# Content Guidance in Formal Problem Solving Processes.

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**Abstract**

In this paper, a formal framework to problem-solving processes is presented. The framework is not complete. Nevertheless, even its present sophistication allows one to see that it is promising.

The framework demonstrably allows one to understand scientific change as content-guided. It will be argued that a formal framework is required in order to make definite and precise statements about the content-guided aspects of scientific problem solving.

## 1 Some Background

On the present-day view of scientific rationality, science is content-guided. As Dudley Shapere phrases it: scientific inquiry relies on "what we have learned, including what we have learnt about how to learn" [62, p. 52]. While the Vienna Circle ultimately reduced scientific methodology to an a priori matter, aiming at delineating all possible science, the present view understands science as essentially relying on contemporary insights in the world as well as in methodological matters. While Kuhn and other members of the 'historicist' movement reduced science to a form of relativism, the present view sees scientific decisions as justified in terms of those contemporary insights.

It may come as a surprise that this view requires a heavy import of logic. In trying to understand scientific reasoning, especially its more creative aspects, heuristic aspects play a central role. One should clearly distinguish between heuristic aspects that pertain to logic and other heuristic aspects. Both kinds are important (and both result from a learning process). Even in order to give the non-logical aspects their due place, the logical aspects should be located and systematically described.

So I shall describe an approach to problem solving that has a remarkable combination of properties. On the one hand, the approach is formal in a specific but plain sense of the term. Yet, at the same time, it leaves ample

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room for content-guidance. Central to the approach are formal problem solving processes—henceforth *fpsps*. These are defined by ingredients of different sorts: one or more logics (for declarative statements), the prospective dynamics of these logics, one or more erotetic logics, an oracle, a procedure (set of instructions), and a set of heuristic instructions. The main aim is obviously the *explication* of actual problem solving processes. The backbone is to solve a problem of the form $\{?\{A, {\sim}A\}\}$ by deriving one of its direct answers, $A$ or ${\sim}A$, from $\Gamma$ by Classical Logic—henceforth **CL**.

The logics for declarative statements should not be seen as alternatives for **CL** (Classical Logic). They are formal explications of methodological reasoning forms. These are handled by adaptive logics, of which there are two sorts: corrective and ampliative. Corrective adaptive logics are invoked for handling premise sets that display unexpected and unwanted properties, such as inconsistency and ambiguity. In the presence of these, **CL** results in triviality, while corrective adaptive logics interpret the premise set as consistently as possible or as unambiguously as possible. Ampliative adaptive logics extend **CL** in view of methodologically justifiable forms of reasoning: inductive generalization, abduction (see Meheus' paper in the present volume), compatibility reasoning (for example for extending a theory), etc.[1]

Most adaptive logics require defeasible inferences. Indeed, the reasoning forms have the peculiarity that there is no positive test for them. This makes their proofs necessarily dynamic: it may be required that a step derived at some point is considered as OUT at a later point and possibly as IN again at a still later point. The reasons for considering steps as IN or OUT are provided by the insights from the ongoing reasoning. Such reasoning forms are explicated by the dynamic proofs of adaptive logics—see [12, 16, 19] for an introduction as well as for technical results, and Section 3 for a brief introduction. In those dynamic proofs, the dynamics is technically controlled by conditions and marking definitions.

A further dynamics is introduced. The prospective dynamics enables one to push formal elements of the proof search into the proof itself. This allows for more or less permissive heuristic procedures and makes the heuristic aspects transparent. The prospective dynamics can be spelled out for the dynamic proofs of adaptive logics.

Erotetic logics (in the style of [66]) are used to write problems (sets of questions) into the proof and to derive problems from given problems and declarative (prospective) statements. The erotetic logics handle the introduction of subproblems and so-called derived problems.

The framework is easily extended (i) with empirical means, observation and experiment, and (ii) with the introduction of available information, which originally was not seen as relevant. Problems that can be solved by empirical means are handled by a specific kind of oracle. Some problems will only be answered in the context of an experimental setup, which may require a fpsp itself.

Interestingly, both extensions are guided by the fpsps: the present state of the fpsp leads to an appeal to empirical means or to the introduction of new information. This guidance has two sources. On the one hand, the prospective dynamics may reveal (in terms of the positive part relation from Section 2)

[1]Whether an adaptive logic is corrective or ampliative is defined, in the present context, with respect to **CL**, which is here taken as the standard of deduction. This is merely a pragmatic convention.

that available information is relevant for the problem. The information may be knowledge that was previously not seen as relevant, or it may be obtained by known empirical means. On the other hand, the derivable disjunctions of abnormalities (jargon to be explained below), which play an essential role in adaptive logics, sensibly guide research in suggesting empirical questions as well as defeasible theoretical choices. Both lead to useful steps towards the solution of the problem.

All these elements are joined and governed by a procedure. The specific road followed to solve a problem is determined by choices that are permitted by the procedure. These choices make it possible to proceed in view of "what we have learnt about how to learn." I shall pay special attention to further aspects that make fpsps content-guided.

In order to keep the discussion within bounds, I shall only consider the technicalities for the propositional case. This is merely a pragmatic decision. The predicative version of the adaptive logics is available, and the same holds for the prospective dynamics.

The background of the approach is formed by many sources of inspiration. Science is seen a problem-solving activity, a widespread view that apparently reached a stable formulation in [36]. From philosophy of science and epistemology, this view is elaborated by a combination of ideas mainly taken from [38, 39, 40, 41, 42, 43], which combine Tom Nickles' view on discovery in terms of constrained problem solving processes, [51, 52, 53, 54, 55, 56, 57] with a contextual view on problem solving, [3, 4, 7, 8, 47]. The central idea is that a problem is determined by constraints that are modified as the problem-solving process proceeds. The constraints can be of three sorts: conditions on the solution, methodological instructions (including heuristic instructions as well as examples of successful problem solutions), and (contextual) certainties (which form the so-called conceptual frame). Some handy sources for the logical aspects: [12, 19, 20, 27] for adaptive logics, [17, 30] for the prospective dynamics, and [65, 66, 67, 68] for erotetic logic.

## 2 Formal Problem-Solving Processes

A fpsp is a chain of stages and a stage (of a fpsp) is a sequence of lines that are written according to a set of instructions: rules with a deontic condition attached to them. The first stage consists of a single line. A subsequent stage is obtained by adding one line to the previous one. The instructions determine which lines may be added at a stage. They obviously depend on the logic or logics of the fpsp.

A fpsp contains problem lines as well as declarative lines. A problem is a non-empty set of questions. In a first approach, one may take the *main* problem to be a single yes–no question $?\{M, {\sim}M\}$, and all derived problems to be sets of yes–no questions. $A$ and ${\sim}A$ are the *direct answers* of the question $?\{A, {\sim}A\}$.

$\Gamma$ will always denote the premise set. The formula of a declarative line may be a member of $\Gamma$ or a formula derived from members of $\Gamma$. Such formula may have the form

$$[B_1, \ldots, B_n]\, A$$

which indicates that $\Gamma \cup \{B_1, \ldots, B_n\} \vdash A$. The set $\{B_1, \ldots, B_n\}$ will be called the *condition* of $A$. A formula of the form $A$ will be said to occur unconditionally.

However, a formula $A$ is also identified with $[\emptyset]\, A$, whence the condition of $A$ is said to be empty.

Given a problem, the problem solver has to chose a *target* (*from* the problem): a direct answer of a question of the problem. A target is noted on a *target line*; it is a formula the problem solver tries to establish. A target $A$ will be written as

$$[A]\, A$$

which is logically redundant but guides the procedure, as we shall see. If $[A]\, A$ occurs on an unmarked line, $A$ will be said to be an *unmarked target*. The procedure is moreover guided by *marking definitions*. Some marks indicate that a line became useless for solving the main problem; others that some action has to be undertaken to further the solution.

