A Formal Explication of the Search for Explanations

The Adaptive Logics Approach to Abductive Reasoning

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Summary. Most logic–based approaches characterize abduction as a kind of *backwards deduction plus additional conditions*, which means that a number of conditions is specified that enable one to decide whether or not a particular abductive inference is sound (one of those conditions may for example be that abductive consequences have to be compatible with the background theory). Despite the fact that these approaches succeed in specifying which formulas count as valid consequences of abductive inference steps, they do not explicate the way people actually reason by means of abductive inferences. This is most clearly shown by the absence of a decent proof theory. Instead, search procedures are provided that enable one to determine the right abductive consequences. However, these do not by far resemble human reasoning.

In order to explicate abductive reasoning more realistically, an alternative approach will be provided in this paper, viz. one that is based on the *adaptive logics programme*. Proof theoretically, this approach interprets the argumentation schema *affirming the consequent* (**AC**: $A \supset B, B \vdash A$) as a defeasible rule of inference. This comes down to the fact that the abductive consequences obtained by means of **AC** are accepted only for as long as certain conditions are satisfied — for example, as long as their negation hasn't been derived from the background theory. In the end, only the *unproblematic* applications of **AC** are retained, while the problematic ones are rejected. In this way, the adaptive logics approach to abduction succeeds to provide a more realistic explication of the way people reason by means of abductive inferences. Moreover, as multiple abduction processes will be characterized, this paper may be considered as the first step in the direction of a general formal approach to abduction based on the adaptive logics programme.

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1 Introduction

When searching an explanation for a (puzzling) phenomenon, people often reason backwards, from the phenomenon to be explained to possible explanations. When they do so, they perform abductive inferences, inferences based on the argumentation schema known as *affirming the consequent*:

(**AC**) $A \supset B, B \vdash A$

Clearly, **AC** is not deductively valid. In the context of *classical logic* (**CL**), its acceptance would even lead to triviality. Nevertheless, people make use of **AC**. Though, to avoid the derivation of unsound consequences, they do so in a defeasible way. As a consequence, abductive explanations remain provisional, and, in the end, some are rejected. The reasons for doing so may be external or internal to the background theory from which these explanations were derived. In the external case, new information is obtained that forces the rejection of some abductive explanations — for example, in case the results of further research are incompatible with these explanations. In formal terms: abductive reasoning is non–monotonic. In the internal case, new (deductive) consequences are derived from the background theory that necessitate the rejection of some abductive explanations — for example, in case it turns out the background theory already provides a perfectly good explanation for the phenomenon at hand. This kind of (internal) defeasibility results from the fact that people lack logical omniscience (people do not have complete insight in the theories they reason from). As such, when a better insight is gained in those theories, some of the earlier drawn consequences might have to be withdrawn.

Most logic–based approaches characterize abduction as a kind of *backwards deduction plus additional conditions* — see e.g. Aliseda–Llera [1, 2], Mayer&Pirri [3, 4], McIlraith [5], and Paul [6]. In these approaches, a number of conditions is specified that enable one to decide whether or not a particular abductive inference is sound. Moreover, different kinds of abduction are characterized by different sets of such conditions. For example, in table 1, the conditions are stated that were given in Aliseda–Llera [1, pp. 48–49] to characterize abductive reasoning that is (in terms of Aliseda–Llera) both consistent and explanatory.

Given Θ (a set of formulae) and ϕ (a sentence), α is a consistent and explanatory abductive explanation of ϕ iff

(i)	$\Theta \cup \{\alpha\} \vdash \phi$
(ii)	$\Theta \nvdash \neg\alpha$
(iii)	$\Theta \nvdash \phi$
(iv)	$\alpha \nvdash \phi$

Table 1. Consistent and Explanatory Abductive Explanation

Although the traditional logic–based approaches to abduction succeed in specifying which formulas may count as valid consequences of abductive inference steps, they do not explicate the way in which people actually reason by means of abductive inferences — hence, the focus is on abductive consequence, not on abductive reasoning. This is most clearly shown by the absence of a decent proof theory, i.e. a proof theory that explicates abductive reasoning steps as describe above, namely as defeasible applications of the inference rule **AC**. Instead, search procedures are provided that enable one to determine the right abductive consequences — for example, the tableaux methods presented in Aliseda–Llera [1, 2] and Mayer&Pirri [3, 4]. However, these explicate **AC** only at the semantic or the metatheoretic level. As a consequence, these search procedures do not resemble human reasoning at all.[2]

In order to explicate abductive reasoning more realistically, an alternative approach will be provided in this paper, viz. one that is based on the *adaptive logics programme*.[3] In the accompanying proof theory, the argumentation schema **AC** is really interpreted as a defeasible rule of inference. More specifically, the consequences obtained by applying **AC** are accepted only for as long as certain conditions are satisfied — for example, as long as their negation hasn't been derived from the background theory. In short, adaptive logics for abduction only retain the unproblematic *applications* of **AC**, while they reject the problematic ones. Hence, in comparison to the traditional logic–based approaches, the adaptive logics approach more realistically captures the way people make abductive inferences. Nonetheless, I will show that all conditions stated by the traditional logic–based approaches are still satisfied by the adaptive logics approach. Finally, as the adaptive logics approach is not restricted to a particular kind of abduction process (multiple kinds of abduction will be explicated), this paper should be considered as the first step in the direction of a general approach towards the explication of abductive reasoning.

2 The Deductive Frame

As spelled out in the previous section, abduction validates some arguments that are not deductively valid — in casu, applications of **AC**. Hence, abductive reasoning goes beyond deductive reasoning. Nevertheless, abduction is constrained by deductive reasoning, for some abductive consequences of a premise set might have to be withdrawn in view of its deductive consequences — for example, in case these abductive consequences are incompatible with the deductive ones. Hence, abduction and deduction go hand in hand, the latter serving as the *deductive frame* of the former.

[2] In some cases, one might even doubt whether these search procedures even obtain the right abductive consequences — see Meheus&Provijn [7].

[3] A thorough introduction into adaptive logics can be found in Batens [8, 9], and an overview of the adaptive logics programme can be found on the Adaptive Logics Homepage (`http://logica.ugent.be/adlog`).

