The body in Renaissance arithmetic: from mnemonics to embodied cognition

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Abstract. In Medieval and Renaissance arithmetic we find several instances of references to body parts or actions involving body parts. In this paper we will address the question on the historical functions of body parts in mathematics and discuss its relation to the currently prevailing practice of symbolic mathematics.

1 INTRODUCTION

There can be no doubt that the earliest human practices of counting and measuring involved body parts, most notably the fingers and hands. The bijective relation between the fingers on our hands and counted objects allows for counting and ordering of ten objects. Our current use of the decimal system is most likely related to this fact. Other systems, such as base-20, practiced during the first millennium among the Maya of Central America refer to other combinations of fingers or body parts. But the art of dactylonomy improves on such basic relations and includes bending, crossing and touching of fingers to represent all possible numbers up to 10,000. We have strong indications that the first symbols used in our script draw their analogy from finger counting practices. It is generally accepted that Chinese numerals are a direct representation of hand gestures [4]. The first three digits 一, 二, 三, obviously represent fingers; 四, four: the thumb is held in palm and four fingers are extended, 五, five: the fingers are extended with the thumb facing upwards, 六, six: the little finger and thumb are extended and other fingers are closed, and so on. For Egyptian or Babylonian mathematics we have no such clues but references exist to the practice of finger reckoning in these cultures. The Latin term *digit* both denotes a cipher as well as a finger and the verb *computare* originally referred to Roman number arithmetic. The first chapter of Bede's *de temporum ratione liber* (c. 730) is entitled "de computo et loquela digitorum" (On computing and speaking with the fingers). Finger reckoning was also practiced in the Arab world and known as *ḥisāb al-'aqd* (literally: arithmetic of the joints). By the time Fibonacci wrote his *Liber abbaci* on reckoning with Hindu-Arabic numerals (1202), the European abacus and finger reckoning were two competing systems, both using a tangible form of practicing mathematics.

2 FUNCTIONS OF BODY PARTS

In this section we discuss the possible functions that body parts had in Renaissance arithmetic. We discern four different functions:

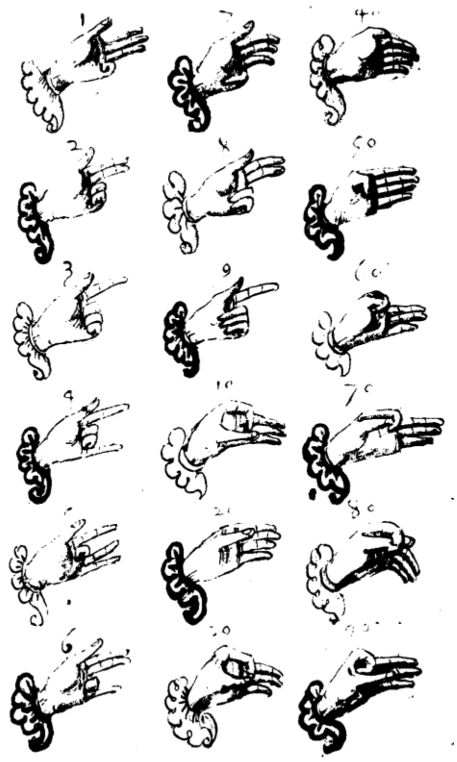

Figure 1. Number representations in abbaco culture (from ms. Add.8784 © The British Library)

- Memorization: though finger reckoning leaves no permanent record as does the abbaco method of calculating with the pen, short-term memorization is the primary and most essential function of body parts in arithmetic. Fingers are used to memorize intermediate results during calculation such as the carry in multiplication procedures. Body parts can also have long-term mnemonic functions. An example is the use of knuckles on the fist to memorize the months of a year which have 31 days.
- Communication: the representation of all numbers up to 10,000 on a single hand was a skill which was taught at the abbaco schools of fourteenth and fifteenth century Italy. As extant witness of this practice we have about 250 abbaco manuscripts of that period. Figure 1 shows the gestures for numbers up to 90, from an anonymous manuscript of 1443. We find the same representation in Pacioli's *Summa* of 1494 (pp. 36-7). The chapter in which the technique is described is called "to set out numbers on the hand" (porre alle mani). The purpose thus is not to calculate but to represent and display numbers. Within the context of abbaco practices such skill functioned as a way of

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communicating and negotiating prices or quantities in merchant transactions. Gestures assured reliable communication across language barriers, functioned in noisy environments and could span short distances as between a ship and shore.

