Aspects of the Dynamics of Discussions and Logics Handling Them*

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Abstract

Although we are all familiar with discussions, spelling out their dynamics in a precise way involves many tough logical problems. This paper reports on a set of logical tools that are useful in this respect. Some concern the arguments produced in a discussion, possibly as a result of interventions of different participants, and the many forms of explicit and implicit agreement that are required to understand what is going on. Others concern the changing positions of participants. Nearly all of the tools are adaptive logics.

1 Aim of this Paper

We all regularly participate in discussions. Discussions vary widely in a number of respects: the number of people participating, the intensity of the interaction, the distribution of competence over the group, the more or less democratic organization, the more or less organized procedure, etc. Moreover, some discussions aim at solving a specific problem, others aim at taking group decisions, still others serve aims that are best described in psychological terms—from friendly entertainment to the 'one man up' show. Most of us are familiar with this variety of discussions, either as participants or as observers. Moreover, when participating or observing, most of us have the idea to understand what is going on, at least if the discussion is a rational one.

Nevertheless, the logical reconstruction of the dynamics of discussions involves many hard problems. The origin of these difficulties is that discussions are affected by several forms of dynamics. One set of problems is related to the fact that, in order to explicate what goes on in a discussion, one has to 'combine' interventions by different participants in rather sophisticated ways. Even if two or more participants contradict each other at some point, they may agree on others and, more importantly, some of their claims may jointly form

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an argument for a statement, and be felt as such by all present. This means that some arguments are produced in a discussion, and affect the outcome (or an intermediate outcome) of a discussion, without being explicitly present in the interventions of any separate participant. Likewise, several forms of explicit and implicit agreement play a central role in discussions. This may obtain even in case any two participants contradict each other in some respects.

Very different problems concern the explication of the position of a participant after this position was changed. If the participants are rational and the discussion is useful, it is likely that at least some participants will change their position. The set of interventions made by such participants are likely to be inconsistent. This means that their latest position does not contain all they affirmed before. However, changing one's position at some point does not entail changing it at all points. So, a participant's last intervention should be combined with those consequences of previous interventions that may be consistently combined with it. To do so is a task that is far from trivial. This is especially so because, as we shall see, we need to retain *consequences* of previous interventions, and not the interventions themselves.

A further complication is that some participants may be confused, and contradict themselves within a single intervention. This does not mean that the whole intervention has to be discarded; maybe the participant contradicted himself or herself only at a specific point. And things may get even more difficult: a dialetheist participant may explicitly defend an inconsistent position.

At this point I have to make an important remark on the involved dynamics. First and foremost, several forms of dynamics occur in the discussion itself—arguments are produced by the interplay between participants, and participants revise their position as a consequence of such arguments. A different dynamics concerns the *interpretation* of the discussion. This is most easily understood by considering a non-participating observer of the discussion that tries to understand what is going on. The reasoning process of this observer will be dynamic in that conclusions drawn at some point will have to be revised later. This applies with respect to forms of agreement, with respect to arguments that derive from the interventions of separate participants (that do not necessarily agree in other respects) and with respect to the changing positions of participants. For (efficient) participants, the matter is even more complex: they have not only to understand what is going on, they also have to change their position in view of the arguments produced and to defend the result. Explicating people's reasoning processes is supposed to be a task of logic. In trying to carry out this task for a discussion, a logician will be bound to apply methods that allow for dynamic interpretations.

In this paper, I shall report on a variety of logics that handle some of the aforementioned features. I shall refer to available materials that are apparently useful. Where these fall short, I shall refer to new results, either by myself or by other members of the Ghent Centre. We shall see that the available materials may be characterized in terms of adaptive logics—even where monotonic logics were available, a more adequate analysis was made possible by devising adaptive logics from them—and that the new results straightforwardly concern adaptive logics. This might be expected. The central aim is the explication of dynamic reasoning processes, and adaptive logics have dynamic proof theories that may be invoked to this end.

This paper does not aim at completeness. Nevertheless, I hope to convince the reader that the instruments presented enable one to analyse important and central dynamic features of discussions, and I hope to convince the reader that the approach is on the right track, or at least on a track that seems to lead in the right direction.

In Section 2, I shall briefly recall some basics of adaptive logics. Section 3 discusses Jaśkowski's **D2** and Meheus' adaptive extension of it. The Rescher–Manor consequence relations and their extension by Meheus' idea are discussed in Section 4. Section 5 is devoted to the difficult problem of the changing positions of participants in discussions. The dynamic proof theories of all these logics cannot be presented here, but an example is presented in Section 6.

2 Adaptive Logics and Dynamic Proof Theories

Adaptive logics and their dynamic proof theories are, first and foremost, a means to obtain a *formally precise grasp* on consequence relations that are (in general) undecidable and for which there is (in general) no positive test.

For all logics, the set of consequences depends on the set of premises. In proof theoretical terms, the dependence is realized, for non-dynamic proof theories, by a fixed set of rules (that characterizes the logic). Adaptive logics are different in that it even depends on the premises whether a specific application of a rule is or is not correct.

The effect is realized by defining an adaptive logic **AL** in terms of a lower limit logic **LLL**, an upper limit logic **ULL** that is an extension of **LLL**, and an adaptive strategy. The lower limit logic provides logical presuppositions that are unquestionable. The upper limit logic defines *normality*: the suppositions of **ULL** are taken to hold unless the set of premises prevents so. An adaptive logic interprets a set of premises "as normally as possible"—that is, as much as possible in agreement with its upper limit logic. Because the phrase "as normally as possible" is not unambiguous, a strategy is required to select a precise interpretation of the phrase.