A Modal Frame

In most logic–based approaches to abduction, deductive reasoning is explicated by means of *classical logic* — see e.g. Aliseda–Llera [1, 2], Meheus&Batens [10], and Meheus [11]. In this paper though, the deductive frame is captured by the modal logic **RBK**.[4] The latter will be characterized in full below, both semantically as well as proof theoretically.

Language Schema. The logic **RBK** is a standard bimodal logic extending (propositional) classical logic with the modal operators $\Box_n$ and $\Box_e$. As a consequence, the modal language $\mathcal{L}^{\mathcal{M}}$ of **RBK** is obtained by adding both these necessity operators, together with the corresponding possibility operators, to the standard propositional language $\mathcal{L}$ (see table 2 for an overview). The set of well–formed formulas $\mathcal{W}^{\mathcal{M}}$ of the language $\mathcal{L}^{\mathcal{M}}$ is defined in the usual way.

Language	Letters	Logical Symbols	Set of Formulas
$\mathcal{L}$	$\mathcal{S}$	$\neg, \wedge, \vee, \supset, \equiv$	$\mathcal{W}$
$\mathcal{L}^{\mathcal{M}}$	$\mathcal{S}$	$\neg, \wedge, \vee, \supset, \equiv, , \Box_n, \Diamond_n, \Box_e, \Diamond_e$	$\mathcal{W}^{\mathcal{M}}$

Table 2. The Languages $\mathcal{L}$ and $\mathcal{L}^{\mathcal{M}}$

In the remaining of this paper, only negation, disjunction, and both necessity operators are taken as primitive. The other logical symbols are defined in the standard way.

Semantic Characterization. An **RBK**–model M is a 5–tuple $< W, w_0, R^n, R^e, v >$. The set W is a (non–empty) set of worlds, with $w_0 \in W$ the actual world. R^n and R^e are both accessibility relations on W, the former of which is both reflexive and transitive, while the latter is merely reflexive. Moreover, the following relation holds between both accessibility relations:

CEI For all $w, w' \in W$: if $R^e ww'$ then also $R^n ww'$.

Finally, v is an assignment function, for which the following condition holds:

C1.1 v: $\mathcal{S} \times W \mapsto \{0, 1\}$.

The valuation function v_M, determined by the model M, is now defined as follows:

C2.0 v_M: $\mathcal{W}^{\mathcal{M}} \times W \mapsto \{0, 1\}$.
C2.1 Where $A \in \mathcal{S}$, $v_M(A, w) = 1$ iff $v(A, w) = 1$.
C2.3 $v_M(\neg A, w) = 1$ iff $v_M(A, w) = 0$.

[4] Actually, in Meheus et al. [12], the deductive frame is captured by the logic $\mathbf{S5^2}$ that is quite similar to the logic **RBK**.

C2.4 $v_M(A \vee B, w) = 1$ iff $v_M(A, w) = 1$ or $v_M(B, w) = 1$.
C2.5 $v_M(\Box_n A, w) = 1$ iff for all $w' \in W$: if $R^n ww'$ then $v_M(A, w') = 1$.
C2.6 $v_M(\Box_e A, w) = 1$ iff for all $w' \in W$: if $R^e ww'$ then $v_M(A, w') = 1$.

A model M verifies a formula $A \in \mathcal{W}^{\mathcal{M}}$ iff $v_M(A, w_0) = 1$. Moreover, a model M is a model of a premise set Γ iff, for all $B \in \Gamma$, $v_M(B, w_0) = 1$.

Definition 1. $\vDash_{\mathbf{RBK}} A$ *(A is valid) iff A is verified by all* **RBK***–models.*

Definition 2. $\Gamma \vDash_{\mathbf{RBK}} A$ *(A is a semantic consequence of Γ) iff all* **RBK***–models of Γ verify A.*

Some remarks seem to be necessary. First of all, as the accessibility relation R^n is both reflexive and transitive, the modal operator $\Box_n$ corresponds to the necessity operator of the (normal) modal logic **S4**. Secondly, the accessibility relation R^e is reflexive but not transitive, meaning that the modal operator $\Box_e$ corresponds to the necessity operator of the (normal) modal logic **KT**. Finally, because of the specific relation between R^n and R^e, as expressed by the condition **CEI**, the truth of a formula $\Box_n A$ in a world w yields the truth of the formula $\Box_e A$ in that world. In the proof theoretic characterization below, this is expressed by the axiom schema **AEI** (see table 3).

Proof Theoretic Characterization. The **RBK**–proof theory is obtained by adding the axiom schemas, inference rules, and definitions stated in table 3 to the axiom system of (propositional) classical logic.

AM1n	$\Box_n(A \supset B) \supset (\Box_n A \supset \Box_n B)$	AM1e	$\Box_e(A \supset B) \supset (\Box_e A \supset \Box_e B)$
AM2n	$\Box_n A \supset A$	AM2e	$\Box_e A \supset A$
AM3n	$\Box_n A \supset \Box_n \Box_n A$		
AEI	$\Box_n A \supset \Box_e A$		
NECn	From $\vdash A$ conclude to $\vdash \Box_n A$	NECe	From $\vdash A$ conclude to $\vdash \Box_e A$
Dfn	$\Diamond_n A =_{df} \neg\Box_n \neg A$	Dfe	$\Diamond_e A =_{df} \neg\Box_e \neg A$

Table 3. Additional Axiom Schemas, Rules, and Definitions of **RBK**

Soundness and Completeness. As both soundness and completeness for **RBK** are proven by standard means, the proofs are left to the reader.

Theorem 1. $\Gamma \vdash_{\mathbf{RBK}} A$ *iff* $\Gamma \vDash_{\mathbf{RBK}} A$.

Representing Abductive Contexts

Because of the higher expressive power of the **RBK**–language $\mathcal{L}^{\mathcal{M}}$ (as compared to the language of classical logic), the logic **RBK** not only enables one to capture deductive reasoning as such, but also enables one to capture some intensional elements of reasoning contexts.

Background Knowledge. The modal operators $\Box_n$ and $\Box_e$ will be used to express both the *nomological* and *empirical background knowledge* held by a reasoner in a reasoning context. First, the nomological background knowledge is represented by elements of the set $\mathcal{W}^{\mathcal{N}} \subset \mathcal{W}^{\mathcal{M}}$ — see definition 3. More specifically, a formula $\Box_n A \in \mathcal{W}^{\mathcal{N}}$ states that A is considered a nomological fact by the reasoner in the reasoning context at hand. Second, the empirical background knowledge is represented by elements of the set $\mathcal{W}^{\mathcal{E}} \subset \mathcal{W}^{\mathcal{M}}$ — see definition 4. A formula $\Box_e A \in \mathcal{W}^{\mathcal{E}}$ states that A is taken to be an empirical fact by the reasoner in the reasoning context.