- Visualization: our current practice of symbolic mathematics allows us to represent a problem or a mathematical structure using a single formula or expression (as the systems of equations (1), discussed below). The importance of visualization in mathematics becomes painfully evident when one is confronted with loss of sight. Nicholas Saunderson, Lucasian professor of mathematics at Cambridge from 1711, was blinded by smallpox. For his own purpose of visualizing arithmetical operations he devised a wooden board and a method called "palpable arithmetic" which is described as a preface to his *Elements of algebra*. A large wooden board was divided into 100 little squares with each nine holes. Digits were represented by combinations of two pens in these holes. While the contrivance could be compared with the abacus, its purpose was basically to envisage large numbers. The tactile interaction of hands and fingers with the wooden board and pegs thus functioned as a way to visualize and arrange mathematical objects and operations.
- Respresentation of procedures: this last function is the most advanced one and will be discussed in the following section. In Lakoff's terminology body parts here use the metonymic mechanism to represent both an unknown quantity of a problem as well as the operations on that quantity prescribed by a proto-algebraic recipe [2, 74].

3 PROBLEM SOLVING PROCEDURES

In previous section we discussed the basic functions of body parts in counting, representation of numbers and calculations in Renaissance arithmetic. But we found references to body parts in more advanced problem-solving procedures. Renaissance mathematics is very much based on the use of what I have coined as proto-algebraic rules [1]. These are procedures which are not algebraic themselves but are based on previous algebraic solutions or procedures. Such rules are usually named and are specific to standard problem types. The book on arithmetic by Johannes Widmann [6] includes no less than 32 such rules.

One of the rules Widman discusses is called *regula coecis* and demonstrates how proto-algebraic rules can be represented by tangible operations and actions. The rule is intended to solve a type of problem best known as "the hundred fowls problem". In European arithmetic it appears more frequently as a problem about a number of men, woman and children who have to pay the bill at a tavern. This formulation, which explains the alternative name *Regula virginum*, first appears in the Parmiers manuscript written around 1430 [6]. The second arithmetic book by Adam Riese [5, 135], lists two problems which can be represented in modern symbolism as follows:

$$\begin{cases} x+y=12 \\ 3x+5y=81 \end{cases} \quad \begin{cases} x+y+z=20 \\ 3x+2y+z=40 \end{cases} \qquad (1)$$

The first problem involves 12 men and woman who consume a total of 81. Men pay 5, woman 3. The question is how many of each is present in the tavern. In a later edition of 1574, the operation is described as *Zech rechnen* (f. 69^v):

> Schreib vor dich gegen der lincken handt die anzahl de Personen. Gegen der rechten handt / wie viel sie vertruncken / und in die mitte / wie vil einjegliche Person / jeglichs geschlechts in sonderheit gibt. Darnach mach das gelt dem menigsten uberall gleich / als dann multiplicir das kleinest an der bezahlung mit den Personen / und nimb von dem das sie vertruncten haben/ Das da bleibt ist die zahl / welche getheilt sol werden.

The procedure prescribes to hold the number of people at the left hand and the consumed total at the right hand; the contribution of each, 5 and 3 in the middle. Then 3 is subtracted from 5 and the difference is called the divisor. The amount of the left hand is multiplied by 3 and subtracted from the amount on the right hand, thus 18. Divided by the divisor 2 the result gives the number of men, thus 9. From this the number of woman can be established.

While the procedure described by the rule can be matched with the one from the *Ganitasārasamgraha* by Mahāvīra (850, Sūtra 143 ½; [3], 303), the embodiment of the proto-algebraic rule is apparently a European invention. The same tanglible procedure is discussed in the Flemish arithmetic of Vander Gucht (1569, ff. 80^v-81^v, 1594, ff. 70^r-71^r) with small differences in formulation.

7 CONCLUSION

We have shown that the use of body parts in Renaissance mathematics goes back to a tradition of finger reckoning which is probably as old as mathematics itself. While basic finger arithmetic functions as an aid for memorization, communication and visualization we find in Renaissance arithmetic examples of embodiment of proto-algebraic procedures. We believe that this form of embodiment compensated a lack of symbolism which developed by the end of the sixteenth century. Most of the functions of tangible procedures have now been replaced by our current practice of symbolic mathematics.

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