We need some preparatory definitions for the formal machinery. Let $*A$ denote the 'complement' of $A$, viz. $B$ if $A$ has the form $\sim B$ and $\sim A$ otherwise. Let us distinguish $\mathfrak{a}$-formulas from $\mathfrak{b}$-formulas, varying on a theme from [63], and assign to each formula two other formulas according to the following table:

| $\mathfrak{a}$ | $\mathfrak{a}_1$ | $\mathfrak{a}_2$ | | $\mathfrak{b}$ | $\mathfrak{b}_1$ | $\mathfrak{b}_2$ |
|---|---|---|---|---|---|---|
| $A \wedge B$ | $A$ | $B$ | | $\sim(A \wedge B)$ | $*A$ | $*B$ |
| $A \equiv B$ | $A \supset B$ | $B \supset A$ | | $\sim(A \equiv B)$ | $\sim(A \supset B)$ | $\sim(B \supset A)$ |
| $\sim(A \vee B)$ | $*A$ | $*B$ | | $A \vee B$ | $A$ | $B$ |
| $\sim(A \supset B)$ | $A$ | $*B$ | | $A \supset B$ | $*A$ | $B$ |
| $\sim\sim A$ | $A$ | $A$ | | | | |

The *positive part* relation is defined recursively by the following clauses:

1. $\mathrm{pp}(A, A)$.
2. $\mathrm{pp}(A, \mathfrak{a})$ if $\mathrm{pp}(A, \mathfrak{a}_1)$ or $\mathrm{pp}(A, \mathfrak{a}_2)$.
3. $\mathrm{pp}(A, \mathfrak{b})$ if $\mathrm{pp}(A, \mathfrak{b}_1)$ or $\mathrm{pp}(A, \mathfrak{b}_2)$.
4. If $\mathrm{pp}(A, B)$ and $\mathrm{pp}(B, C)$, then $\mathrm{pp}(A, C)$.

The *instructions* rely on the prospective rules presented in [30]. A fpsp starts with an application of Main, which introduces the main problem. In the instructions, $k$ denotes a suitable line number.

Main Start a fpsp with the line:

| | | |
|---|---|---|
| 1 | $\{?\{M, \sim M\}\}$ | Main |

Target If $\mathsf{P}$ is the problem of an unmarked problem line, and $A$ is a direct answer of a member of $\mathsf{P}$, then one may add:

| | | |
|---|---|---|
| $k$ | $[A]\, A$ | Target |

Prem If $A$ is an unmarked target, $B \in \Gamma$, and $\mathrm{pp}(A, B)$, then one may add:

| | | |
|---|---|---|
| $k$ | $B$ | Prem |

The formula analysing rules of **CL** (see [30]) may be summarized as follows:[2]

[2]The left rule states that both $[\Delta]\, \mathfrak{a}_1$ and $[\Delta]\, \mathfrak{a}_2$ may be derived (separately) from $[\Delta]\, \mathfrak{a}$ (notational abuse here and in the text).

$$\frac{[\Delta]\,\mathfrak{a}}{[\Delta]\,\mathfrak{a}_1 \quad [\Delta]\,\mathfrak{a}_2} \qquad\qquad \frac{[\Delta]\,\mathfrak{b}}{[\Delta, *\mathfrak{b}_2]\,\mathfrak{b}_1 \quad [\Delta, *\mathfrak{b}_1]\,\mathfrak{b}_2}$$

The general form of the rules is $[\Delta]\,A \ /\ [\Delta \cup \Delta']\,B$. The following instruction handles their application:

FAR If $C$ is an unmarked target, $[\Delta]\,A$ is the formula of an unmarked line $i$, $[\Delta]\,A \ /\ [\Delta \cup \Delta']\,B$ is a formula analysing rule, and $\mathrm{pp}(C, B)$, then one may add:

| | | |
|---|---|---|
| $k$ | $[\Delta \cup \Delta']\,B$ | $i$; R |

in which R is the name of the formula analysing rule.

The condition analysing rules of **CL** are summarized by:

$$\frac{[\Delta \cup \{\mathfrak{a}\}]\,A}{[\Delta \cup \{\mathfrak{a}_1, \mathfrak{a}_2\}]\,A} \qquad\qquad \frac{[\Delta \cup \{\mathfrak{b}\}]\,A}{[\Delta \cup \{\mathfrak{b}_1\}]\,A \quad [\Delta \cup \{\mathfrak{b}_2\}]\,A}$$

They all have the form $[\Delta \cup \{B\}]\,A \ /\ [\Delta \cup \Delta']\,A$. The following instruction refers to them:

CAR If $A$ is an unmarked target, $[\Delta \cup \{B\}]\,A$ is the formula of an unmarked line $i$, and $[\Delta \cup \{B\}]\,A \ /\ [\Delta \cup \Delta']\,A$ is a condition analysing rule, then one may add:

| | | |
|---|---|---|
| $k$ | $[\Delta \cup \Delta']\,A$ | $i$; R |

in which R is the name of the condition analysing rule.

The instructions EM (excluded middle) and EM0 allow one to eliminate certain problems without answering them.

EM0 If $[\Delta \cup \{*A\}]\,A$ is the formula of a line $i$ that is neither R-marked nor I-marked, then one may add:

| | | |
|---|---|---|
| $k$ | $[\Delta]\,A$ | $i$; EM0 |

EM If $A$ is an unmarked target, $[\Delta \cup \{B\}]\,A$ and $[\Delta' \cup \{\sim B\}]\,A$ are the respective formulas of the unmarked or only D-marked lines $i$ and $j$, and $\Delta \subseteq \Delta'$ or $\Delta' \subseteq \Delta$, then one may add:

| | | |
|---|---|---|
| $k$ | $[\Delta \cup \Delta']\,A$ | $i$, $j$; EM |

Transitivity is essential for eliminating solved questions as well as for summarizing the remaining problems (and paths) in a problem-solving process.

Trans If $A$ is an unmarked target, and $[\Delta \cup \{B\}]\,A$ and $[\Delta']\,B$ are the respective formulas of the at most S-marked[3] lines $i$ and $j$, then one may add:

| | | |
|---|---|---|
| $k$ | $[\Delta \cup \Delta']\,A$ | $i$, $j$; Trans |

The last instruction handles derived problems:

[3]A line is at most S-marked iff (if and only if) it is not R-marked, not I-marked and not D-marked.

DP If $A$ is an unmarked target from problem line $i$ and $[B_1, \ldots, B_n]\, A$ is the formula of an unmarked line $j$, then one may add:

$k \qquad \{?\{B_1, \sim B_1\}, \ldots, ?\{B_n, \sim B_n\}\} \quad i, j$; DP

In view of the intended applications (deriving predictions, explanations, etc.) the procedural system has no instruction for applying EFQ—the paraconsistent procedural variant logic of **CL** is studied in [22, 23].

I now list the *marking definitions*. We shall need several kinds of marks, which have distinct effects on the procedure. Each kind is governed by a definition, which applies to the stages of the fpsp—marks may come and go with each new stage. An R-mark indicates that a line is *redundant*.