Definition 3. $\mathcal{W}^{\mathcal{N}} = \{\Box_n A \mid A \in \mathcal{W}\}$.

Definition 4. $\mathcal{W}^{\mathcal{E}} = \{\Box_e A \mid A \in \mathcal{S} \cup \mathcal{S}^{\neg}\}$.[5]

Two remarks are needed at this point. Firstly, in view of axiom schema **AEI** (see table 3), nomological background knowledge can be combined with empirical background knowledge in order to derive further empirical information — for example, in the process of making predictions. Hence, nomological information may be said to have *empirical impact*.

More importantly, one might wonder what the modal operators are taken to express. In accordance with the epistemological framework presented in Batens [13, 14], the elements of the background knowledge are classified as (part of the) *contextual certainties* and *relevant premises* of a given context (which is defined as a problem–solving situation). Without going into the details, both the contextual certainties and the relevant premises of a context are considered as true in that context, and helpful in order to solve the problem at hand (for more details, the reader is referred to the cited literature). Hence, the necessity operators occurring in the elements of $\mathcal{W}^{\mathcal{N}}$ and $\mathcal{W}^{\mathcal{E}}$ capture the fact that the elements of the background knowledge are considered as unproblematic in the given context (thus, the necessities have to be interpreted epistemologically, not ontologically).

Abductive Contexts. The contexts considered in this paper are *abductive contexts*, viz. problem–solving situations in which possible explanations are sought for puzzling (empirical) phenomena. Given the above elaboration of the meaning of the modal operators, the elements of the background knowledge are

[5] Firstly, The elements of the set $\mathcal{S}^{\neg}$ are the negations of the elements of the set $\mathcal{S}$ — $\mathcal{S}^{\neg} =_{df} \{\neg A \mid A \in \mathcal{S}\}$. Secondly, one of the anonymous referees rightly remarked that also conjunctions of elements of the set $\mathcal{S} \cup \mathcal{S}^{\neg}$ may be considered as empirical facts, so that the empirical background knowledge is better explicated by the set $\mathcal{W}^{\mathcal{E}'} = \{\Box_e(A_1 \wedge ... \wedge A_n) \mid A_1, ..., A_n \in \mathcal{S} \cup \mathcal{S}^{\neg}\}$. However, the elements of $\mathcal{W}^{\mathcal{E}'}$ are all derivable from the elements of $\mathcal{W}^{\mathcal{E}}$ by means of the logics presented in this paper. For the purposes of this paper, this implies that replacing $\mathcal{W}^{\mathcal{E}}$ by $\mathcal{W}^{\mathcal{E}'}$ wouldn't make a difference. Hence, to keep things as simple as possible, I will stick to $\mathcal{W}^{\mathcal{E}}$.

considered unproblematic, which in abductive contexts means that they are not in need of any explanation.

Besides background knowledge, abductive contexts obviously also contain some elements that are in need of an explanation. The latter will be represented by elements of the set $\mathcal{W}^{\mathcal{O}}$, i.e. the set of observed (empirical) phenomena — see definition 5.

Definition 5. $\mathcal{W}^{\mathcal{O}} = \mathcal{S} \cup \mathcal{S}^{\neg}$.

Surely, not all elements of $\mathcal{W}^{\mathcal{O}}$ express (empirical) phenomena in need of an explanation (henceforth, puzzling phenomena). Nonetheless, a formula $A \in \mathcal{W}^{\mathcal{O}}$ will be taken to express a puzzling phenomenon in an abductive context in case A is not considered as unproblematic by the reasoner in that abductive context — in other words, in case $\Box_e A$ is not derivable in that abductive context (for otherwise, A would be considered as unproblematic by the reasoner).

Final Remark. In the remaining of this paper, premise sets will be taken to express abductive contexts. As such, premise sets will be restricted to formulas that express the background knowledge of a reasoner (i.e. elements of $\mathcal{W}^{\mathcal{N}} \cup \mathcal{W}^{\mathcal{E}}$) and formulas that express observed (empirical) phenomena (i.e. elements of $\mathcal{W}^{\mathcal{O}}$). Obviously, in case the latter express puzzling phenomena, they will trigger abductive inferences in the adaptive logics I will propose later on (see section 4). However, I first need to discuss the abductive inference steps themselves.

3 On Defeasible Inference

As stated in section 1, abductive inference steps are formally captured by the (defeasible) inference rule **AC**. However, because the deductive frame is captured in modal terms (as set out in the previous section), abductive inference has to be captured in modal terms as well. As a consequence, the inference rule **AC** will be restricted to the following schema (A and B are formulas, and Δ is a set of formulas):

($\mathbf{AC^m}$) $\Box_n(A \supset B),\ B,\ \Delta \vdash A$

Some clarification is called for. First of all, $\mathbf{AC^m}$ expresses that a formula A may only be considered as a possible explanation for a phenomenon B, in case there is a nomological statement $\Box_n(A \supset B)$ expressing the dependence of B upon A. As a consequence, to capture abductive inference by means of the inference rule $\mathbf{AC^m}$ in a sense resembles Hempel's account of explanation — see Hempel&Oppenheim [15].

Secondly, the explanandum B may not be part of the empirical background knowledge (i.e. B isn't allowed to be a modal formula of the form $\Box_e C$), for otherwise it cannot be considered to trigger abductive inferences (remember that the background knowledge is here taken to be accepted beyond doubt, and hence, in no need of explanation)!