Before proceeding to a more precise characterization, let us consider two examples. If a theory $\langle\Gamma, \mathbf{CL}\rangle$, in which Γ is the set of non-logical axioms and **CL** is Classical Logic, turns out to be inconsistent, we shall usually want to interpret it as consistently as possible. So, although the inconsistency of Γ forces us to give up **CL** in order to reason from Γ, we shall nevertheless want to interpret Γ as much as possible in accordance with **CL**. Typically, **CL** is the upper limit logic in this case—it defines normality. The lower limit logic will be a suitable paraconsistent logic, and the adaptive strategy will specify what it means to interpret Γ as normally as possible. Adaptive logics of this kind are called corrective. Γ was originally intended to be interpreted in terms of **CL**, but, as this turns out impossible, we want to interpret it as much as possible in terms of **CL**.

Next consider the case in which we have to reason from a set of data Γ and a set of expectancies Δ. The reliability of the expectancies is obviously weaker than those of the data: where data and expectancies conflict, the latter are given up. If all expectancies are considered to have the same reliability, the set of premises may be formalized as $\Gamma \cup \{\Diamond A \mid A \in \Delta\}$. If we close this set by, say the modal logic **T**,[1] we do not obtain the desired result: whenever B is

[1]The choice of **T** is justified with respect to the required adaptive logic—see [7].

CL-derivable from the from the data and expectancies, and not from the data alone, at best $\Diamond B$ will be **T**-derivable from $\Gamma \cup \{\Diamond A \mid A \in \Delta\}$.[2] This clearly is not what we want. We want to interpret $\{\Diamond A \mid A \in \Delta\}$ 'as normally as possible': we want the expectancies to be true whenever the premises allow so. In other words, we want to interpret $\{\Diamond A \mid A \in \Delta\}$ as much as possible in agreement with **Triv**, which validates $\Diamond A \supset A$.

I now present a more precise characterization of adaptive logics. Let me start with the semantics. For any set of premises Γ, there is a set of **LLL**-models of Γ. From these, the adaptive strategy selects the models that are as normally as possible. The **AL**-consequences of Γ are the formulas verified by all selected models—see [1] for a first implementation of this idea.

In proof theoretic terms, an adaptive logic has two kinds of rules. The unconditional rules are those of the lower limit logic **LLL**. The conditional rules are those rules of the upper limit logic **ULL** that are not validated by **LLL**. Whenever a conditional rule is applied, a condition (a set of formulas) is attached to the derived formula. If an unconditional rule is applied to derive B from $A_1, \ldots, A_n$, then the union the conditions of $A_1, \ldots, A_n$ are all attached to B. If a conditional rule is applied, the condition of B is the union the conditions of $A_1, \ldots, A_n$ and of a further set of formulas—we shall see all this at work in Section 6.[3] The adaptive strategy determines which formulas are marked, and a marked formula is considered as not derived. Whether a formula is marked depends on its condition and on the set of formulas that occur in the proof.

To see the full import of the dynamics of the proofs, I need to make explicit a feature that is implicit in the previous paragraph. Which formulas occur in the proof obviously depends in the *stage* of the proof. Marks may *come* and *go* as the proof proceeds to a subsequent stage. And this of course leads to several questions. The proof theory should connect derivability to semantic consequence. Obviously, derivability at a stage cannot be complete with respect to the semantics. However, it is possible to define "final derivability" and prove it (sound and) complete with respect to the semantics. But there is another question as well: How is a proof at a stage (of A from Γ) related to semantic consequence (to $\Gamma \vDash A$)? The answer is quite interesting. Subsequent stages of a proof gradually reveal the meaning of the premises—see [2] for a precise formulation of the claim and for the argumentation. As the proof proceeds, one obtains (in general) a better estimate (and never a worse one) of the final consequences of the premises. In other words, the proof theory enables one to *define* a notion of final derivability that is sound and complete with respect to the semantics, and, from a computational point of view, provides a sensible means to approximate final derivability.[4]

A peculiar feature of adaptive logics is that, to a normal set of premises, they assign the same consequence set as the upper limit logic. While this may sound impressive, the most important application contexts of adaptive logics obviously concern abnormal sets of premises.

[2] I say "at best" because two different members of Δ may be required to **CL**-derive B.

[3] A somewhat different terminology will be used there because it helps to make things precise. Conditions will be attached to lines of the proof and lines will be marked or unmarked—see below in the text.

[4] The propositional fragment (and many other fragments) are decidable. Even when a fragment is not decidable, there may be certain criteria that enable one to conclude from a specific dynamic proof of A from Γ that A is finally derivable from Γ.

3 Jaśkowski's D2 and Meheus' AJ

The underlying idea of Jaśkowski's paraconsistent logics relates directly to discussions—see [12] and earlier papers in Polish. Jaśkowski noted that participants in discussions may contradict each other, and hence that the set of interventions cannot be handled by **CL**. The backbone of his approach is to define a paraconsistent logic from a modal logic. The most popular case is **D2**, defined as follows—where $\Gamma^\Diamond = \{\Diamond A \mid A \in \Gamma\}$.

$$\Gamma \vdash_{\mathbf{D2}} A =_{df} \Gamma^\Diamond \vdash_{\mathbf{S5}} \Diamond A$$

For present purposes, the important feature of Jaśkowski's approach is that it provides a way to handle statements made in discussions in terms of **S5**. $\Gamma^\Diamond \vdash_{\mathbf{S5}} \Diamond A$ denotes that A has been stated in the discussion, i.e., that it follows from the position of at least one participant. $\Gamma^\Diamond \vdash_{\mathbf{S5}} \Box A$ denotes that A is **CL**-derivable from the position of each participant—remark that this is a special case of (a very strong form of) agreement.