**Definition 1** *An at most S-marked declarative line $i$ that has $[\Delta]\, A$ as its formula is R-marked at a stage iff, at that stage, $[\Theta]\, A$ is the formula of a line for some $\Theta \subset \Delta$.*

*An unmarked problem line $i$ is R-marked at a stage iff, at that stage, a direct answer $A$ of a question of line $i$ is the formula of a line.*

Remember that a target is a target *from* a problem line (and from its problem). If $A$ is a target, every line in which $[\Delta]\, A$ is derived for some $\Delta \neq \{A\}$ will be called a *resolution* line (*for* target $A$). The line called $j$ in instruction DP will be said to *generate* the problem line introduced by DP. $A$ is *a direct target from* $[\Delta]\, B$ iff $[\Delta]\, B$ is the formula of the resolution line that generates problem $\mathsf{P}$, $A$ is a target from $\mathsf{P}$, and $A \in \Delta$—note that some targets from $\mathsf{P}$ are not members of $\Delta$.[4] A *target sequence* is a sequence $\langle [\Delta^1]\, A^1, \ldots, [\Delta^n]\, A^n \rangle$ in which every $A^{i+1}$ ($1 \leq i < n$) is a direct target from $[\Delta^i]\, A^i$. A target sequence $\langle [\Delta^1]\, A^1, \ldots, [\Delta^n]\, A^n \rangle$ is *grounded* iff $A^1$ is not a direct target from any unmarked $[\Theta]\, B$ derived in the fpsp. A set is flatly inconsistent iff it contains $A$ as well as $\sim A$ for some $A$. I-marked lines are *inoperative*: acting on the line is not useful for solving any extant problem.

**Definition 2** *An at most S-marked target line that has $[A]\, A$ as its formula is I-marked at a stage iff every problem line from which $A$ is a target is marked at that stage.*

*An at most S-marked resolution line of which $[\Delta^1]\, A^1$ is the formula for some $\Delta^1 \neq \emptyset$ is* I-marked *at a stage iff, at that stage, for every grounded target sequence $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$,*

*(i) some target $[A^i]\, A^i$ ($1 \leq i \leq n$) is marked, or*
*(ii) $\{A^n, \ldots, A^1\} \cap \Delta^1 \neq \emptyset$, or*
*(iii) $\Delta^1 \cup \ldots \cup \Delta^n \cup \Gamma_s^\circ$ is flatly inconsistent.*

*An unmarked problem line is I-marked iff no unmarked resolution line generates it.*

$A$ is a *dead end* iff $A$ is (in the present propositional context) a literal and $A$ is not a positive part of a premise. If $A$ is not a literal, then CAR leads from the unmarked $[\Delta \cup \{A\}]\, B$ to one or more $[\Delta \cup \Delta']\, B$. The latter is called a CAR-descendant of $[\Delta \cup \{A\}]\, B$. D-marks indicate that no further action can be taken in view of a line.

[4] A resolution line containing $[q]\, p$ leads to the derived problem $?\{p, \sim p\}$ and $\sim p$ is a target from this.

**Definition 3** *An at most S-marked resolution line with formula $[\Delta]\, A$ is D-marked at a stage iff some $B \in \Delta$ is a dead end or, at that stage, all CAR-descendants of $[\Delta]\, A$ occur in the fpsp and are D-marked.*

*An at most S-marked target line with formula $[A]\, A$ is D-marked at a stage iff $A$ is a dead end or no further action can be taken in view of target $A$.*[5]

The procedure may be *sped up* (in a way that derives from a straightforward insight in the fpsp) by S-marks. Let $\Gamma^{\circ}_{s}$ be the union of the set of premises and the set of unconditional formulas that occur at stage $s$ of the fpsp.

**Definition 4** *A R-unmarked resolution line in which $[\Delta^1]\, A^1$ is derived is S-marked iff*

*(i)* $\Delta^1 \cap \Gamma^{\circ}_{s} \neq \emptyset$, *or*
*(ii) for some target sequence* $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$, $\{A^n\} \cup \Delta^1$ *is flatly inconsistent whereas* $\Delta^1$ *is not flatly inconsistent, or*
*(iii)* $\Delta_1 \subset \Delta^n \cup \ldots \cup \Delta^2$ *for some target sequence* $\langle [\Delta^n]\, A^n, \ldots, [\Delta^1]\, A^1 \rangle$.

What precedes describes a rather permissive procedure in which the only declarative logic is **CL**. Below I illustrate the procedure by means of an extremely simple example. The prospective dynamics for adaptive logics is hardly more complicated—see [17].

Let $?\{p \vee q, {\sim}(p \vee q)\}$ be the main problem and let $\{t \supset u, {\sim}u \vee r, (r \wedge t) \vee s, {\sim}s, (q \vee u) \supset ({\sim}t \vee q)\}$ be the premise set. The superscripts of the marks name the stage at which the line is (thus) marked—this convention saves rewriting.

| | | | |
|---|---|---|---|
| 1 | $\{?\{p \vee q, {\sim}(p \vee q)\}\}$ | Main | |
| 2 | $[{\sim}(p \vee q)]\, {\sim}(p \vee q)$ | Target | $\mathrm{D}^3$ |
| 3 | $[{\sim}p, {\sim}q)]\, {\sim}(p \vee q)$ | 2; C${\sim}\vee$E | $\mathrm{D}^3$ |

As ${\sim}p$ is not a positive part of any premise, line 3 is D-marked, and hence so is line 2.

| | | | |
|---|---|---|---|
| 4 | $[p \vee q]\, p \vee q$ | Target | |
| 5 | $[p]\, p \vee q$ | 4; C$\vee$E | $\mathrm{D}^5$ |
| 6 | $[q]\, p \vee q$ | 4; C$\vee$E | |
| 7 | $\{?\{q, {\sim}q\}\}$ | 4, 6; DP | |
| 8 | $[q]\, q$ | Target | |
| 9 | $(q \vee u) \supset ({\sim}t \vee q)$ | Prem | |
| 10 | $[q \vee u]\, {\sim}t \vee q$ | 9; $\supset$E | |
| 11 | $[q]\, {\sim}t \vee q$ | 10; C$\vee$E | |
| 12 | $[q, t]\, q$ | 11; $\vee$E | $\mathrm{I}^{12}$ |
| 13 | $[u]\, {\sim}t \vee q$ | 10; C$\vee$E | |
| 14 | $[u, t]\, q$ | 13; $\vee$E | |
| 15 | $\{?\{u, {\sim}u\}, ?\{t, {\sim}t\}\}$ | 8, 14; DP | |
| 16 | $[t]\, t$ | Target | |
| 17 | $(r \wedge t) \vee s$ | Prem | |
| 18 | $[{\sim}s]\, r \wedge t$ | 17; $\vee$E | |

[5] Whether an action that can be taken in view of target $A$ is obvious in view of the instructions. If no further action can be taken in view of target $A$, all instructions that could be applied in view of that target were taken and resulted in lines that are R-marked, I-marked or D-marked.

| | | | |
|---|---|---|---|
| 19 | $[{\sim}s]\, t$ | 18; $\wedge$E | $\mathrm{S}^{22}$ |
| 20 | $\{?\{s, {\sim}s\}\}$ | 16, 19; DP | $\mathrm{R}^{22}$ |
| 21 | $[{\sim}s]\, {\sim}s$ | Target | $\mathrm{R}^{22}$ |
| 22 | ${\sim}s$ | Prem | |

The S-mark of line 19 indicates that Trans can be applied. I repeat part of the fpsp:

| | | | |
|---|---|---|---|
| 14 | $[u, t]\, q$ | 13; $\vee$E | $\mathrm{S}^{23}$ |
| 15 | $\{?\{u, {\sim}u\}, ?\{t, {\sim}t\}\}$ | 8, 14; DP | $\mathrm{R}^{23}$ |
| 16 | $[t]\, t$ | Target | $\mathrm{R}^{23}$ |
| 17 | $(r \wedge t) \vee s$ | Prem | |
| 18 | $[{\sim}s]\, r \wedge t$ | 17; $\vee$E | |
| 19 | $[{\sim}s]\, t$ | 18; $\wedge$E | $\mathrm{S}^{22}$ $\mathrm{R}^{23}$ |
| 20 | $\{?\{s, {\sim}s\}\}$ | 16, 19; DP | $\mathrm{R}^{22}$ |
| 21 | $[{\sim}s]\, {\sim}s$ | Target | $\mathrm{R}^{22}$ |
| 22 | ${\sim}s$ | Prem | |
| 23 | $t$ | 19, 22; Trans | |

