in mathematics education, particularly geometry, the learner can make links between the arts and mathematics. Our students generally ask where is mathematics used, then we can state and analyse such valuable samples to improve their mathematical thinking, and so foster their creativity power. By using these patterns, we will teach abstract subjects as applied subjects. In addition, by using the geometrical patterns, students could have a better understanding about geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.

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Narges Assarzadegan, a high school teacher from Isfahan, Iran has written before (Newsletter 62) about the Persian manuscript *Lubb Al-Hisab* (Book of Reckoning). Here she describes some cut and paste activities she designed for her class (Grade 11, age 14), based on a book by Abu'l- Wafa Buzdjani (4th Hejira/ AD 10th century).

An introduction to wasan, native Japanese mathematics

Japanese culture is much indebted to the Chinese one, including its mathematics. However, at the beginning of the seventeenth century, under the Tokugawa shogunate, Japan entered a period of *sakuko*, seclusion from the outside world. Japanese were not allowed to go abroad and foreigners were either forbidden in Japan or very restricted in their movement. The Dutch were limited in number and confined to a small island before Nagasaki. Remaining Chinese and Koreans were placed under strict supervision. Despite this seclusion, but thanks to this period of peace and security, Japan enjoyed a period of development of arts and crafts. It is in this era that *wasan* developed as a native kind of mathematics distinct from its Chinese heritage. Some philosophers have raised the question: could mathematics develop different from the way it did in history? In a way, because of its isolated development, the study of *wasan* provides an almost experimental situation to explore this question. There were many schools in the major cities of Japan teaching *wasan* in a way somewhat similar to European *abbaco* schools or *rechenmeister* in Germany and Flanders. The best known schools are those of Seki and later Aida. Contrary to the European context however, *wasan* served little practical purpose. It was a sort of mathematics mainly pursued for its intellectual and esthetic aspects. Good examples of this are the so-called *sangaku* problems. These are problems of geometrical nature which were displayed on wooden tablets at the entrance of Shinto shrines and Buddhist temples. An impressive collection of them including a mathematical analysis was published by Fukugawa and Pedoe (1989). See figure 1 for an example.

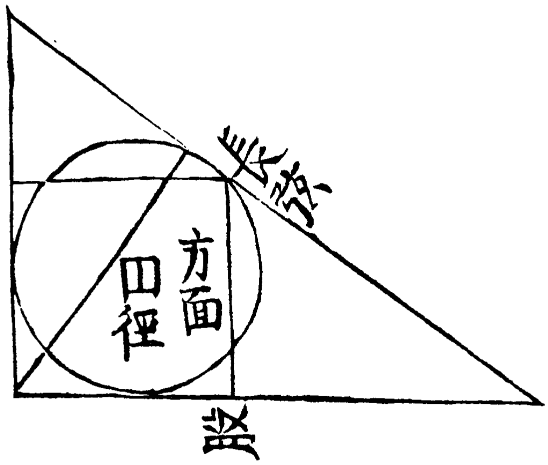

FIGURE 1: A SIMPLE SANGAKU PROBLEM. GIVEN THE SUM OF THE LENGTH OF THE BASE AND THE LONGER PART OF THE HYPOTHENUSE, AND GIVEN THE SUM OF THE LENGTH OF THE LEG, THE DIAMETER AND THE SIDE OF THE SQUARE, FIND THESE INDIVIDUAL VALUES (PROBLEM 40 OF SEKI'S *KENKI SANPŌ* 1683).

Wasan thrived until the second half of the nineteenth century. At the beginning of the eighteenth century under Yoshimune

Tokugawa,[1] who reigned from 1716 to 1744, the prohibition of foreign books became somewhat relaxed. The shogun himself showed an interest in Western astronomy and calendar composition and started building a library, of which later a catalogue as been found (Heeffer 2008). In the middle of the nineteenth century Kyō Uchida taught mathematics to several hundreds of students in Tokyo, adopting the Dutch word *Mathematische* for the name of his school. Any Dutch book that could be found on one of the ships in Nagasaki became a precious item and circulated within a small group of Japanese scholars called *rangakusha*. These consisted of medical doctors, astronomers, gunnery instructors and mathematicians of the samurai class in service of feudal lords. Early nineteenth century there even was a book dealer in Yedo (now Tokyo) who specialized in Dutch books. In 1857 the *Yōsan Yōhō* (洋算用法, *The method of Western arithmetic*) was published by Yanagawa Shunzō. It introduces Japan to the Western (Dutch) method of calculating. Shortly after the Meiji restoration of 1868 it was decided that only *yōzan* (Western mathematics) was to be taught at Japanese schools. In a matter of a few years *wasan* became completely neglected and *wasan* books were replaced by schoolbooks translated from French, English and German. Many *wasan* books were discarded and only used for their paper. However, some scholars collected these precious books and manuscripts, One of them being Tsuruichi Hayashi (1873 – 1935) who became the first historian to publish on *wasan* in a Western language.[2] He left an enormous collection of 14,470 *wasan* books and manuscripts, now the bulk part of the *wasan* collection at Tohoku University. Together with two other minor collections, Tohoku university library now holds the largest collection of *wasan* books in the world with 18,335 items. It is estimated that this accounts for two-thirds of the books produced on this subject. It gives us an idea of the spread of this tradition in Japanese society and the influence it must have had.