Incidentally, if some member of Γ is inconsistent, $Cn_{\mathbf{S5}}(\Gamma^\Diamond)$ is trivial.[5] In the previous paragraphs, I implicitly supposed that each member of Γ represents the position of some participant—the conjunction of all statements made by a participant. Of course, nothing prevents one from reinterpreting the members of Γ as interventions—each member of Γ is then the conjunction of the statements made by a participant during one intervention. The advantage of this interpretation is that triviality does not ensue when some participant changes his or her position between interventions. The price to pay is double. First, the interventions of a participant become disconnected. So, if A is **CL**-derivable from a participant's intervention, and $A \supset B$ is **CL**-derivable from a different intervention of the same participant, then it is very well possible that $\Gamma^\Diamond \nvdash_{\mathbf{S5}} \Diamond B$. Next, $\Gamma^\Diamond \vdash_{\mathbf{S5}} \Box A$ now denotes an even stronger (and hardly useful) form of agreement: that A is **CL**-derivable from each intervention of each participant. To prevent inconsistent interventions from leading to triviality, Γ may be interpreted as the set of statements made in interventions of participants. The price goes up accordingly. So, let us return to the interpretation according to which each member of Γ corresponds to a participant's position.

In [13], Joke Meheus extends Jaśkowski's approach. Her idea is to introduce the *consistent core* of $\Gamma^\Diamond$. Statements derivable from the position of some participant may be said to be (at least) implicitly affirmed during the discussion. The consistent core of $\Gamma^\Diamond$ comprises those statements that are at least implicitly affirmed and are moreover compatible with the position of all participants—see [6] for two logics of compatibility.

The consistent core may be seen as the statements that all participants agree about in the discussion. "Agreement" is here meant in a weaker (and more useful) sense than the one expressed by $\Gamma^\Diamond \vdash_{\mathbf{S5}} \Box A$. Some people may complain that, if A belongs to the consistent core but is not derivable from a participant's position, then one cannot be absolutely certain that the participant agrees with it. This is correct, but rather immaterial. We are not trying to analyse the beliefs or convictions of the participants, but their positions in the discussion. For example, participants may very well defend a position they do not subscribe to. They may do so because they want or need to play the role

[5] $Cn_{\mathbf{L}}(\Gamma) = \{A \mid \Gamma \vdash_{\mathbf{L}} A\}$ as expected.

of the *advocatus diaboli*, or for some other reason. So, a participant's position depends only on what the participant states during his or her interventions. In the light of this, it is sensible to say that any participant implicitly agrees with any statement derivable from another participant's position, unless (and until) this statement has been (implicitly or explicitly) contradicted (by at least some participant).[6] Of course, one may refrain from contradicting a statement because one considers it unimportant or irrelevant (not worth disagreeing with). As far as one's position in the discussion is concerned, one still agrees with it.[7]

Remark that some disagreements may be *connected.* A simple example is where one participant affirms p, another q, and a third $\sim p \vee \sim q$. Although no two participants contradict each other, their statements are jointly incompatible. And indeed, $\Diamond p, \Diamond q, \Diamond(\sim p \vee \sim q) \vdash_{\mathbf{S5}} (\Diamond p \wedge \Diamond \sim p) \vee (\Diamond q \wedge \Diamond \sim q)$. In the predicative case disagreements are expressed by formulas of the form $\exists(\Diamond A \wedge \Diamond \sim A)$ in which "$\exists$" abbreviates a sequence of existential quantifiers (in some preferred order) of an existential quantifier over any individual variable free in A. Where Δ is a finite set, let $Dab(\Delta)$ abbreviate $\bigvee\{\exists(\Diamond A \wedge \Diamond\sim A) \mid A \in \Delta\}$. Moreover, let $\mathcal{F}^p$ be the set of primitive formulas (sentential letters and primitive predicative formulas including identities).

Definition 1 *$Dab(\Delta)$ is a minimal Dab-consequence of $\Gamma^\Diamond$ iff $\Delta \subset \mathcal{F}^p$, $\Gamma^\Diamond \vdash_{\mathbf{S5}} Dab(\Delta)$, and there is no $\Theta \subset \Delta$ such that $\Gamma^\Diamond \vdash_{\mathbf{S5}} Dab(\Theta)$.*

If $Dab(\Delta)$ is a minimal Dab-consequence of $\Gamma^\Diamond$, all $A \in \Delta$ will be called *unreliable*: there is disagreement about at least one of them and it is not determined which one.

Definition 2 $U(\Gamma) =_{df} \bigcup\{\Delta \mid Dab(\Delta)$ *is a minimal Dab-consequence of* $\Gamma^\Diamond\}$.

The abnormal part of a **S5**-model[8] is defined as follows:[9]

Definition 3 $Ab_p(M) = \{A \in \mathcal{F}^p \mid M \models \exists(\Diamond A \wedge \Diamond\sim A)\}$.

Meheus' logic **AJ** is semantically defined as follows.

Definition 4 *M is an* **AJ***-model of $\Gamma^\Diamond$ iff M is an* **S5***-model of $\Gamma^\Diamond$ and $Ab_p(M) \subseteq U(\Gamma)$.*

Definition 5 *$\Gamma^\Diamond \models_{\mathbf{AJ}} A$ iff all* **AJ***-models of $\Gamma^\Diamond$ verify A.*

It is easily seen that, expressed in terms of the intended application context, **AJ** extends **S5** in that the statements agreed about is extended to the consistent core, and hence that the consistent core is joined to the position of

[6] If the number of participants is small, all of them may be expected to take a stand on any statement made during the discussion. If more participants are involved, one may at best expect nodding or yea-saying, and no participant or observer is able to observe the thus expressed stands of all participants.