Again, the S-mark of line 14 indicates how to proceed.

| | | | |
|---|---|---|---|
| 24 | $[u]\, q$ | 14, 23; Trans | $\mathrm{S}^{29}$ $\mathrm{R}^{30}$ |
| 25 | $\{?\{u, {\sim}u\}\}$ | 8, 24; DP | $\mathrm{R}^{29}$ |
| 26 | $[u]\, u$ | Target | $\mathrm{R}^{29}$ |
| 27 | $t \supset u$ | Prem | |
| 28 | $[t]\, u$ | 27; $\supset$E | $\mathrm{S}^{28}$ $\mathrm{R}^{29}$ |
| 29 | $u$ | 23, 28; Trans | |
| 30 | $q$ | 24, 29; Trans | |

Line 14 is R-marked at stage 24. The S-mark of line 24 triggers Trans, which results in 30. Several marks are added at stage 30, most importantly, line 6 is S-marked, which leads to

| | | |
|---|---|---|
| 31 | $p \vee q$ | 6, 30; Trans |

whence the problem is solved (and line 1 is R-marked).

The example does not present the shortest way to derive an answer to $?\{p \vee q, {\sim}(p \vee q)\}$ from the premise set. Rather, it illustrates most of the features of (these simple) fpsps. Most of the heuristics is pushed into the fpsp and each step of the fpsp is sensible in view of the previous stage. Note that several premises were not introduced and cannot be introduced into the fpsp for this main problem.

# 3 Adaptive Logics

Here is a loose characterization: a logic is adaptive iff it adapts itself to the specific premises to which it is applied. The *logic* adapts itself to the premises: it depends on the premises which instances of inference rules are correct. And the logic adapts *itself* to the premises: the reasoner does not interfere in this.

The first studied adaptive logics were inconsistency-adaptive—see [5, 6, 9]. As a growing multiplicity of adaptive logics were described, the need for systematization presented itself. The idea was to find a common formal characterization, which was called *the standard format*. As the basic mechanism behind all adaptive logics is the same, the standard format should do most if not all of the work. It does.

In the sequel, I shall consider only adaptive logics in standard format. I shall try to avoid to complex technicalities, for which I refer to other papers, but rather try to clarify the way in which adaptive logics characterize defeasible reasoning processes.

The standard format is both simple and perspicuous. It was first presented in [12] and was studied more thoroughly in [19]. An adaptive logic **AL** is in standard format if it is characterized by a triple consisting of the following elements:

(i) **LLL**, a lower limit logic,
(ii) $\Omega$, a set of abnormalities that all have the same logical form,
(iii) an adaptive strategy.

Each of these elements has a specific technical function.

The lower limit logic **LLL** determines the part of the adaptive logic that is not subject to adaptation. From a proof theoretic point of view, **LLL** delineates the rules of inference that hold unexceptionally. From a semantic point of view, the adaptive models of $\Gamma$ are a selection of the **LLL**-models of $\Gamma$. It follows that $Cn_{\mathbf{LLL}}(\Gamma) \subseteq Cn_{\mathbf{AL}}(\Gamma)$. In principle, the lower limit logic is a monotonic and compact logic.

Abnormalities are formulas that are presupposed to be false, unless and until proven otherwise. "Abnormality" is a technical term. The abnormalities of corrective adaptive logics are logical falsehoods of **CL**, but the abnormalities of ampliative adaptive logics are not. Thus, in a context in which abductive reasoning is appropriate, it will be supposed that one of the known possible 'causes' of a fact is indeed the cause of this fact. Similarly, in a context in which inductive generalization is appropriate, the presence of instances of $Px \wedge Qx$ and the absence of instances of $Px \wedge \sim Qx$ will be taken a reason to consider $\exists x(Px \supset Qx)$ as true—there are some complications, but these need not concern us here.

$\Omega$ comprises all formulas of a certain logical form, which may be restricted. For many inconsistency-adaptive logics, $\Omega$ is the set of formulas of the form $\exists(A \wedge \sim A)$, the existential closure of $(A \wedge \sim A)$. For other inconsistency-adaptive logics, the set is restricted, for example, to formulas in which $A$ is a primitive formula—a formula that contains no logical symbols except for identity. Similar restrictions are imposed on many ampliative adaptive logics. See, for example, [26] and [25]. Where introduced, the restriction is justifiable and desirable—if it were not introduced, a flip-flop logic would result.[6] Examples of such inconsistency-adaptive logics are those that have as their lower limit logic, for example, Schütte's $\mathbf{\Phi}_v$ from [61] (called **CLuNs** in Ghent—see [24]), Priest's **LP** from [58], or Meheus' **AN**$\emptyset$ from [44], and modal adaptive logics that characterize paraconsistent inference relations under a translation, for ex-

[6]A flip-flip logic is an extremely simple adaptive logic **AL** for which $Cn_{\mathbf{AL}}(\Gamma)$ is identical to $Cn_{\mathbf{LLL}}(\Gamma)$ if $\Gamma$ requires some abnormalities to be true and is identical to $Cn_{\mathbf{ULL}}(\Gamma)$ if $\Gamma$ requires no abnormality to be true—see below in the text on **ULL**. Most flip-flop logics have no sensible applications

ample [46, 49, 50]. Incidentally, if the logical form characterizing $\Omega$ is restricted, every formula of the unrestricted form entails a disjunction of formulas of the restricted form.

To illustrate the variety of sets of abnormalities, I list just a few other logical forms that characterize, possibly with a restriction, the set of abnormalities of some adaptive logics: $\exists A \wedge \exists{\sim}A$, $\Box A$, $\Diamond A \wedge {\sim}A$, and $(A \supset B) \wedge (A \wedge {\sim}B)$.[7]

That $\Omega$ is characterized by a logical form is not unimportant. It enables one to consider adaptive logics as formal logics. They are defined by **LLL**, which is a formal logic in the standard sense, and by the supposition that all formulas of a certain logical form are false until and unless proven otherwise.[8]

Extending the lower limit logic with the requirement that no abnormality is logically possible, results in the *upper limit logic* **ULL**. Syntactically, **ULL** is obtained by extending **LLL** with an axiom stating that members of $\Omega$ entail triviality. Semantically, the upper limit logic is characterized by the lower limit models that verify no abnormality. **ULL** requires premise sets to be normal, and 'explodes' abnormal premise sets (assigns the trivial consequence set to them).

So **ULL** is the logic that is appropriate for premise sets that do not require any abnormalities to hold true. For corrective adaptive logics have a lower limit logic that is weaker than **CL**; most of them have **CL** as their upper limit logic. One normally would apply **CL**, but if the premises do not allow this (because their **CL**-consequence set is trivial), the adaptive logic interprets the premise set as much as possible according to **CL**. Ampliative adaptive logics have an upper limit logic that has often no sensible application contexts itself. This does not make the adaptive logic meaningless. For example, we all know that the world we live in is not completely uniform: not all objects have the same properties. Nevertheless, it makes sense to interpret the world 'as uniform as possible'. The logic of inductive generalization does precisely this: it takes the world to be as uniform as the empirical data permit.