Hayashi published a well-received series of articles in the Dutch mathematical journal *Nieuw Archief voor Wiskunde*. His “Brief History of the Japanese Mathematics” (sic) is the first published account of the *wasan* tradition in a Western language (Hayashi 1905a, b). Mikami (1909) published a reaction to Hayashi correcting him on several points, later followed by a more extensive history on *wasan* (Mikami 1913). This publication was further augmented by Smith (1914) and turned into a popular book, at this time still the only English book on the subject. There are not so many Western publications on *wasan* since then.[3] The most extensive study to date is in French by Annick Horiuchi (1994) who primarily studied the works of Seki and his student Katahiro Takebe. While Martzloff (1997) writes on Chinese mathematics, he also provides valuable links to *wasan*, especially on its Chinese heritage. Despite its limited appearance in Western publications on the history of mathematics the study of *wasan* in Japan is blossoming more than ever since the Meiji restoration. There are several academic centers in Japan where scholars study the original sources and there is The Wasan Institute in Tokyo which publishes a dedicated journal and regular books with facsimiles and commentaries. In addition, there is a lively scene of amateurs, collecting and studying *wasan* literature.[4]

[1] Contrary to Japanese custom, I will list Japanese names with their first name first.

[2] The parallel with Western book collectors and historians of mathematics is obvious. Guillaume Libri, who published the classic *History of mathematics in Italy* (Libri 1838) owned thousands of books and manuscripts, most of which were sold during his lifetime. At some point he was accused of having stolen them from libraries. The many appendices in this three-volume work contain transcriptions from books and manuscripts of his own library, including the *Trattato d'abaco* by Pierro della Francesca. The other example is David Eugene Smith who wrote the two volume work *History of mathematics*. According to an inventory made in 1940 he owned over 20,000 items on mathematics, including 11,000 rare books, numerous manuscripts, autographs and portraits, now part of the Columbia university library collection (Simons 1945).

[3] For a list of *wasan* publications in Western languages see http://logica.ugent.be/albrecht/wasanbib.htm.

[4] This combination of academic and amateur efforts may seem exceptional but there is also a similar

So what is now so special about *wasan*? What makes it different from Western mathematics? Let us first look at the subjects treated. They can be broadly divided into three categories: arithmetic and algebra, geometry and recreational problems. Typical arithmetical problems include merchant problems as we also know from Renaissance arithmetic. However, within *wasan* they are mostly related to the use of the *soroban*, the Japanese equivalent of the abacus. Calculation with Hindu-Arabic numerals was not generally known before the 1880's. I have seen the *soroban* still in use in today's high-tech Japan. Other recurring themes are the approximation of fractions, infinite series and Bernoulli numbers. Geometrical problems include the calculation of the area of triangles, polygons and rings. In the process of devising techniques for the calculation of the area under a curve, Seki became close to what developed as calculus in the West. Recreational problems concern mostly magic squares and circles of a intricate nature and the *mamakodate* problem, known in the West as the Josephus or Christians and Turks problem. Possibly this last one has been imported into Japan by Portuguese Jesuits but it appears that the problem was known in Japan already from the tenth century! And finally there is algebra which deserves our closer attention.

We can rephrase our question about an alternative mathematics with respect to algebra. Did algebraic solutions to arithmetical and geometrical problems within *wasan* develop into a symbolic algebra as it did in Europe? Opinions on this differ widely, also among Japanese scholars. Ken'Ichi Sato devoted an article to the comparison of *wasan* algebra with its Chinese predecessor and concludes that the Japanese version cannot be considered algebra at all and that it "should be recognized as a category independent on that of Western mathematics" (Sato 1995, 65). Chikara Sasaki, on the other hand, in an article comparing Japanese with Western algebra, concludes that precisely because of its symbolic characteristics "Seki's school of algebra can be compared with that of Viète" (Sasaki 1999, 19). We tend closer to the second position but the situation is more complex than that.