[7] A more thorough analysis may reveal the distinction between agreement and irrelevance, but if the statement is indeed irrelevant, it would not have much import for the outcome of the discussion anyway.

[8] I mean a predicative model that allows for $\Diamond a = b \wedge \Diamond\sim a = b$. See [6] or [13] for details.

[9] I shall need several kinds of abnormal parts of models and sets of premises. They will be distinguished by superscripts and subscripts the choice of which is obvious from the context.

all participants. Thus $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Box A$ iff A belongs to the consistent core of $\Gamma^\Diamond$. If $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Diamond B$ and $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Box A$, then $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Diamond(A \wedge B)$.

Obviously, **AJ** is not intended to replace **S5** with respect to the present application. $\Gamma^\Diamond \vDash_{\mathbf{S5}} \Box A$ and $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Box A$ denote distinct forms of agreement, both of which are sensible. Similarly, $\Gamma^\Diamond \vDash_{\mathbf{S5}} \Diamond A$ and $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Diamond A$ denote two distinct ways in which A may be affirmed in the discussion, and both are sensible. The **AJ**-versions denote (at least) implicit agreement and (at least) implicit affirmation. In the presence of $\Gamma^\Diamond \nvDash_{\mathbf{S5}} \Box A$, $\Gamma^\Diamond \vDash_{\mathbf{AJ}} \Box A$ denotes implicit agreement in the strict sense. Similarly for implicit affirmation in the strict sense. As we shall see in the next section, there are further sensible forms of implicit agreement and implicit affirmation. All of these are essential for understanding the typical dynamics that characterizes discussions.

AJ is an ampliative adaptive logic: it upgrades some possibilities to necessities, and hence allows for consequences that go beyond the standard of deductive inference—**S5** in the present context. Of course, one may define an inconsistency-adaptive logic from **AJ**, but this is not essential for our present enterprise.

4 Rescher–Manor Consequence Relations

The underlying idea of the Rescher–Manor consequence relations is that inconsistent sets of sentences are divided into maximal consistent subsets—henceforth *MCS*—and that what 'follows' from the inconsistent set is defined in terms of the classical consequences of (a selection of) the *MCS*. Some consequence relations were implicitly present already in [14], and were articulated in [17]. Extensions and applications appeared in [15], [16], and elsewhere. Later, further consequence relations were defined within the same approach. Some of these are called "prioritized" because they depend on non-logical preferences. The non-prioritized ones are called "flat". A survey and comparative study is presented in [10] and [11]. In the present section I concentrate on the flat consequence relations.

That these consequence relations are characterized in terms of adaptive logics was first shown in [4]. There, they are characterized in terms of inconsistency-adaptive logics. Direct dynamic proofs (that proceed directly in terms of **CL**) are presented in [9]. In [5], it is shown that the Rescher–Manor consequence relations may also be characterized in terms of adaptive logics that have **S5** as their lower limit logic.[10] Given the relation with Jaśkowski's approach, one naturally comes to the question whether the flat Rescher–Manor consequence relations are relevant for the analysis of discussions. The answer to this question is definitely positive. This answers provokes two further questions: (i) Is it possible to extend the flat Rescher–Manor consequence relations by the trick Joke Meheus applied to obtain **AJ**? (ii) Are the so obtained results useful for the understanding of the dynamics of discussions? Again, the answers to both questions are positive. Remark, however, that, unlike what is the case for extending **S5** to **AJ**, we are here superimposing an adaptive logic on top of another adaptive logic.

In the remaining part of this section, I shall first define the flat Rescher–Manor consequence relations. Next, I shall present their characterizations in

[10] See [3] for some difficult problems about the relation between both characterizations.

terms of adaptive logics that have **S5** as their lower limit logic. Finally, I shall consider their extensions obtained by introducing Meheus' consistent core—that is, by upgrading a specific and justifiable selection of possibilities to necessities.

Δ is a *MCS* of the set of formulas Γ iff (i) $\Delta \subseteq \Gamma$, (ii) $\Delta \nvdash_{\mathbf{CL}} \bot$, and (iii) for all $A \in \Gamma - \Delta$, $\Delta \cup \{A\} \vdash_{\mathbf{CL}} \bot$. Members of Γ that belong to all *MCS* of Γ are called *free* members of Γ. The *largest MCS* of Γ are those the cardinality of which is not smaller than the cardinality of any other *MCS* of Γ. The definitions of the flat consequence relations are as follows:

Definition 6 *$\Gamma \vdash_{Free} A$ iff A is a* **CL***-consequence of the free members of* Γ*.*

Definition 7 *$\Gamma \vdash_{Strong} A$ iff A is a* **CL***-consequence of all* MCS *of* Γ*.*

Definition 8 *$\Gamma \vdash_{Weak} A$ iff A is a* **CL***-consequence of some* MCS *of* Γ*.*

Definition 9 *$\Gamma \vdash_{C\text{-}Based} A$ iff A is a* **CL***-consequence of all largest* MCS *of* Γ*.*

Definition 10 *$\Gamma \vdash_{Argued} A$ iff A is a* **CL***-consequence of some* MCS *of Γ and $\sim A$ is not a* **CL***-consequence of any* MCS *of* Γ*.*

The **S5**-models that verify all consistent members of $\Gamma^{\Diamond}$ will be called the **C**-models of $\Gamma^{\Diamond}$. These may themselves be characterized as minimally abnormal models:

Definition 11 $Ab^{c}_{\Gamma}(M) = \{A \mid A \in \Gamma; M \nvDash \Diamond A\}$

Definition 12 *An* **S5***-model M is a* **C***-model of $\Gamma^{\Diamond}$ iff there is no* **S5***-model M' such that $Ab^{c}_{\Gamma}(M') \subset Ab^{c}_{\Gamma}(M)$.*

Where $\mathcal{W}$ is the set of closed formulas (wffs) of the non-modal language, a world w of a model will be said to verify $\Delta \subseteq \mathcal{W}$ iff $V(A, w) = 1$ for all $A \in \Delta$. The minimally abnormal worlds with respect to Γ are defined as follows:

Definition 13 $Ab_{\Gamma}(w) = \{A \in \Gamma \mid w \text{ does not verify } A\}$ *.*

Definition 14 *A world w of a* **S5***-model M is minimally abnormal with respect to Γ iff no world w' of any* **S5***-model M' is such that $Ab_{\Gamma}(w') \subset Ab_{\Gamma}(w)$.*

Definition 15 *A* **C***-model M of $\Gamma^{\Diamond}$ is a* **MA***-model of $\Gamma^{\Diamond}$ iff all worlds of M are minimally abnormal with respect to* Γ*.*

From the **MA**-models of $\Gamma^{\Diamond}$ we define the **RM**-models of $\Gamma^{\Diamond}$ in terms of their abnormal parts with respect to the possibility of conjunctions of premises:

Definition 16 $Ab^{\wedge}_{\Gamma}(M) = \{\{A_1, \ldots, A_n\} \mid n > 1; A_1, \ldots, A_n \in \Gamma; M \nvDash \Diamond(A_1 \wedge \ldots \wedge A_n)\}$ *.*

Definition 17 *M is a* **RM***-model of $\Gamma^{\Diamond}$ iff it is a* **MA***-model of $\Gamma^{\Diamond}$ and there is no* **MA***-model M' of $\Gamma^{\Diamond}$ such that $Ab^{\wedge}_{\Gamma}(M') \subset Ab^{\wedge}_{\Gamma}(M)$.*

Definition 18 *$\Gamma^{\Diamond} \vDash_{\mathbf{RM}} A$ iff all* **RM***-models of $\Gamma^{\Diamond}$ verify A.*

The following theorems are proved in [5]:

Theorem 1 $\Gamma \vdash_{Weak} A$ *iff* $\Gamma^{\lozenge} \models_{\mathbf{RM}} \lozenge A$.

Theorem 2 $\Gamma \vdash_{Argued} A$ *iff* $\Gamma^{\lozenge} \models_{\mathbf{RM}} \lozenge A$ *and* $\Gamma^{\lozenge} \not\models_{\mathbf{RM}} \lozenge{\sim}A$.

Theorem 3 $\Gamma \vdash_{Strong} A$ *iff* $\Gamma^{\lozenge} \models_{\mathbf{RM}} \Box A$.

The two other consequence relations require special treatment. The C-based consequence relation is most easily incorporated by introducing a special (but simple) modality $\Box_\Gamma$ (where $\Gamma \subset \mathcal{W}$). Let $\#(w, \Gamma)$ be the cardinality of the set of members of Γ verified by w, and let $w \in m(\Gamma)$ iff there is no $w' \in W$ such that $\#(w', \Gamma) > \#(w, \Gamma)$. Extend the **S5**-semantics with the clause:

$$V(\Box_\Gamma A, w) = 1 \text{ iff } V(A, w') = 1 \text{ for all } w' \in m(\Gamma)$$

Theorem 4 $\Gamma \vdash_{C\text{-}based} A$ *iff* $\Gamma^{\lozenge} \models_{\mathbf{RM}} \Box_\Gamma A$.

For the Free consequence relation, we need a different selection of the models. First we define the abnormal part of Γ as the set of non-free members of Γ:

Definition 19 $Ab_F(\Gamma) = \{A \mid A \in \Gamma;$ *for some* $B_1, \ldots, B_n \in \Gamma : B_1, \ldots, B_n \nvdash_{\mathbf{CL}} \bot$ *and* $A, B_1, \ldots, B_n \vdash_{\mathbf{CL}} \bot\}$

Next, we define the abnormal part of the models with respect to the necessity of members of Γ:

Definition 20 $Ab^{\Box}_{\Gamma}(M) = \{A \mid A \in \Gamma; M \not\models \Box A\}$

Definition 21 *M is a $\mathbf{F}$-model of $\Gamma^{\lozenge}$ iff M is a $\mathbf{C}$-model of $\Gamma^{\lozenge}$ and* $Ab^{\Box}_{\Gamma}(M) = Ab_F(\Gamma)$.

And I repeat from [5]:

Theorem 5 $\Gamma \vdash_{Free} A$ *iff* $\Gamma^{\lozenge} \models_{\mathbf{F}} \Box A$.

So, I have defined all Rescher–Manor consequence relations in terms adaptive logics that have **S5** as their lower limit logic. The next step is to present the promised *extensions*. The most important motivation for them lies in the fact that they enable one to spell out *implicit* agreements. Precisely these are revealed by the *consistent core*. Suppose that one participant states $p \wedge q$, another participant states ${\sim}p$, and no further participant makes any claims about either p or q. As there is disagreement about p, we are unable to tell which stand on p is taken by a participant that does not explicitly make a statement about it. However, as q was affirmed, and no participant denied it, q will count as an implicit agreement for all.