Although corrective adaptive logics have a lower limit logic that is weaker than **CL**, it is possible to add new logical symbols that have exactly the same meanings as the original symbols of **CL**. These new connectives are extremely helpful from a technical point of view, but obviously do not occur in the premise set. They are means to handle the premises, to interpret them as normally as possible, even if the premises require abnormalities to hold true.

As was suggested before, if the premise set does not require any abnormality to obtain, then the adaptive logic delivers the same consequences as the upper limit logic. If the premise set requires some abnormalities to obtain, the adaptive logic will still deliver more consequences than the lower limit logic, viz. all upper limit consequences that are not 'blocked' by those abnormalities.[9] In sum, the adaptive logic interprets the set of premises 'as normally as possible'; it takes abnormalities to be false 'in as far as' the premises permit.

An *adaptive strategy* is required because many premise sets **LLL**-entail a disjunction of abnormalities (members of $\Omega$) without entailing any of its disjuncts.

[7] The abnormalities are the formulas that have the (possibly restricted) form and are **LLL**-contingent.

[8] This is one of the central differences with the so-called formula-preferential systems from [2, 37], which refer to an arbitrary set of formulas of which as many members as possible are considered as true. See also [27] for the relation between adaptive logics and formula-preferential systems.

[9] Flip-flop logics are an exception—see footnote 6.

Disjunctions of abnormalities will be called *Dab-formulas*. In the sequel, any expression of the form $Dab(\Delta)$ will refer to the (classical) disjunction of the members of a finite $\Delta \subseteq \Omega$. *Dab*-formulas that are derivable by the lower limit logic from the premise set $\Gamma$ will be called *Dab-consequences* of $\Gamma$. If $Dab(\Delta)$ is a *Dab*-consequence of $\Gamma$, then so is $Dab(\Delta \cup \Theta)$ for any finite $\Theta \subset \Omega$. For this reason, only the *minimal* *Dab*-consequences of the premise set are relevant: $Dab(\Delta)$ is a *minimal* *Dab*-consequence of $\Gamma$ iff $\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta)$ and there is no $\Theta \subset \Delta$ such that $\Gamma \vdash_{\mathbf{LLL}} Dab(\Theta)$. If $Dab(\Delta)$ is a minimal *Dab*-consequence of $\Gamma$, then $\Gamma$ determines that some member of $\Delta$ behaves abnormally, but fails to determine which member of $\Delta$ behaves abnormally. We have seen that adaptive logics interpret a premise set 'as normally as possible'. As some minimal *Dab*-consequences of $\Gamma$ may contain more than one disjunct, this phrase is ambiguous. It is disambiguated by choosing a specific adaptive strategy.

*Reliability* from [6] is the oldest known strategy. The minimal abnormality strategy, first presented in [5], delivers at least the same consequences as the Reliability strategy. For some lower limit logics and sets of abnormalities $\Delta$ is a singleton whenever $Dab(\Delta)$ is a minimal *Dab*-consequence of a premise set. If this is the case, the Reliability strategy and the Minimal Abnormality strategy lead to the same result and coincide with what is called the *Simple* strategy—see [26, 44, 45] for examples. Most other strategies were needed to characterize an existing consequence relation by an adaptive logic—see [11, 14, 28, 34, 64] for examples. In this paper, I only mention the marking definition of the Reliability strategy, referring to [19] for the marking definitions of the Minimal Abnormality strategy and of the Simple strategy (and to the specific papers for the making definitions of the other strategies).

Adaptive logics handle defeasible inferences. Some formulas derived at a point have to be considered as OUT at a later point, and possibly as IN at a still later point. This dynamics should be controlled. The means to do so in the dynamic proofs of adaptive logics are the marking definitions. They are phrased in terms of the conditions that are attached to lines of annotated proofs.[10]

A line of an annotated dynamic proof consists of a line number, a formula, a justification, and a (possibly empty) condition. The proofs are governed by three (generic) rules and a marking definition. The rules, which depend on **LLL** and $\Omega$ only, determine which lines may be added to the proof; the marking definitions, which depend on $\Omega$ and the strategy, determine which formulas are IN or OUT at some stage of the proof. Let

$$A \qquad \Delta$$

abbreviate that $A$ occurs in the proof on the condition $\Delta$, the rules may then be phrased as follows:

[10]The definitions may be adjusted for non-annotated proofs, which are just lists of formulas as usual.

PREM If $A \in \Gamma$:
$$\frac{\begin{array}{ll} \ldots & \ldots \end{array}}{\begin{array}{ll} A & \emptyset \end{array}}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:
$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$
$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Only the conditional rule, RC, introduces non-empty conditions. The unconditional rule, RU, simply carries the conditions over from given formulas to formulas that are derived from them.

The marking definitions require some preparation. $Dab(\Delta)$ is a *minimal Dab-formula* at stage $s$ of the proof if, at stage $s$, $Dab(\Delta)$ occurs in the proof on the empty condition and, for any $\Delta' \subset \Delta$, $Dab(\Delta')$ does not occur in the proof on the empty condition. Where $Dab(\Delta_1)$, ..., $Dab(\Delta_n)$ are the minimal *Dab*-formulas at stage $s$ of the proof, $U_s(\Gamma) = \Delta_1 \cup \ldots \cup \Delta_n$ is the set of unreliable formulas at stage $s$. So, whenever $Dab(\Delta)$ is a *minimal Dab*-formula at stage $s$, all members of $\Delta$ are considered as unreliable at that stage (at least one of them is true, but it is not determined which one). Further *Dab*-formulas may be derived at a later stage $s'$. So $U_{s'}(\Gamma)$ may comprise abnormalities that are not a member of $U_s(\Gamma)$. The opposite is also possiblen viz. if $Dab(\Delta)$ is derived at stage $s$ and, for some $\Theta \subset \Delta$, $Dab(\Theta)$ is derived at stage $s'$. A line is marked (for Reliability) at stage $s$ iff a member of its condition is unreliable at that stage:

**Definition 5** *Marking for Reliability: Line $i$ is marked at stage $s$ iff, where $\Theta$ is the condition of line $i$, $\Theta \cap U_s(\Gamma) \neq \emptyset$.*

A formula is *derived* from $\Gamma$ *at a stage* of the proof iff it is the formula of a line that is unmarked at that stage. As the proof proceeds, unmarked lines may be marked and vice versa. So, it is important that one defines a different, stable, kind of derivability:

**Definition 6** *$A$ is* finally derived *from $\Gamma$ on line $i$ of a proof at stage $s$ iff (i) $A$ is the formula of line $i$, (ii) line $i$ is not marked at stage $s$, and (iii) any extension of the proof in which line $i$ is marked may be further extended in such a way that line $i$ is unmarked.*

A different definition goes as follows: $A$ is *finally derived* on line $i$ of a (possibly infinite) proof from $\Gamma$ iff $i$ is unmarked and the proof is *stable* with respect to line $i$, viz. line $i$ is unmarked in all extensions of the proof. The previous definition is more appealing. The only way to establish the existence of a proof in which $A$ is finally derived is by a metalinguistic reasoning anyway. Moreover, the definition has a nice game-theoretic interpretation: if an opponent is able to extend the proof in such a way that line $i$ is marked, then the proponent is able to extend it further in such a way that line $i$ is unmarked.