Japanese algebra is derived from the so-called *celestial element method* in China. It first appeared in a Chinese work of 1248 by Li Ye (or Li Shi), *Sea-mirror of the circle measurements*. More important for Japan however was a book written in 1299 and titled The *Introduction to mathematical studies* (*Suan xue qi meng* (SQ), 算学啓蒙) (Lam Lay-Yong, 1979). The Chinese edition of the *SQ* was lost for a long time until 1839 when a Korean reprint of 1660 was discovered. Though lost in China, it was translated into Japanese in 1658, printed in several editions and widely read. The original Chinese method solves higher degree equations by laying out and manipulating the coefficients of the unknown terms using colored rods, called *sangi*, on a grid. Black rods represent positive values, red ones negatives. In Japan the method was called *tengen jutsu*. Early Japanese texts follow the Chinese method but print the rods as Chinese characters for numbers. For example, the equation $11520 - 432x - 236x^2 + 4x^3 + x^4 = 0$ is represented (using Hindu-Arabic numerals) as in table 1.

TABEL 1: THE REPRESENTATION OF AN EQUATION BY COEFFICIENTS ON A GRID

10000	1000	100	10	1		
1	1	5	2	0		実
	–	4	3	2	x	方
	–	2	3	6	x^2	廉
				4	x^3	偶
				1	x^4	

situation in Germany on the study of old arithmetic books, concentrated around the activities of the Adam Ries museum in Annaberg-Buchholz.

So, moving from a tangible form of performing algebra to a printed representation involves already a step towards symbolization. The rods, as material means, were replaced by Chinese characters. The red colored rods become replaced by a negative sign (a diagonal dash).

A further development introduced by Seki was the method of side writing. The idea probably evolved from the custom of annotating Chinese characters with hiragana (the Japanese phonetic script) to show how to pronounce them. At some point Seki started adding literals to the numerical coefficients in the grids to tackle more general problems. For example: given a rectangle with an area of A and the sum of its length and width being B, how to determine the length and width. Take x (the celestial element) to be the length, then the width will be $(B - x)$ and the area $x(B - x) = A$. The coefficients to be placed on the grid would then be -1, 1B and -1A respectively, representing the equation $-x^2 + Bx - A = 0$. While the method allowed expressing complex general problems in equations, it was kept secret within Seki's school. In de *Hatsubi Sanpō* of 1674 Seki solved 15 problems posed in an earlier book without revealing his method of side writing. It was his student Katahiro Takebe who would first explain the method in a commentary published in 1687.

It may be all too easy to draw similarities between Seki and Viète on basis of their use of generalized coefficients. Important differences withhold us to attribute the qualification of symbolic algebra to *wasan*. With Viète, European algebra evolved away from problem solving to the study of the structure of equations. Important discoveries such as the relations between de roots of an equation and its coefficients, the number of roots and the degree of an equation, and the relation between algebra and geometry are missing in *wasan*. By the end of the sixteenth century the foundations of European algebra became completely transformed as part of the humanist program of providing more solid foundations based on Greek roots. Algebra became a normative model for a *mathesis universalis*, a universal symbolic language to approach the laws of nature, and as such had a decisive influence on the development of Western science. In contrast, *wasan* contributed nothing to the understanding of nature. Despite its ability to solve complex problems in an efficient way, the *wasan* tradition failed to produce a single law on mechanics, hydraulics or kinematics. Even when Dutch books on Newtonian physics became available during the eighteenth century, Japanese scholars did not resort to *wasan* methods of algebra for its mathematical foundations and instead struggled with ways to represent Western symbols (Ravina 1993, 213). The failure of embedding *wasan* within other intellectual traditions made it possible to replace it by Western mathematics in a very short period during the Meiji restoration.

The current growth of interest in wasan is not only from a historical perspective. The rich history of *wasan* and its enormous legacy of woodblock printed books provide ample opportunities of using historical material in the classroom. Being an integral part of Japanese culture and its close relation with religion (*sangaku*) makes *wasan* an excellent example of the use of history for mathematics eduction. Several projects have been started along this line (Leung e.a. 2006). Specific lessons have been developed by specialized institutes as the CRICED from Tsukuba University.[5] Placing traditional *wasan* methods of problem solving against their Western counterparts provides students with a better understanding of the development of mathematics and the way mathematics is part of our social and cultural legacy.

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Announcements of events

ICME 11 Mexico 2008
11th International Congress on Mathematical E

ICME-11
July 6-13, 2008
Monterrey, Mexico
http://www.icme11.org.mx/icme11/

of special interest is the
TSG 23: The role of history of mathematics in mathematics education
See HPM Newsletter 67 or
http://tsg.icme11.org/tsg/show/24
for more information on this.

HPM 2008
History and Pedagogy of Mathematics
The HPM Satellite Meeting of ICME 11
July 14-18, 2008
Mexico City, Mexico
See First Announcement in Newsletter 66. The Second Announcement is available on the website: http://www.red-cimates.org.mx/HPM2008.htm

Plenary Activities:
The following plenary lectures will be given:

David Pengelley (New Mexico State University, USA): *Teaching mathematics with primary historical sources: Should it go mainstream? Can it?*

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