Not all consequences of statements made during the discussion will enter the consistent core. For example, $p \vee r$ is a **CL**-consequence of the first participant's statement, and no participant explicitly denied it. But if $p \vee r$ did belong to the consistent core, it should be concluded that r belongs to the position of the second participant because this participant stated ${\sim}p$. Such a conclusion would obviously be foolish, as r might be any statement whatsoever.

Implicit agreements are far more important for understanding the dynamics of a discussion than explicit agreements. This is the more so as the implicit agreements among all participants enter all of their positions, whereas implicit agreements between (overlapping and varying) groups of participants enter the positions of the members of these groups.

The idea underlying the following proposals is that one should not concentrate on implicit agreements, but on explicit disagreements, that disagreements may be 'connected', and that one should express these connections in terms of the simplest formulas that cause the disagreements.

To characterize the extensions, we need $U(\Gamma)$ and $Ab_p(M)$ from Section 3, together with the following two definitions.

Definition 22 *M is a $\mathbf{RM}^\star$-model of $\Gamma^\Diamond$ iff it is a $\mathbf{RM}$-model of $\Gamma^\Diamond$ and $Ab_p(M) \subseteq U(\Gamma)$.*

Definition 23 *M is a $\mathbf{F}^\star$-model of $\Gamma^\Diamond$ iff it is a $\mathbf{F}$-model of $\Gamma^\Diamond$ and $Ab_p(M) \subseteq U(\Gamma)$.*

The extended Rescher–Manor consequence relations are then characterized as follows.[11]

Definition 24 $\Gamma \vdash_{Weak^\star} A$ *iff* $\Gamma^\Diamond \models_{\mathbf{RM}^\star} \Diamond A$.

Definition 25 $\Gamma \vdash_{Strong^\star} A$ *iff* $\Gamma^\Diamond \models_{\mathbf{RM}^\star} \Box A$.

Definition 26 $\Gamma \vdash_{C\text{-}based^\star} A$ *iff* $\Gamma^\Diamond \models_{\mathbf{RM}^\star} \Box_\Gamma A$.

Definition 27 $\Gamma \vdash_{Free^\star} A$ *iff* $\Gamma^\Diamond \models_{\mathbf{F}^\star} \Box A$.

The extensions are best understood by concentrating first on the *Strong**-consequence relation. A *Strong* consequence of Γ is one explicitly agreed upon by all (overlapping) consistent parties (groups of participants the positions of which are compatible) in the discussion—the consequence is **CL**-derivable from the statements made by each such party. A *further Strong** consequence of Γ is one that is implicitly agreed upon by all consistent parties. *Free* consequences represent a very different explicit agreement. They are the **CL**-consequences of the interventions of those participants that do not contradict anyone—some of these participants may be wise, others diplomatic, still others may lack a clear view. The *Free** extend the *Free* consequences by including the consistent core. A *Weak* consequence is one is affirmed by some consistent party—**CL**-derivable from statements made by that party—but may be denied by another consistent party. Unlike what one may expect, the *Weak** consequence relation extends the *Weak* one. If some consistent party affirms A but does not affirm some B that belongs to the consistent core, then $A \wedge B$ is a *Weak** consequence. So, even where consistent parties contradict each other, their implicit positions are extended with the statements that are affirmed by some and denied by none. The *C-based** consequences extend the *C-Based* consequences in a similar way. This basically means that the content of the majority position comprises the consistent core, which is obviously desirable.

[11] I do not consider an extension of the *Argued* consequence relation because adding the consistent core to each position does not warrant an extension of that consequence relation. Incidentally, it seems to me that the purpose served by the *Argued* consequence is much better served by the *Strong** consequence relation.

5 Changing Positions

Before, I have supposed that the positions of participants are stable. This is clearly an idealization. As arguments are presented in the discussion, is is likely that participants change their positions, especially if the discussion is a rational one.

In this section, I discuss those changing positions. It should be stressed, however, that, by allowing for changing positions of participants, one arrives at the true dynamics of a discussion. As positions chance, the arguments and agreements have to be updated. The most sensible approach to that problem is that one updates the arguments and agreements after any intervention of a participant. After each intervention, the position of the intervening participant is updated and the arguments and agreements are settled in accordance with the mechanisms described in previous sections. So, I shall restrict my attention to the changing positions of participants.

The set of interventions of a participant may be represented by a sequence of sets of statements:

$$\Sigma = \langle \Gamma_1, \ldots, \Gamma_n \rangle$$

Γ_1 represents the participant's first intervention, etc. As stated in Section 1, I presuppose that any participant intends to defend a consistent position, but do not rule out that some interventions are inconsistent due to confusion.

In [18], Liza Verhoeven has studied the use of the prioritized Rescher–Manor consequence relations with respect to the present problem. The result is negative: none of those consequence relations is able to adequately characterize a participant's changing position. The basic reason for this is that the changing position of a participant can only be understood in terms of the consequences of the participant's interventions, not in terms of the statements that were explicitly made. So, we need to look for a new approach. The underlying idea is that a participant's position depends, first and foremost on his or her last intervention Γ_n. As this may be consistent, we have extract 'the consistent part' of it. Let the result be $C(\Gamma_n)$. Next, we add the consequences of the next to last intervention that are compatible with $C(\Gamma_n)$. And so on.

I shall briefly sketch two approaches to this problem. I consider the first one, forthcoming in [8], as the most promising, but the related research is not completely finished. So, I merely present an outline. Next, I shall describe the second road.