**Definition 7** $\Gamma \vdash_{\mathbf{AL}} A$ *($A$ is* finally **AL**-derivable *from $\Gamma$) iff $A$ is finally derived on a line of a proof from $\Gamma$.*

To describe the *semantics*, let $M \models A$ denote that $M$ assigns a designated value to $A$, in other words that $M$ verifies $A$, and let the abnormal part of a **LLL**-model $M$ be defined as follows:

**Definition 8** $Ab(M) = \{A \in \Omega \mid M \models A\}$

Where $Dab(\Delta_1),\ Dab(\Delta_2),\ \ldots$ are the minimal minimal $Dab$-formulas that are verified by all **LLL** models of $\Gamma$, $U(\Gamma) = \Delta_1 \cup \Delta_2 \cup \ldots$ is the set of formulas that are unreliable with respect to $\Gamma$.[11] Let $\mathbf{AL}^r$ and $\mathbf{AL}^m$ be the adaptive logics defined by a **LLL**, an $\Omega$, and the Reliability strategy, respectively the Minimal Abnormality strategy.

**Definition 9** *A* **LLL**-*model $M$ of $\Gamma$ is* reliable *iff $Ab(M) \subseteq U(\Gamma)$.*

**Definition 10** $\Gamma \vDash_{\mathbf{AL}^r} A$ *iff $A$ is verified by all reliable models of $\Gamma$.*

**Definition 11** *A* **LLL**-*model $M$ of $\Gamma$ is* minimally abnormal *iff there is no* **LLL**-*model $M'$ of $\Gamma$ such that $Ab(M') \subset Ab(M)$.*

**Definition 12** $\Gamma \vDash_{\mathbf{AL}^m} A$ *iff $A$ is verified by all minimally abnormal models of $\Gamma$.*

Note that the proof theory as well as the semantics of an adaptive logic are fixed by the standard format. There is more. Many metatheoretic properties of adaptive logics can be proved from the format itself, rather than from the specific properties of the logic. Among such properties are: the *Soundness* and *Completeness* proof (given that the lower limit logic is sound and complete with respect to its semantics), the *Derivability Adjustment* Theorem ($\Gamma \vdash_{\mathbf{ULL}} A$ iff there is a finite $\Delta \subset \Omega$ such that $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$), *Proof Invariance* (a formula finally derived in some proof from $\Gamma$ can be finally derived in every proof from $\Gamma$), *Strong Reassurance* (if $M$ is a **LLL**-model of $\Gamma$ but not an **AL**-model of $\Gamma$, then there is an **AL**-model $M'$ such that $Ab(M') \subset Ab(M)$), Cautious Cut, Cautious Monotonicity, and many further properties—see especially [19].

Nearly all known inconsistency-adaptive logics have a characterization in standard format.[12] In some cases, forging a consequence relation into standard format may require a translation, for example to a modal language. Even where it is useful to provide 'direct dynamic proofs' in untranslated terms (see [32] or [48] for examples) the formulation in standard format has the advantage to provide the proof theory, semantics and metatheoretic properties, and to warrant (by an easy demonstration) that the direct proof theory is correct.

In many situations one needs to combine adaptive logics. This too may be handled in a generic way. Two general stratagems for combining adaptive logics were designed. Apparently they serve all needs. Again, the combination stratagems, rather than specific properties of the combined logics, warrant that the combination does the desired job. The matter is studied, for example, in [12].

[11] So $U(\Gamma)$ is like $U_s(\Gamma)$, except that it refers to final derivability (or rather to its semantic counterpart) and not to a stage of a proof.

[12] An exception is Priest's $\mathbf{LP}^m$ from [59] and emended in [60]. This adaptive logic proceeds in terms of properties of the model, rather than in terms of the formulas verified by the model.

# 4 Some Extra-Logical Extensions

Some questions can be directly answered by standard means, for example questions that are answered (obviously at a cost) by observational or experimental means. Building this into fpsps is a bit more tricky than is usually thought (and than I supposed before). Hintikka, for example in [35], tries to handle this in terms of a set of formulas. If a direct answer to a question is a member of this set, and the question is posed, then the answer is provided by the oracle. This will not do because some questions are only answered in specific situations and this cannot be expressed by means of **CL**-formulas. Consider an example.

Let $p$ be "the shutter of the window is open" and let $q$ be "the sun shines". Suppose that I know that the position of the sun is such that, if the shutter is open and the sun shines, then I see a spot of sunlight on the floor, but that I do not know (and cannot observe in the present situation) whether the shutter is open and that I do not know (and cannot observe in the present situation) whether the sun is shining. So, in the present situation, the oracle informs me that $\sim p \vee \sim q$ (either the shutter is closed or the sun does *not* shine). Suppose, moreover, that the sun is actually shining, $p$, and that the shutter of the window is closed. So, if the shutter were open, I would see that the sun is shining. The trouble is that this cannot be expressed by any statement in the oracle set. Indeed, the only candidate seems to be $\sim p \vee q$. But if that were in the oracle set, I could force the oracle in the present situation to answer both $\sim p \vee \sim q$ and $\sim p \vee q$, from which $\sim p$ follows. However, the oracle cannot provide me with this information in the present situation (we supposed that I cannot observe in the present situation whether the shutter is open). So if plain formulas go into the oracle set, its contents has to vary with changes in the situation, contrary to Hintikka's implicit setup. The proposal I made in [13], viz. that the set contains answerable questions, is affected by the same problem.[13]

How should the oracle be handled? The researcher is convinced that a certain question $Q$ is answered if certain circumstances $\Delta$ obtain. Let this belief be expressed by $(\Delta : Q)$. In the example from the previous paragraph, the formula is $(\{p\} :?\{q, \sim q\})$. Such a formula, respectively statement, may be handled as a premise.

Obviously, the statement $(\Delta : Q)$ may be false. But if the researcher is convinced of its truth (which may be seen from its occurrence in the fpsps) and if he or she is convinced that all members of $\Delta$ hold true (because they occur in the fpsp), then he or she will *take* his or her observation as an answer to the question. Consider again the example from two paragraphs ago. Suppose I press a button which I believe to open the shutter and that I do not see a spot of sunlight thereafter. I will then conclude that the sun is not shining. If the button did not function and the shutter is still closed, I shall still believe that the sun is not shining, because I believe the shutter to be open.

So the oracle is an epistemic device, not an ontological one. Formally, it may be represented by a function that maps couples composed of a question and a set of statements to a direct answer of the question, for example $f(Q, \Delta) = A$, in which $A$ is a direct answer to $Q$. If $(\Delta : Q)$ and all members of $\Delta$ occurs in the fpsp, the researcher obtains one of the direct answers of $Q$. So the connected instruction is as follows:

[13] That $\sim p \vee q$ cannot be put into the set is a result of the fact that it does not express the intended counterfactual or conditional.

New If $A$ is an unmarked target, $\mathrm{pp}(A, B)$ for some direct answer $B$ of $Q$, $(\Delta : Q)$ and all members of $\Delta$ occur in the fpsp, then one should add, for some direct answer $C$ of $Q$:

$k$ $C$ $i$; New

Of course, the researcher does not know beforehand which new premise will be introduced if $(\Delta : Q)$ and the members of $\Delta$ occur in the fpsp. So the heuristics cannot proceed in terms of the answers that would be provided if the conditions for New were realized, but only in terms of the statements of the form $(\Delta : Q)$ that are held true. Precisely such statements are essential for setting up experiments and for interpreting their outcome. It is instructive to reconstruct for example Galilei's inclined plane experiment in terms of such knowledge. Planning an experiment may be seen as itself resulting from a fpsp.

I have supposed that a new premise is provided as soon as $(\Delta : Q)$ and all members of $\Delta$ occur in the fpsp. One might object that posing the question to the oracle, viz. actually making the relevant observation, may come at some expense, and hence that it should be left to the researcher's decision whether the question $Q$ is actually introduced. To modify the formal machinery accordingly is clearly not difficult, but I take it to be superfluous. Indeed, the expense might be associated to the members of the condition $\Delta$ in $(\Delta : Q)$. If the shutter is open and if I turn my head in the right direction and if I pay the required attention to the presence of a spot of sunlight on the floor, then I have the answer to the question whether the sun is shining.

