berkeley, Los Angeles and London, 1999), 382. The part continues: "It is to these ends that Books 1 and 2 are directed, while in Book 3 our explanation [*explicationem Systematis mundani*] of the system of the world illustrates these propositions. For in Book 3, by means of propositions demonstrated mathematically in Books 1 and 2, we derive from celestial phenomena the gravitational forces by which bodies tend toward the Sun and toward the individual planets. Then the motions of the planets, the comets, the moon, and the sea are deduced from these forces by propositions that are also mathematical." *Ibid*., 382.
61. *Ibid.*, 415.
62. In the case of the motion of the Moon, Newton declared as follows: "I wished to show by these computations of the lunar motions that the lunar motions can be computed from their causes by the theory of gravity", *ibid*., 869. In Book III, Proposition 24, Newton declared: "Hitherto I have given the causes of the motions of the moon and seas", *ibid*., 839.
63. In the conclusion of his *Experimental philosophy* (London, 1664), 192, Henry Power used the expression "*deducing the Causes of things*". I do not know in what way Power might have been important to Newton's idea of science. Newton owned a copy of the first edition, see Harrison, *op. cit.* (ref. 39), 221.
64. See Isaac Newton, *Opticks or a treatise of reflections, refractions, inflections and colours of light* (New York, 1979), 369.
65. *Ibid*., 404–5.
66. Newton, *op. cit.* (ref. 60), 943.
67. *Ibid*., 53.
68. Athanasios Raftapoulos calls the immediate cause a "property" and the highest level cause a "cause". Athanasios Raftopoulos, "Newton's experimental proofs as eliminative reasoning", *Erkenntnis*, l (1999), 91–121, pp. 107–8.
69. I. Bernard Cohen (ed.), *Isaac Newton's papers and letters on natural philosophy* (Cambridge, 1978), 54. In the *Opticks*, immediately after the famous analysis–synthesis piece, Newton declares that he used the method of analysis to discover and prove the "original Differences of the Rays of Light in respect of their Refrangibility, Reflexibility, and Colour, and their alternate Fits of easy Reflection and easy Transmission, and the Properties of Bodies both opake and pellucid, on which their Reflexions and Colours depend" (Newton, *op. cit.* (ref. 64), 405; for causal statements on these matters see e.g., *ibid*., 57, 113, 119, 244). Thereafter he used the method of composition for explaining the phenomena arising from them (e.g. the rainbow).
70. Isaac Newton, *The optical papers of Isaac Newton*, i: *The optical lectures 1670–1672*, ed. by Alan E. Shapiro (Cambridge, 1984), 433; see also 525.
71. *Ibid*., 603.
72. *Ibid*., 523; for Newton's explanation of the rainbow, see *ibid*., 593–601.