Suppose that $C(\Gamma)$ is the (or the suitable) consistent part of $Cn_{\mathbf{L}}(\Gamma)$, in which $\mathbf{L}$ is some suitable logic. As Γ may be inconsistent, $\mathbf{L}$ better be paraconsistent. As we supposed that the participant was defending a consistent position, $Cn_{\mathbf{L}}(\Gamma)$ will have to constitute an interpretation of Γ that is 'as consistently as possible'. In other words, $\mathbf{L}$ will have to be an inconsistency-adaptive logic. Once all this is available, the most sensible reconstruction of a participant's position (after his or her n-th intervention) is clearly as follows:

$$C(\Gamma_1 \cup \ldots \cup \Gamma_n) \cup C(\Gamma_2 \cup \ldots \cup \Gamma_n) \cup \ldots \cup C(\Gamma_n)$$

This definition proceeds in terms of $C(\Gamma)$. The idea that the position contains only those $\mathbf{L}$-consequences of the next to last intervention that are compatible with $C(\Gamma_n)$ is realized indirectly, viz. because the consistent part of the last two interventions taken together, $C(\Gamma_{n-1} \cup \Gamma_n)$, is defined in such a way that it is compatible with the consistent part of the last intervention, $C(\Gamma_n)$. This approach seems the right one. The problem is to offer a suitable definition of $C(\Gamma)$. At present, the apparently best solution to this problem is not proved adequate.

The second approach[12] is a halfway house between the previous approach and prioritized Rescher–Manor consequence relations. It takes the Feys–von Wright modal logic **T** as its lower limit logic, its extension with the axiom $\Diamond A \supset A$ (hence **Triv**) as its upper limit logic, and follows either the Reliability strategy or the Minimal Abnormality strategy—I chose the Reliability strategy for the present paper.

First, the sequence of interventions

$$\Sigma = \langle \Gamma_1, \ldots, \Gamma_n \rangle$$

is turned in modal terms as follows

$$\Sigma^{\Diamond} = \{\Diamond^{n-i+1} A \mid A \in \Gamma_i \ (1 \leq i \leq n)\}$$

in which $\Diamond^{n-i+1}A$ is A preceded by $n-i+1$ diamonds—thus $\Diamond^1 A = \Diamond A$, $\Diamond^2 A = \Diamond\Diamond A$, etc.

Let $\mathcal{F}^a$ be the set of atoms (primitive open and closed formulas and their negations). Let $Dab^i(\Delta)$ denote the disjunction $\bigvee\{\exists(\Diamond^i A \wedge \neg A) \mid A \in \Delta\}$. We shall say that $Dab^i(\Delta)$ is a Dab^i-consequence of Σ iff all **T**-models of Σ verify $Dab^i(\Delta)$. A Dab^i-consequence $Dab^i(\Delta)$ of Σ will be called *minimal* iff (i) $\Delta \subseteq \mathcal{F}^a$ and (ii) there is no $\Delta' \subset \Delta$ such that $Dab^i(\Delta')$ is a Dab^i-consequence of Σ.

Definition 28 $U^i(\Sigma) = \bigcup\{\Delta \mid Dab^{n-i+1}(\Delta)$ *is a minimal* Dab^{n-i+1}*-consequence of* $\Sigma\}$

Remark that $U^1(\Sigma)$ depends only on Γ_n, $U^2(\Sigma)$ depends on $\Gamma_{n-1} \cup \Gamma_n$, etc. Given these sets of *unreliable* formulas with respect to $\Sigma^{\Diamond}$, we now turn to the abnormal parts of the models.

Definition 29 $Ab^i(M) =_{df} \{A \in \mathcal{F}^a \mid v_M(\exists(\Diamond^{n-i+1} A \wedge \neg A), w_0) = 1\}$

Where $\mathcal{M}^0_\Sigma$ is the set of all **T**-models of $\Sigma^{\Diamond}$, we stepwise define models that are reliable with respect to the different layers of abnormalities in the obvious way:

$$\mathcal{M}^{i+1}_\Sigma = \{M \in \mathcal{M}^i_\Sigma \mid Ab^{i+1}(M) \supseteq U^{i+1}(\Sigma)\}$$

and call $\mathcal{M}^n_\Sigma$ the **AT**-models of $\Sigma^{\Diamond}$. Finally, we define:

Definition 30 $\Sigma^{\Diamond} \vDash_{\mathbf{AT}} A$ *iff all* $M \in \mathcal{M}^n_\Sigma$ *verify* A.

It seems useful to present a simple propositional example at this point. Where

$$\Sigma = \langle \{\sim s\}, \{p \wedge r, s\}, \{\sim p, q\} \rangle$$

the reader can easily verify that all models in $\mathcal{M}^3_\Sigma$ verify $\sim p$, q, r, as well as s, as desired. The example illustrates that **AT** is a halfway house between the Rescher–Manor consequence relations and an approach that proceeds in terms of consequences of the interventions. Thus, the fact that $p \wedge r \in \Gamma_2$ is contradicted by $\sim p \in \Gamma_3$ does not prevent r from being derivable.

[12] This approach is studied in [7] in the context of diagnosis.

6 An Example of a Dynamic Proof Theory

To illustrate dynamic proof theories, I shall briefly spell out the one for **AT**. I present the rules and the marking definition, as well as a simple example of a proof, but skip the whole metatheory. Here are the (generic) rules:

PREM If $A \in \Sigma^{\Diamond}$, then one may add a line consisting of
- (i) the appropriate line number,
- (ii) A,
- (iii) "–",
- (iv) "PREM", and
- (v) $\emptyset$.

RU If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta^1_{j_1}, \ldots \Delta^m_{j_m}$ respectively, then one may add a line consisting of
- (i) the appropriate line number,
- (ii) A,
- (iii) the line numbers of the B_i,
- (iv) "RU", and
- (v) $(\Delta^1 \cup \ldots \cup \Delta^m)_{\max(j_1,\ldots,j_m)}$.