Finally, dependent on the particular abduction process one intends to capture, certain additional conditions have to be satisfied before the defeasible inference rule $\mathbf{AC^m}$ may be applied. In the representation of $\mathbf{AC^m}$ above, these conditions are represented by the elements of Δ. Some important remarks have to be made with respect to Δ. Firstly, the elements of Δ capture some of the conditions stated by the traditional logic–based approaches to abduction (see section 1). This is not surprising as both approaches tend to capture the same reasoning patterns, albeit by distinct means. Secondly, the elements of Δ can only be presumed in a defeasible way themselves, viz. for as long as there is no information that forces us to reject them. As a consequence, the elements of Δ are also obtained by means of defeasible inference rules. In this paper, I will only discuss the inference rules below, the consequences of which will be used to express respectively that an abductive explanation is minimal (**NNN**), and that the explanandum is not derivable from the background knowledge alone (**NEN**):

(**NNN**) $\vdash \neg\Box_n(A \supset B)$
(**NEN**) $\vdash \neg\Box_e A$

How the consequences of these inference rules are used to capture some of the conditions put forward by the traditional logic–based approaches to abduction, will be explicated in full later on (in section 4). Finally, remark that the defeasible inferences above are actually prior to abductive inferences, for the consequences of these additional inference steps are necessary to be able to apply $\mathbf{AC^m}$. Hence, abduction processes are characterized as layered processes, viz. as specific combinations of multiple defeasible inference steps. This means that the adaptive logics that will be characterized in the following section and that explicate these abduction processes will be so–called *prioritized adaptive logics*, i.e. adaptive logics that are obtained by combining defeasible inference rules in a certain way — see e.g. Batens [8], and Batens et al. [16].

4 Enter Adaptive Logics

In order to present the adaptive logics approach to abductive reasoning, some prioritized adaptive logics capturing specific abduction processes will be characterized, viz. the prioritized adaptive logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$. The difference between these logics comes down to the following: in case there are multiple possible explanations for a phenomenon, $\mathbf{AbL^p}$ will merely enable one to derive the disjunction of these possible explanations (*practical abduction*), while $\mathbf{AbL^t}$ will enable one to derive all possible explanations (*theoretical abduction*).[6]

[6] For an intuitive justification of these abductive processes, see Meheus&Batens [10, pp. 224–225].

Previous Attempts. This is not the first attempt to explicate abductive reasoning by means of the adaptive logics programme. Despite the fact that some nice results were obtained, the earlier attempts remained unsatisfactory. In Meheus et al. [12], a proof theory was provided for the traditional logic–based approaches to abduction. It is based on the characteristics of the adaptive logics–proof theory, but also incorporates some extra–logical features. As such, only a proof theory for abduction was provided, not a formal logic. On the other hand, in Meheus&Batens [10] and Meheus [11], two (actual) adaptive logics were provided to explicate abduction, viz. the logics $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$. Nonetheless, $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$ only capture abductive reasoning in a limited way. First of all, these logics don't allow abductive inferences at the purely propositional level. Secondly, only practical abduction could be characterized by the approach presented in [10] and [11], which is most likely due to the fact that the deductive frame of $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$ is based on classical logic. Thirdly, the logics $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$ lack some properties that seem to be necessary to capture abductive explanation in a decent way. Most importantly, in case an explanandum is explained by the background theory alone, $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$ go on to provide possible abductive explanations, despite the fact that none are needed.[7] Neither of these shortcomings also applies to the approach presented in this paper.

The Standard Format

All adaptive logics (**AL**) have a uniform characterization. This characterization is called the *standard format* of adaptive logics and was presented most thoroughly in Batens [8, 9]. The main advantage of the standard format consists in the fact that all **AL** characterized accordingly have a common semantic and proof theoretic characterization. Moreover, a lot of metatheoretic properties have been proven for **AL** in standard format (most importantly, soundness and completeness).[8] Below, I will first give a general characterization of the standard format. Afterwards, I will present the proof theory of **AL** in standard format. The semantics of **AL** in standard format will not be spelled out. Nothing fundamental is lost though, for the focus of this paper is on the proof theory (see section 1). Moreover, the interested reader can find the semantic characterization of **AL** in standard format in Batens [8, 9] and Batens et al. [17].

General Characterization

All *flat* adaptive logics in standard format are characterized by means of the following three elements:

[7] To be fair, in a lecture at the University of Utrecht (20 Octobre 2009), Joke Meheus showed how to overcome the second shortcoming of $\mathbf{LA^r}$ and $\mathbf{LA^r_s}$. Nevertheless, the other shortcomings still remain and are not likely to be overcome anytime.

[8] Proofs for these metatheoretic properties are provided in Batens [9].

- A *lower limit logic* (**LLL**): a reflexive, transitive, monotonic, and compact logic that has a characteristic semantics (with no trivial models) and contains classical logic.
- A *set of abnormalities* Ω: a set of formulas characterized by a (possibly restricted) logical form `F` that is **LLL**–contingent and contains at least one logical symbol.
- An *adaptive strategy*.

Remark that the **AL** that will be presented later on are not flat **AL**, but prioritized adaptive logics (**PAL**). The latter can also be characterized by means of the standard format, though some slight modifications are necessary.[9] More specifically, the set of abnormalities Ω is replaced by a structurally ordered sequence $\Omega_>$ of sets of abnormalities:

Definition 6. $\Omega_> = \Omega_1 > \Omega_2 > ...$[10]

The order imposed on the sequence $\Omega_>$ expresses a priority relation: in case $\Omega_i > \Omega_j$, the priority of the elements of Ω_i is higher than the priority of the elements of Ω_j. For reasons of convenience, I will use Ω to refer to the union of the sets $\Omega_1, \Omega_2, ...$, while $\Omega_>$ will be used to refer to the sequence $\Omega_1 > \Omega_2 > ....$

The Adaptive Consequence Relation. The adaptive consequences of a premise set are obtained by the interplay between the three constituting elements of a (prioritized) adaptive logic. This will be explicated by characterizing the **PAL**–consequence relation in general. Where the expression $Dab(\Delta)$ is used to represent a finite disjunction of abnormalities (elements of Ω), the **PAL**–consequence relation is defined as follows:

Definition 7. *$\Gamma \vdash_{\mathbf{PAL}} A$ iff there is a finite $\Delta \subset \Omega$ such that $\Gamma \vdash_{\mathbf{LLL}} A \vee Dab(\Delta)$ and $\mathrm{FALSE}_{\mathrm{AS}}(\Delta)$.*

The above definition tells us that a formula A is **PAL**–derivable from a premise set Γ iff $A \vee Dab(\Delta)$ is **LLL**–derivable from Γ and Δ satisfies the additional condition $\mathrm{FALSE}_{\mathrm{AS}}(\Delta)$. Intuitively, the latter means that one is allowed to derive A from $A \vee Dab(\Delta)$ in case all elements of Δ can safely be interpreted as *false* — metaphorically, one might consider this as a metatheoretic application of disjunctive syllogism. As a consequence, abnormalities are falsified as much as possible. In other words, premise sets are interpreted as normally as possible with respect to some standard of normality.