The expense for realizing certain statements to hold is obviously an important factor. If the expense for answering the main problem by direct means is near to nil, to do so would be preferred over engaging in logical deduction. If the expense is extremely high, one may renounce from seeking a direct answer, even where this entails that the main problem is left unsolved. However, such considerations will be neglected in the present context.

The presence of $(\Delta : Q)$ and of the members of $\Delta$ in the fpsp triggers an application of New as soon as some unmarked target $A$ is a positive part of a direct answer of $Q$. If there is no such target $A$, the answer to $Q$ is, for all one knows, irrelevant to the extant problems and hence should not be introduced. However, this should be refined in future work. Suppose that $Q$ is $?\{B, {\sim}B\}$, that $B$ is present in the fpsp, but that we observe ${\sim}B$. This reveals that there is a problem with the fpsps, and hence that it has to be restructured in order to solve the problem (possibly by introducing an inconsistency-adaptive logic).

The present extension requires that one redefines: $A$ is a *dead end* iff $A$ is (in the present propositional context) a literal and $A$ is not a positive part of a premise or of a direct answer to a question $Q$ which is such that $(\Delta : Q)$ is among the premises for some $\Delta$. It also requires an instruction for introducing premises of the form $(\Delta : Q)$. This instruction should obviously require that $(\Delta : Q)$ is only introduced if some unmarked target is a positive part of a direct answer to $Q$. Finally, handling the matter adequately requires that fpsps are extended with a feature indicating that the researcher performs a certain action in an attempt to realize that the members of $\Delta$ are true, and that he or she reasons about such actions, taking their cost into account. Nothing of this involves serious difficulties. Once it is built into the fpsps, the latter will be able to guide action and the ensuing observations.

A second extralogical extension was announced, but this is easier to handle. It would be very unrealistic to suppose that a problem-solving process proceeds from all available knowledge. It actually proceeds from a set of knowledge that is judged relevant by the researcher and that he or she is sufficiently acquainted with. Apart from the knowledge that forms the premise set, lots of knowledge is available within the scientific community. Some of it may turn out relevant as the problem-solving process proceeds, for example because the problem may be suspected not to be solvable from the present premise set and because some targets are positive parts of available knowledge.

If a body of available knowledge is added to the premise set, a researcher may need to study this knowledge extensively or may invoke the help of a specialist in the domain. The reasons for this are double. On the one hand, a specialist will know many consequences of the body of knowledge which are directly relevant for the extant problem, whereas it may take the researcher lots of (logical) work to see what follows and what of this is relevant. Moreover, the specialist will be able to determine which changes should be made to a fpsp in order to accommodate the new body of knowledge. The changes may pertain to different elements of the fpsp: the logics, the procedure, the heuristic instructions, and the questions that are answerable by standard means.

# 5 On Heuristics

In [35] and elsewhere, Hintikka draws a sharp distinction between the rules of the logic and the heuristics by which a desired proof is obtained—I avoid Hintikka's "strategy" because of the technical sense of this term in Section 3. Hintikka's favoured comparison is with games of chess: the rules determine the possible moves whereas the heuristics is directed towards applying the rules in such a way that the game is won. According to this view, the forms of reasoning that are explicated by adaptive logics are not a matter of logic at all, but a matter of heuristics and hence should be explicated in heuristic terms.

The least one can say is that the comparison does not add much bite to the distinction. For one thing, the normal outcome of a game of chess is a checkmate king, whereas a proof exhibits a reasoning that leads from (a subset of) the premises to the conclusion. So, in the case of proofs, unlike in the case of chess, both the heuristics and its result (even according to the standard view) are of the same kind, viz. concern reasoning. That the heuristics concerns reasoning clarifies at once why it is possible, for example in terms of the prospective dynamics, to push part of the heuristics into the proof itself. In other words, the bookkeeping of part of the heuristic reasoning, which is usually done outside of the proof, can very well be done within the proof.

Prospective proofs still require a heuristics. However, as a result of their proof format, the delineation between rules and heuristics lies at a different point than with more usual proofs. The matter is even modified more drastically if prospective proofs are phrased in terms of a procedure—remember that this is composed by instructions: rules with permissions and obligations attached to them. The more restrictive the deontic qualifications of the instructions, the less room is left for heuristic considerations. Note that the extreme case, viz. a deterministic procedure, is by no means a universal ideal. The room that should be reserved for the heuristics depends on the specific application.

The difference of opinion with Hintikka has a principled side and a pragmatic one. From a pragmatic point of view, I see it as an advantage of my approach that part of the usual heuristics is fixed by the chosen adaptive logics, the prospective dynamics, and the (more or less restrictive) procedure by which the latter is governed. These three elements provide means to articulate those parts of the heuristics that are incorporated into them. Once articulated, they can be evaluated, discussed, and replaced where necessary. In other words, it is a pragmatic advantage of my view that the claim that "it is a matter of heuristics" can be replaced by definite and precise statements about an articulated framework.[14]

However, there was also a principled side. We have seen above that the delineation between rules and heuristics is not a fixed one. It is instructive to see why this is so. Consider again Hintikka's view. In view of a given purpose, a specific heuristics is applied in the framework of **CL**. For every premise set $\Gamma$ (possibly combined with a problem, say to find an explanation for $Pb$), there will be a set $f(\Gamma)$ of formulas that can be obtained by the heuristics. So the heuristics can be seen as a function that maps every set $\Gamma$ to a set $f(\Gamma)$. So it is a logic, a mapping $\wp(\mathcal{W}) \mapsto \wp(\mathcal{W})$ (in which $\mathcal{W}$ is the set of closed formulas and $\wp(\mathcal{W})$ its power set). Obviously, this logic may have a number of unusual properties; it may for example be non-monotonic, or even non-reflexive. But is is a logic in the above sense. So it makes sense to study its proof theory and to use it as an explication for the connected form of reasoning, and it makes sense to study its semantics and the properties of its models. One may still try to argue that the resulting logic is not the standard of deduction. I am not sure that one will succeed in establishing that some other logic is the standard of deduction, but I grant that the attempt to do so may be sensible. Yet, even if the logic is not the standard of deduction and some other logic is, it still is a logic and this was the point I had to make.

# 6 On Content Guidance

A first, general, and hence non-specific form of content guidance lies with the language of a scientific discipline and with the connected conceptual system. Both are the result of the development that led to the present state of the discipline. They incorporate a set of presuppositions about the domain. They determine the way in which problems are phrased and tackled.

A clear and specific form of content-guidance resides in the use of premises of the form $(\Delta : Q)$, which was discussed in Section 4. There we have seen that the oracle is actually an epistemic device, referring to current knowledge. Similarly, the fact that the premise set may be extended with available knowledge that was judged irrelevant before, constitutes a clear form of content-guidance.

A point that pertains to all adaptive logics is that these logics validate certain *applications* of rules that transcend their lower limit logic. It depends on the content of the premise set, as determined by the lower limit logic, whether certain consequences are drawn from the premises. This important point deserves to be spelled out clearly. It is not that the consequence set is a function

[14]Needless to say, there still is a need to make more precise claims about the heuristics that remains once the adaptive logics, their prospective dynamics and the governing procedure have been fixed.

of the premise set—a feature that is shared by nearly all logics. The point is that the logics adapt *themselves* to the premises: whether a conclusion follows from some premises does not merely depend on properties of the logic, but also on the other premises. So this form of content guidance is built in into adaptive logics.