RC If $B_1, \ldots, B_m \vdash_{\mathbf{T}} A \vee Dab^k(\Theta)$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta^1_{j_1}, \ldots \Delta^m_{j_m}$ respectively, then one may add a line consisting of
- (i) the appropriate line number,
- (ii) A,
- (iii) the line numbers of the B_i,
- (iv) "RC", and
- (v) $(\Theta \cup \Delta^1 \cup \ldots \cup \Delta^m)_{\max(k,j_1,\ldots,j_m)}$.

Remark that the fifth element of a line (unless this element is $\emptyset$) receives a unique index i that refers to some set $\Gamma_{n-i+1} \cup \ldots \cup \Gamma_n$. The meaning of this index is that the formula (second element of the line) is derivable from the premises provided the members of that fifth element are reliable at level i, viz. are not members of $U^i(\Sigma)$. As (at the predicative level) $U^i(\Sigma)$ is undecidable, the marking definition (below) proceeds in terms of sets of formulas that are unreliable at stage s of the proof, $U^i_s(\Sigma)$. These sets are defined in the same way as the $U^i(\Sigma)$, but from the Dab^{n-i+1}-formulas that are derived in the proof with a condition that has an index *lower* than i, and are minimal at stage s.

The marking definition for Reliability is as follows:

Definition 31 *Line i is marked at stage s iff, where Δ_j is its fifth element, $\Delta_j \cap U^j_s(\Sigma) \neq \emptyset$.*

A is *finally* **AT**-derived on line i at a stage s of a proof from $\Sigma^{\Diamond}$ iff line i is not marked at stage s, and any extension of the proof in which line i is marked, may be further extended in such a way that line i is unmarked. A is finally **AT**-derivable from $\Sigma^{\Diamond}$ iff A is finally **AT**-derived at a stage of a proof from $\Sigma^{\Diamond}$. As we are only interested in the non-modal formulas that are finally derivable from $\Sigma^{\Diamond}$, I define:

Definition 32 *Where $A \in \mathcal{W}$, $\Sigma^{\Diamond} \vdash_{\mathbf{AT}} A$ iff A is finally derivable from $\Sigma^{\Diamond}$.*

I now present a simple dynamic proof from the propositional $\Sigma^{\Diamond}$:

$$\langle\{\Diamond\Diamond\Diamond{\sim}s\}, \{\Diamond\Diamond(p \wedge r), \Diamond\Diamond s\}, \{\Diamond{\sim}p, \Diamond q\}\rangle$$

As, at the propositional level, **AT** is decidable, the dynamics could have been avoided. This is obviously not possible at the predicative level. Still, the propositional example nicely illustrates the matter. The boxed "13" at the end of line 8 indicates that line 8 is marked in view line 13. Similarly for other boxed numbers.

1	$\Diamond{\sim}p$		PREM	$\emptyset$	
2	$\Diamond q$		PREM	$\emptyset$	
3	$\Diamond\Diamond(p \wedge r)$		PREM	$\emptyset$	
4	$\Diamond\Diamond s$		PREM	$\emptyset$	
5	$\Diamond\Diamond\Diamond{\sim}s$		PREM	$\emptyset$	
6	${\sim}p$	1	RC	$\{{\sim}p\}_1$	
7	q	2	RC	$\{q\}_1$	
8	$p \wedge r$	3	RC	$\{p, r\}_2$	$\boxed{13}$
9	p	8	RU	$\{p, r\}_2$	$\boxed{13}$
10	r	8	RU	$\{p, r\}_2$	$\boxed{13}$
11	$\Diamond\Diamond p$	3	RU	$\emptyset$	
12	$\Diamond\Diamond r$	3	RU	$\emptyset$	
13	$\Diamond\Diamond p \wedge {\sim}p$	6, 11	RU	$\{{\sim}p\}_1$	
14	r	12	RU	$\{r\}_2$	
15	s	4	RU	$\{s\}_2$	

The reader can easily check each of the following. If p were derived from line 11, it would receive the condition $\{p\}_2$ and hence would be marked in view of line 13. The formula $\Diamond{\sim}p \wedge p$ can only be derived on the condition $\{p\}_2$, and hence ${\sim}p \notin U^1(\Sigma)$. Of course, one may derive $\Diamond\Diamond\Diamond{\sim}p \wedge p$ on the condition $\{p\}_2$, and hence ${\sim}p \in U^3(\Sigma)$, but this has no effect on line 6. Lines 6, 7, 14, and 15 will not be marked in any extension of the proof and hence are finally derived.

7 Open Problems

It is astonishing that, even if we rely on the relatively unknown dynamic proof theories, we are just at the beginning of developing the tools for analysing discussions. There seem to be four urgent tasks in this respect. The first is to articulate the approach in terms of the consistent part of sets that are closed by some paraconsistent logic. This will provide a more adequate analysis of changing positions. The next task is to remove the idealizing presuppositions of the logics presented in previous sections. A third task consists in devising a set of heuristic rules to speed up the dynamic proofs (at the predicative level) and perhaps even to develop computer implementations. Finally, one should dig deeper into the dynamics of discussions. More particularly, one should study the ways in which agreement may be reached, the relative import of implicit and explicit forms of agreement, and the relation between, on the one hand, the reasons for a participant to change his or her position and, on the other hand, the effect of those reasons on the outcome of the discussion—Who wins or looses? Which decision results from the available agreements?

The aim of the present paper was mainly to attract attention to an available line of research and to illustrate the dynamic proof procedure. People interested should study the papers referred to.[13]

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