Definition 7 has some interesting consequences. In case $\Delta = \emptyset$, no abnormalities have to be falsified in order to derive the formula A from the premise

[9] Prioritized adaptive logics are well–studied in the literature, see e.g. Batens [8], Batens et al. [16], and Verhoeven [18].

[10] No upper bound is necessary, as is most clearly explained in [8, pp. 52–54]. However, all **PAL** that will be considered in this paper do have an upper bound.

set Γ. Hence, in case the formula A is a **LLL**–consequence of Γ, it is an adaptive consequence of Γ as well. In general, this implies that a (prioritized) adaptive logic derives more consequences from a premise set than the lower limit logic it is based on (more specifically, the adaptive consequence set of a premise set is a superset of the **LLL**–consequence set of that premise set). On the other hand, in case $\Delta \neq \emptyset$, the formula A is only an adaptive consequence of the premise set, in case all elements of Δ may be interpreted as false (if not, the formula A cannot safely be interpreted as true). For as long as it hasn't been determined whether or not all elements of Δ may indeed be interpreted as false, the formula A is called a *conditional consequence* of the premise set Γ. Obviously, this intermediate phase of conditional acceptance of consequences corresponds to the proof theoretic derivation of consequences by means of defeasible inference rules.

Whether a conditional consequence of a premise set is a final consequence as well, depends on the condition $\text{FALSE}_{\text{AS}}(\Delta)$. Whether this condition is satisfied for a particular Δ, is determined by the adaptive strategy of an adaptive logic. For the adaptive logics I will present below, this is either the *reliability* strategy or the *normal selections* strategy.[11] Both strategies base the decision to reject (or to retain) a conditional consequence of a premise set on the minimal *Dab*–consequences of that premise set — see definition 8. In advance though, it is important to notice that the minimal *Dab*–consequences of a premise set are defined in a *stepwise* manner: where the expression $Dab^i(\Delta)$ is used to represent finite disjunctions of abnormalities of priority i (elements of Ω_i), the minimal *Dab*–consequences of the form $Dab^1(\Delta)$ are determined first, then the minimal *Dab*–consequences of the form $Dab^2(\Delta)$,...

Definition 8. *$Dab^i(\Delta)$ is a minimal Dab–consequence of a premise set Γ iff (1) there is a finite $\Theta \subset \Omega_1 \cup ... \cup \Omega_{i-1}$ such that $\Gamma \vdash_{\mathbf{LLL}} Dab^i(\Delta) \vee Dab(\Theta)$, (2) there is no $\Sigma \subset \Omega_j$ such that $\Omega_j > \Omega_i$, $Dab^j(\Sigma)$ is a minimal Dab–consequence of Γ, and $\Sigma \cap \Theta \neq \emptyset$, and (3) there is no $\Delta' \subset \Delta$ such that (1) and (2) apply to $Dab^i(\Delta')$ as well.*

Not all abnormalities occurring in a minimal *Dab*-consequence of a premise set, may be interpreted as false. Hence, some of the conditional consequences derived by interpreting certain of these abnormalities as false, have to be rejected. First of all, the reliability strategy will reject all conditional consequences that were derived by interpreting some of the abnormalities occurring in a minimal *Dab*–consequence as false. As a consequence, the condition $\text{FALSE}_{\text{AS}}(\Delta)$ for the reliability strategy (henceforth, $\text{FALSE}_{\text{R}}(\Delta)$) is defined as follows:

[11] For more information on alternative adaptive strategies, see e.g. Batens [9] (for the *minimal abnormality* strategy), and Meheus&Primiero [19] (for the *counting* strategy).

Definition 9. *For $\Delta \subset \Omega$,* $\text{FALSE}_{\text{R}}(\Delta)$ *iff, for all Ω_i in $\Omega_>$, there is no finite $\Theta \subset \Omega_i$ such that $Dab^i(\Theta)$ is a minimal Dab–consequence of Γ and $\Theta \cap \Delta \neq \emptyset$.*

On the other hand, the normal selections strategy will only reject those conditional consequences that were derived by interpreting as false all abnormalities of a minimal *Dab*–consequence of the premise set. In other words, the condition $\text{FALSE}_{\text{AS}}(\Delta)$ for the normal selections strategy (henceforth, $\text{FALSE}_{\text{NS}}(\Delta)$) is defined as follows:

Definition 10. *For $\Delta \subset \Omega$,* $\text{FALSE}_{\text{NS}}(\Delta)$ *iff, for all Ω_i in $\Omega_>$, there is no finite $\Theta \subset \Omega_i$ such that $Dab^i(\Theta)$ is a minimal Dab–consequence of Γ and $\Theta \subset \Delta$.*

Dynamic Behavior. Because of the conditional status of some of the **PAL**–consequences, **PAL** display an external as well as an internal dynamics. Firstly, the external dynamics comes down to non–monotonicity: if the premise set is extended, some conditionally derived **PAL**–consequences of the premise set may not be derivable anymore. Secondly, the internal dynamics is a strictly proof theoretic feature: growing insights in the premises, obtained by deriving new consequences from the premises (in casu *Dab*–consequences), may lead to the withdrawal of earlier reached conclusions, or to the rehabilitation of earlier withdrawn conclusions.

The dynamic behavior of **PAL** resembles the dynamics present in abductive reasoning (see section 1). Consequently, **PAL** seem particularly well–suited to explicate abductive reasoning.

Proof Theory

As **PAL** are standard adaptive logics, the **PAL**–proof theory has some characteristic features shared by all adaptive logics. First of all, a **PAL**–proof is a succession of stages, each consisting of a sequence of lines. Adding a line to a proof means to move on to the next stage of the proof. Secondly, the lines of a **PAL**–proof consist of four elements (instead of the usual three): a line number, a formula, a justification, and an adaptive condition. The latter is a finite subset of Ω (the union of the sets of abnormalities of a prioritized adaptive logic). Finally, the **PAL**–proof theory consists of both *deduction rules* and a *marking criterion.* Both of these will be discussed below.

Deduction Rules. The deduction rules determine how new lines may be added to a proof. Below, the deduction rules are listed in shorthand notation, with

$$A \qquad \Delta$$

expressing that the formula A occurs in the proof on a line with condition Δ.