73. Ernan McMullin, "The impact of Newton's *Principia* on the philosophy of science", *Philosophy of science*, lxviii (2001), 279–310, esp. pp. 288–9.
74. Alan E. Shapiro, *Fits, passions, and paroxysms: Physics, method, and chemistry and Newton's theories of colored bodies and fits of easy reflection* (Cambridge, 1993), 23.
75. McMullin, *op. cit.* (ref. 73), 289.
76. I. Bernard Cohen, *The Newtonian Revolution, with illustrations of the transformation of scientific ideas* (Cambridge, 1980), 28, 37.
77. Cohen, *op. cit.* (ref. 76), 63.
78. Newton, *op. cit.* (ref. 60), 444.
79. Bruce Brackenridge, *The key to Newton's dynamics, the Kepler problem and the Principia* (Berkeley, 1995), 26. Hence a centripetal force is a necessary and sufficient condition for the area law. As Newton writes somewhat further: "Since the uniform description of areas indicates the center towards which that force is directed by which a body is most affected and by which it is drawn away from rectilinear motion and kept in orbit, why should we not in what follows use uniform description of areas as a criterion for a center about which all orbital motion takes place in free spaces?" (Newton, *op. cit.* (ref. 60), 449).
80. Newton, *op. cit.* (ref. 60), 446.
81. He further writes: "The more the law of force departs from the law there supposed, the more the bodies will perturb their mutual motions; nor can it happen that bodies will move exactly in ellipses while attracting one another according to the law here supposed, except by maintaining a fixed proportion of distances one from another. In the following cases, however, the orbits will not be very different from ellipses" (Newton, *op. cit.* (ref. 37), 568).
82. Newton, *op. cit.* (ref. 60), 451.
83. François De Gandt, *Force and geometry in Newton's Principia*, transl. by Curtis Wilson (Princeton, 1995), 267.
84. Newton, *op. cit.* (ref. 60), 588–9.
85. Newton, *op. cit.* (ref. 60), 415; Newton, *op. cit.* (ref. 64), 369.
86. Cohen, *op. cit.* (ref. 76), 28, 37.
87. See Steffen Ducheyne, "Mathematical models in Newton's Principia: A new view of the 'Newtonian Style'", *International studies in the philosophy of science*, xix (2005), 1–19. This paper contains a critique of I. Bernard Cohen's "Newtonian Style".
88. Smith, *op. cit.* (ref. 16).
89. Newton, *op. cit.* (ref. 60), 567–9.
90. Newton, *op. cit.* (ref. 60), 818, 819, 832, 840, 841, 845, 847, 864, 867.
91. Newton, *op. cit.* (ref. 60), 943.
92. Gerd Buchdahl, "Gravity and intelligibility: Newton to Kant", in R. E. Butts and J. W. Davis (eds), *The methodological heritage of Newton* (Bristol, 1970), 74–102, esp. p. 81.
93. See especially Andrew Janiak, "Newton and the reality of force", *Journal of the history of philosophy* (forthcoming).
94. Yves Gingras, "What did mathematics do to physics", *History of science*, xxxix (2001), 383–416.
95. Robert Iliffe, "Abstract considerations: Disciplines and the incoherence of Newton's natural philosophy", *Studies in history and philosophy of science*, Part A, xxxv (2004), 427–54, p. 439.
96. For a thorough account of Newton's concept and practice of unification see Steffen Ducheyne, "Newton's idea and practice of unification", *Studies in history and philosophy of science*, Part A, xxxvi (2005), 61–78.
97. See Newton, *op. cit.* (ref. 37), 588–9.

98. See Barbara Shapiro, *Probability and certainty in seventeenth-century England: A study of the relationships between natural science, religion, history, law, and literature* (Princeton, 1983), 58.

99. See Henry G. Van Leeuwen, *The problem of certainty in English thought 1630–1690* (The Hague, 1963). For a general discussion on the emergence of probability in the seventeenth century, see Ian Hacking, *The emergence of probability: A philosophical study of early ideas about probability, induction and statistical inference* (Cambridge, 1975); Richard Popkin, *The history of scepticism from Erasmus to Spinoza* (Berkeley, Los Angeles and London, 1979); and Shapiro, *op. cit.* (ref. 78).

100. See Michael Hunter, *Science and society in Restoration England* (Cambridge, 1981), 180; Hunter points to the "significant methodological differences" between the members. See also his *Establishing the new science: The experience of the early Royal Society* (Woodbridge, 1989), 207–8. The corresponding chapter (pp. 185–244) in Hunter's books is a reprint of the original paper: Paul Wood and Michael Hunter, "Towards Solomon's House: Rival strategies for reforming the early Royal Society", *History of science*, xxiv (1986), 49–108.

101. Joseph Glanvill, quoted in Van Leeuwen, *op. cit.* (ref. 99), 76.

102. Schmitt suggested that it could be argued that there was a significant Aristotelian component to Newton's thought. See Schmitt, *op. cit.* (ref. 3), 28, and *op. cit.* (ref. 2), 7. He did not provide a thorough elaboration of this and did not specify wherein this Aristotelian component consisted.

103. For a rough sketch on the preceding period see Schmitt, *op. cit.* (ref. 3), especially Chapter I.