There is a multiplicity of adaptive logics for every purpose. Consider handling inconsistency as an example. Many paraconsistent logics can be taken as the lower limit logic and combined with a suitable set of abnormalities and a strategy. Moreover, some forms of handling inconsistency are characterized by an adaptive logic under a translation. A good example form the Rescher–Manor mechanisms. They are characterized by an adaptive logic in standard format, but under a translation—see for example [11]. They also have a so-called direct proof theory, as was shown in [32], but this is not connected to an adaptive logic in standard format. The situation is similar for the so-called signed systems for paraconsistent reasoning from [33], as was shown in [28]. The variety is even larger than that. An inconsistency-adaptive logic need not be the most suitable instrument to handle a theory that turned out to be inconsistent. In some cases, it is more fitting to apply a logic that is adaptive with respect to properties of a different logical symbol. This point was already made in [10] and a case study is forthcoming in [31].

Given the multiplicity of adaptive logics for nearly every purpose, the choice has to be justified. The justification can only be obtained by logically analysing the concrete case one is confronting—see again [31] for an example. In other words, the choice of the adaptive logics that should be applied in a fpsp is typically content dependent.

The selection of an adaptive logic is typically a *local* matter. This means that the result of a fpsp will lead to certain changes in the setup of the fpsp. Consider the case where one tries to obtain an explanation for a certain fact. If one hopes that a theory is able to provide the explanation, the suitable way to handle the problem is by a fpsp in which a logic is abduction is set to work. But suppose that, at some point, it becomes dubious whether the theory will provide any explanation. One will then first try to obtain a new generalization that may provide the explanation. So one has to extend the adaptive logics of the fpsp by a logic of inductive generalization. Another example of a local choice occurs when the theory from which one tries to obtain the explanation turns out inconsistent. This insight, possibly gained by the fpsp itself, may require that **CL** is replaced by an inconsistency-adaptive logic as the underlying logic of the theory—see [18] for an analysis of the effect of such a move. In still other cases an invoked theory may prove to be contradicted by available data, and hence need to be introduced as plausible only—there is a multiplicity of adaptive logics handling plausible premises (see for example [25]). So, again, the choice has to be justified in a contextual way.

The situation is similar for the choice of an erotetic logic: one has to justify the specific choice in terms of the insights provided by the problem-solving process. The situation is also similar for the choice of the specific procedure that governs the prospective dynamics.

A further form of content guidance relates to invoked background theories. In fpsps, background theories can be introduced in a defeasible way (as plausible only) and (as was said before) there is a multiplicity of available means for handling plausible premises. So background theories are taken serious but nevertheless can be rejected or revised where necessary.

It is well-known that the results of scientific discovery and creativity are not fully determined by the situation of the discipline. Even if scientific discovery is seen as a form a problem solving, there should be left ample room for the world-view, personal constraints, ..., of individual researchers and research groups—see for example [47]. While it may be difficult to formalize such elements, or even to make them explicit, it is easy enough to take their consequences into account in fpsps. The central feature is that solutions may be arrived at by eliminating certain disjuncts from disjunctions of abnormalities. How this is done is discussed at length in [21] (in the context of inconsistency-adaptive logics) and in [15] (in the context of inductive generalization), and the mechanism is easily generalized to all adaptive logics. A derived (non-singleton) disjunction of abnormalities leads to the question which of its disjuncts is abnormal. This question can typically be answered, fully or partially, but always in a defeasible way, by analysing the situation in the discipline, but also by world-view considerations, by personal constraints, or even by blind guesses. So fpsps provide a framework that locate the problems, suggest ways out (by clarifying which defeasible statements may provide a solution), and logically guide the handling of such statements. The origin of the statements is obviously extra-logical. This is a form of content-guidance. However, but the logic guides the problem-solving process.

Finally, I come to the heuristics that remains after all other elements of the fpsp have been fixed. There are logical as well as non-logical aspects to this heuristics. The logical ones are mainly related to the road along which a statement that is derivable by the available means is actually derived. This too is largely a matter of learning. Suppose that one wants to derive theorems from the Peano Axioms. Whatever the target at a specific point in the fpsp, it will always be a positive part of an instance of the axiom of mathematical induction. One will soon find out, however, that the introduction of such instances will only occasionally lead to a successful search path. This insight may lead to the conclusion that one first tries to operate on the other axioms, and only introduces an instance of the induction axiom when that fails. This is just an example out of many. Every logician that has made a set of axiomatic proofs has experienced that each new axiomatic system, even for the same logic, requires another heuristics.

The non-logical aspects are especially related to the kind of move that is invoked at a specific point. Given a target or set of targets, there often is the choice between attempting a theoretical derivation, making an observation, and (devising and) performing an experiment. Which is the right move will have to be determined on the basis of one's insight in the situation and on the analogy between the situation and analogical cases that were successfully solved in the past.

# 7 Concluding Comments

Let me begin with some warnings. The first is that, in the end, all knowledge is defeasible. It does not follow that all declarative premises of a fpsp should be introduced in a conditional way. This, indeed, would only make sense in a futile attempt to incorporate into the approach all possible outcomes of scientific reasoning—see the comments on the Vienna Circle in the first paragraph. So it cannot and should not be avoided that fpsps are seen as themselves subject to revision. As we have seen in Section 6, the insights provided by a fpsp may lead to changes to the elements of the fpsp. Such changes are justified in view of insights in the situation and in view of insights from the history of science—"what we have learnt about how to learn".

The second warning concerns abduction. There are many ampliative reasoning forms. To reduce all of them to abduction would come to a serious overestimation of the latter and to a simplistic view on scientific discovery and creativity. A further warning concerning abduction itself is appropriate. It is fairly clear what it means that an explanation for a fact is abduced in the presence of a theory—the most recent book on the topic, [1], handles precisely that. A very different matter is an abduction in the absence of a theory. I can only confess that I have read nothing on this that was sufficiently clear to me. Some authors give the impression that such abductions rely on implicit beliefs, which may be connected to action habits rather than to statements. This seems acceptable enough. Only, as the origins of such implicit beliefs are clearly located in our observing and handling of everyday objects, it seems highly unlikely that 'abductions' relying on implicit beliefs could be responsible for creative science or for novel insights.

In this paper, I avoided taking a stand in such philosophical debates as the one opposing realism to positivism. The reason for this is that such debates may be relevant from a general epistemological point of view, but their impact on methodological decisions is often overestimated, at least from a contemporary point of view. Moreover, in as far as they do play a role in problem solving, they may be handled in the same way as world-views or personal constraints. Even if they might occasionally shape a fpsp, the formal framework of fpsps is sufficiently malleable (see Section 6) not to exclude the actual impact of such philosophical stands.

The last warning is that the fpsps approach outlined in the previous sections needs (obviously) further elaboration. Nothing was said, for example, on model-based reasoning. Also, while fpsps demonstrably result in certain conceptual changes, the problem of conceptual change requires a much deeper study. So the approach is far from complete. It was presented here because it is promising and hence deserves further study.

The most important comment is that seeing science as a content-guided problem-solving activity does not exclude a formal approach. Quite to the contrary. Precisely in order to be able to express and understand the content-guidance, one needs a maximally formal approach. A fpsp can be seen as a frame with many open slots. The slots have to be filled with (declarative as well as erotetic) logics, prospective dynamics, a procedure, and a set of heuristic instructions. The formal framework will not deliver any results unless the slots are filled and to fill them requires a justification. The more structure there is in the entities that may be plugged into the slot, the more it will be possible to make the content-guidance explicit and to justify it. As I see it, this is the great advantage of the fpsps approach over Hintikka's—see Section 5. Rather than keeping the heuristics free from the intrusion of rules and instructions, one has to maximize the intrusion. One has to make an effort to turn all possible aspects of the heuristics into alternative sets of transparent formal entities. Only in doing so, one will be able to make sense of the content-guided aspects of problem solving.

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