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, ..., A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, ..., A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

The adaptive condition of a line i expresses that as long as all its elements can be considered as false, the formula on that line may be considered as derivable from the premise set. Secondly, in order to indicate that not all elements of the adaptive condition of a line i can be considered as false, line i is marked — formally, this is done by placing the symbol ✓ next to the adaptive condition. Obviously, when a line is marked, the formula on that line may not be considered as derivable anymore. Finally, the marking in a **PAL**–proof is dynamic: at some stage of the proof, a line might be marked (resp. unmarked), while at a later stage, it might become unmarked (resp. marked) again.

Marking Criterion. At every stage of a **PAL**–proof, the marking criterion determines which lines have to be marked. To determine whether a line has to be marked at a stage s of a **PAL**–proof, both the reliability strategy as well as the normal selections strategy first determine the *minimal Dab–consequences* of the premise set at stage s.

Definition 11. *$Dab^i(\Delta)$ is a minimal Dab–consequence of a premise set Γ at stage s of a proof, iff (1) $Dab^i(\Delta)$ occurs on an unmarked line k at stage s, (2) all members of the adaptive condition of line k belong to some Ω_j in the sequence $\Omega_>$ such that $\Omega_j > \Omega_i$, and (3) there is no $\Delta' \subset \Delta$ such that (1) and (2) apply to $Dab(\Delta')$ as well.*

It is important to notice that the minimal *Dab*–consequences of a premise set at a stage s are determined in a *stepwise* manner: first for priority level 1, then for priority level 2,...

Well now, the marking definitions for **PAL** based on the reliability strategy and the normal selections strategy are the following:

Definition 12 (Reliability). *Line i with adaptive condition Δ is marked at stage s iff $Dab^i(\Theta)$ is a minimal Dab–consequence of Γ at stage s, and $\Theta \cap \Delta \neq \emptyset$.*

Definition 13 (Normal Selections). *Line i with adaptive condition Δ is marked at stage s iff $Dab^i(\Theta)$ is a minimal Dab–consequence of Γ at stage s, and $\Theta \subset \Delta$.*

Defining Derivability. A formula is considered as derivable from a premise set Γ, in case it occurs as the second element of an unmarked line in a proof from Γ.

Definition 14 (Derivability). *The formula A is* derived *from Γ at stage s of a* **PAL***–proof iff A is the second element of an unmarked line at stage s.*

Because of the dynamic nature of adaptive proofs, markings may change at every stage. Hence, at every stage of a proof, it has to be reconsidered whether or not a formula is derivable. In other words, derivability is stage–dependent. Although this may seem problematic at first, it nevertheless reflects the way people treat abductive consequences. For, given that abductive consequences are provisional consequences, conclusions drawn by relying on abductive inference steps are hardly ever conclusive. Hence, at any moment, two options are available to people, viz. to keep on reasoning until conclusiveness has been reached or to base one's actions on the provisional conclusions. As the first option may take more time than available, the latter option will be the only viable one in a lot of cases.[12]

Besides a stage–dependent notion of derivability, a stable notion of derivability can be defined as well. It is called *final derivability*, which refers to the fact that, for some formulas, derivability is only decided at the final stage of a proof.

Definition 15 (Final Derivability). *The formula A is* finally derived *from Γ on line i of a* **PAL***–proof at stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) every extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked again.*

Because of its stability, the notion of final derivability is used to define **PAL**–derivability.

Definition 16. *$\Gamma \vdash_{\mathbf{PAL}} A$ (A is* **PAL***–derivable from Γ) iff A is finally derived on a line of a* **PAL***–proof from Γ.*

The Prioritized Adaptive Logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$

In this final section, I will characterize the prioritized adaptive logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$. First, I will show how these logics fit the standard format. Secondly, I will show that both logics characterize abductive reasoning as a combination of multiple defeasible inference rules. Thirdly, I will argue that the

[12] For a more extensive justification of this claim, see Batens et al. [20].

abductive consequences of both logics still satisfy the conditions put forward by the traditional logic–based approaches to abduction. To conclude, I will also point out the main difference between both logics.

Preliminary Remark. I will limit myself to the propositional fragment of both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$. However, extending these logics to their full predicate versions is completely straightforward, and hence, can safely be left to the reader.

Characterizing $\mathbf{AbL^p}$ *and* $\mathbf{AbL^t}$

As both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ are prioritized adaptive logics in standard format, they are characterized by means of a lower limit logic, an ordered sequence of sets of abnormalities, and an adaptive strategy. First, consider those characterizing the logic $\mathbf{AbL^p}$:

- The **LLL** of $\mathbf{AbL^p}$ is the modal logic **RBK** (see section 2).
- The abnormalities of $\mathbf{AbL^p}$ are characterized by the ordered sequence $\Omega_>$ $= \Omega_{bk} > \Omega_p$, with

$$\Omega_{bk} = \{ \Box_x A \mid x \in \{n, e\} \text{ and } A \in \mathcal{W} \}.$$
$$\Omega_p = \{ \Box_n(A \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg A \mid B \in \mathcal{S} \cup \mathcal{S}^\neg,\ A \text{ in Conjunctive Normal Form, and } B \text{ is not a subformula of } A \}.$$

- The adaptive strategy of $\mathbf{AbL^p}$ is the reliability strategy.

Next, consider the elements characterizing the logic $\mathbf{AbL^t}$. These differ from those of $\mathbf{AbL^p}$ with respect to the sequence $\Omega_>$ as well as with respect to the adaptive strategy.

- The **LLL** of $\mathbf{AbL^t}$ is the modal logic **RBK** (again, see section 2).
- The abnormalities of $\mathbf{AbL^t}$ are characterized by the ordered sequence $\Omega_>$ $= \Omega_{bk} > \Omega_t$, with

$$\Omega_{bk} = \{ \Box_x A \mid x \in \{n, e\} \text{ and } A \in \mathcal{W} \}.$$
$$\Omega_t = \{ \Box_n(A \supset B) \wedge \neg\Box_n B \wedge B \wedge \neg\Box_e B \wedge \neg A \mid A, B \in \mathcal{S} \cup \mathcal{S}^\neg, \text{ and } B \text{ is not a subformula of } A \} \cup \{ \Box_n((A_1 \wedge ... \wedge A_n) \supset B) \wedge \neg\Box_n((A_2 \wedge ... \wedge A_n) \supset B) \wedge \neg\Box_n((A_1 \wedge A_3 \wedge ... \wedge A_n) \supset B) \wedge ... \wedge \neg\Box_n((A_1 \wedge ... \wedge A_{n-1}) \supset B) \wedge B \wedge \neg\Box_e B \wedge \neg(A_1 \wedge ... \wedge A_n) \mid A_1, ..., A_n, B \in \mathcal{S} \cup \mathcal{S}^\neg, \text{ and } B \text{ is not a subformula of } A_1 \wedge ... \wedge A_n \}.$$

- The adaptive strategy of $\mathbf{AbL^t}$ is the normal selections strategy.

In view of the standard format of (prioritized) adaptive logics outlined above, a semantic or proof theoretic characterization for these logics need not be provided anymore.

A Formal Explication of Abductive Explanation

Earlier on, I stated that adaptive logics characterize abduction processes proof theoretically as layered processes — more specifically, as specific combinations of multiple defeasible inference rules. For the kinds of abductive explanation explicated by the logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$, these defeasible inference rules are **NEN**, **NNN**, and $\mathbf{AC^m}$ (see section 3).

Preliminary Remarks. Because of space limitations, all proofs presented below are $\mathbf{AbL^p}$– as well as $\mathbf{AbL^t}$–proofs. In order to make a clear distinction between both kinds of proofs, lines in a proof are given two adaptive conditions, one for each logic. Markings related to the logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ are placed next to the corresponding adaptive condition. Some abbreviations are introduced as well. First of all, $\Box_n(A \supset B) \wedge ... \in \Omega_p$ is abbreviated as $\langle A, B\rangle^p$. Analogously, $\Box_n(A \supset B) \wedge ... \in \Omega_t$ is abbreviated as $\langle A, B\rangle^t$. When the ambiguous $\langle A, B\rangle^{p/t}$ is used, $\langle A, B\rangle^p$ is meant in the $\mathbf{AbL^p}$–proof, while $\langle A, B\rangle^t$ is meant in the $\mathbf{AbL^t}$–proof. Finally, $\Omega_{i_1,...,i_n}$ is used to refer to the union of the adaptive conditions of lines $i_1, ..., i_n$.

Combining Defeasible Inference Rules. In order to show how $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ combine multiple defeasible inference rules to explicate abductive explanation, consider the adaptive proof below. It is based on the premise set $\Gamma = \{\Box_n(p \supset q), q\}$.

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	$\emptyset$
2	q	–;PREM	$\emptyset$	$\emptyset$

At this stage of the proof, the premises have been introduced. These clearly show that p is a possible explanation for q. In order to derive p as an abductive consequence of Γ, the formulas $\neg\Box_e q$ and $\neg\Box_n q$ have to be derived first. This is done as follows:

3	$\Box_e q \vee \neg\Box_e q$	–;RU	$\emptyset$	$\emptyset$
4	$\Box_n q \vee \neg\Box_n q$	–;RU	$\emptyset$	$\emptyset$
5	$\neg\Box_e q$	3;RC	$\{\Box_e q\}$	$\{\Box_e q\}$
6	$\neg\Box_n q$	4;RC	$\{\Box_n q\}$	$\{\Box_n q\}$

Both $\neg\Box_e q$ and $\neg\Box_n q$ are conditional consequences of the premise set Γ. As is shown below, their derivation is necessary in order to derive p as an abductive consequence of the premise set Γ.[13]

7	$p \vee \neg p$	–;RU	$\emptyset$	$\emptyset$
8	$p \vee \langle p, q\rangle^{p/t}$	1,2,5,(6,)7;RU	Ω_5	$\Omega_{5,6}$
9	p	8;RC	$\Omega_5 \cup \{\langle p, q\rangle^p\}$	$\Omega_{5,6} \cup \{\langle p, q\rangle^t\}$

[13] In the justification of line 8, the reference to line 6 is placed between brackets in order to express that it is only necessary in the $\mathbf{AbL^t}$–version of the proof.

At this stage of the proof, the formula p has been conditionally derived on line 9. Hence, as long as all elements of the adaptive condition of line 9 may be interpreted as false, p may be considered a (conditional) abductive consequence of the premise set Γ.

The first two applications of **RC** in the proof above, viz. those resulting in the conditional derivation of the formulas $\neg\Box_e q$ and $\neg\Box_n q$, clearly correspond to applications of the defeasible inference rules **NEN** and **NNN**. On the other hand, the third application of **RC** in the proof, viz. the one resulting in the conditional derivation of the formula p, corresponds to an application of the defeasible inference rule $\mathbf{AC^m}$. Remark that the formulas obtained by the first two applications of **RC** are required for the third application of **RC**. In other words, the formulas obtained by applying the inference rules **NEN** and **NNN** correspond to some of the conditions that have to be satisfied before the inference rule $\mathbf{AC^m}$ may be applied (see section 3). However, notice that $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ require slightly different conditions to be satisfied. For $\mathbf{AbL^p}$ only formulas of the form $\neg\Box_e A$ are required. For $\mathbf{AbL^t}$ on the other hand, also formulas of the form $\neg\Box_n A$ are required. In the proof above, this is clear from the fact that in the $\mathbf{AbL^t}$–version the formula on line 6 is necessary for the derivation of p on line 9, while it is not in the $\mathbf{AbL^p}$–version.

Meaning of the Additional Conditions. Let's have a closer look at the additional conditions captured by the consequences of the inference rules **NEN** and **NNN**. Firstly, consider the formulas of the form $\neg\Box_e A$ obtained by means of the defeasible inference rule **NEN**. Both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ only allow applications of $\mathbf{AC^m}$ in case $\neg\Box_e A$ is derivable for the explanandum A. The reason for this is quite simple: the formula $\neg\Box_e A$ guarantees that the explanandum A cannot be explained by means of the background theory alone. Hence, A is in need of an explanation and is allowed to trigger abductive inferences. It is easily verified that in case the explanandum A is derivable from the background theory alone, $\Box_e A$ will be derivable from the premises. As a consequence, $\neg\Box_e A$ will be withdrawn, as will all abductive consequences triggered by A. To illustrate this, consider again the proof above, but suppose that the formula $\Box_e p$ is added to the premise set Γ. As a consequence, the proof can be extended in the following way:

...	...	...	...		...	
5	$\neg\Box_e q$	3;RC	$\{\Box_e q\}$	✓	$\{\Box_e q\}$	✓
...	...	...	...		...	
8	$p \vee \langle p, q\rangle^{p/t}$	1,2,5,(6,)7;RU	Ω_5	✓	$\Omega_{5,6}$	✓
9	p	8;RC	$\Omega_5 \cup \{\langle p, q\rangle^p\}$	✓	$\Omega_{5,6} \cup \{\langle p, q\rangle^t\}$	✓
10	$\Box_e p$	–;PREM	$\emptyset$		$\emptyset$	
11	$\Box_e q$	1,10;RU	$\emptyset$		$\emptyset$	

At this stage of the proof, the formula $\Box_e q$ has been derived on line 11. As this is a minimal *Dab*–consequence of Γ at stage 11, lines 5, 8, and 9 are marked.

Consequently, the formula $\neg\Box_e q$ cannot be considered as derivable anymore, and neither can the formula p.

Secondly, consider the formulas of the form $\neg\Box_n A$ obtained by means of the defeasible inference rule **NNN**. These are used in the logic $\mathbf{AbL^t}$ to guarantee that the inference step known as *strengthening the antecedent* doesn't enable one to derive abductive explanations containing irrelevant parts. For, the logic $\mathbf{AbL^t}$ only validates an abductive inference based on a nomological formula $\Box_n((A_1 \wedge ... \wedge A_n) \supset q)$ in case the formulas $\neg\Box_n((A_2 \wedge ... \wedge A_n) \supset q)$, $\neg\Box_n((A_1 \wedge A_3 \wedge ... \wedge A_n) \supset q)$,...,$\neg\Box_n((A_1 \wedge ... \wedge A_{n-1}) \supset q)$ are derivable. It is easily verified that this will never be the case when the formula $\Box_n((A_1 \wedge ... \wedge A_n) \supset q)$ has been obtained by means of strengthening the antecedent. To illustrate this, suppose the proof above (the original proof, i.e. lines 1–9) is extended in the following way:

10	$\Box_n((p \wedge r) \supset q)$	1;RU	$\emptyset$		$\emptyset$	
11	$\neg\Box_n(p \supset q)$	–;RC	$\{\Box_n(p \supset q)\}$	✓	$\{\Box_n(p \supset q)\}$	✓
12	$\neg\Box_n(r \supset q)$	–;RC	$\{\Box_n(r \supset q)\}$		$\{\Box_n(r \supset q)\}$	
13	$p \wedge r$	2,5,10(,11,12);RC	$\Omega_5 \cup \{\langle p \wedge r, q\rangle^p\}$		$\Omega_{5,11,12} \cup \{\langle p \wedge r, q\rangle^t\}$	✓

Clearly, the antecedent of $\Box_n((p \wedge r) \supset q)$ contains an irrelevant part, viz. r. The latter has been added to the antecedent of the nomological formula on line 1 by an application of strengthening the antecedent. However, as $\Box_n(p \supset q)$ is a minimal *Dab*–consequence of the premise set Γ, lines 11 and 13 are marked. Hence, neither $\neg\Box_n(p \supset q)$, nor $p \wedge r$ are considered as derivable from Γ at stage 13 of the proof (and at all later stages of the proof). A small digression is necessary at this point. Line 13 is only marked for the logic $\mathbf{AbL^t}$. Hence, the formula $p \wedge r$ seems to be derivable from Γ by the logic $\mathbf{AbL^p}$. This is not the case though, for $\mathbf{AbL^p}$ also blocks abductive inferences based on nomological statements obtained by strengthening the antecedent. Only, $\mathbf{AbL^p}$ doesn't need formulas of the form $\neg\Box_n A$ to do so, as the following extension of the proof above shows.

13	$p \wedge r$	2,5,10(,11,12);RC	$\Omega_5 \cup \{\langle p \wedge r, q\rangle^p\}$	✓	$\Omega_{5,11,12} \cup \{\langle p \wedge r, q\rangle^t\}$	✓
14	$\langle p \wedge r, q\rangle^p \vee \langle p \wedge \neg r, q\rangle^p$	1,5;RC	Ω_3		Ω_3	

Comparison with the 'Backwards Deduction'–Approaches

In order to show that both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ capture abductive explanation in an adequate way, I will show that both logics satisfy the conditions for abductive explanation put forward by the traditional 'backwards deduction'–approaches to abduction (see section 1, table 1).

Condition (i) states that the background knowledge extended by an abductive explanation has to yield the explanandum. Given the dependency of the defeasible inference rule $\mathbf{AC^m}$ on nomological statements derivable from the background knowledge, this condition is satisfied a priori.

Condition (ii) states that an abductive explanation has to be compatible with the background knowledge. It is easily verified that this will be the case for both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$. For, in case an abductive consequence derived on a line i in a proof is incompatible with the background knowledge, line i will irrevocably be marked at some stage of the proof. For example, consider the premise set $\Gamma = \{\Box_n(p \supset q), q, \Box_e \neg p\}$. As in the proof above (lines 1–9), the formula p is conditionally derivable from the premise set Γ. However, the proof can be extended in such a way that line 9 is marked.

...	...	...	...	...
9	p	8;RC	$\Omega_5 \cup \{\langle p, q\rangle^p\}$ ✓	$\Omega_{5,6} \cup \{\langle p, q\rangle^t\}$ ✓
10	$\Box_e \neg p$	–;PREM	$\emptyset$	$\emptyset$
11	$\langle p, q\rangle^{p/t}$	1,2,5,(6,)10;RU	Ω_5	$\Omega_{5,6}$

At stage 11 of the proof, the formula $\langle p, q\rangle^{p/t}$ is a minimal *Dab*–consequence of the premise set Γ. As a consequence, line 9 is marked.

Condition (iii) states that the explanandum may not be derivable from the background knowledge alone. As I have shown above, this condition is satisfied for both $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$.

Condition (iv) states that an abductive explanation may not yield the explanandum by itself. Actually, this is satisfied by the fact that applications of $\mathbf{AC^m}$ are only validated conditionally in $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ in case the nomological statements involved are of a specific syntactic form. This is a consequence of the way Ω_p and Ω_t were defined. For example, the elements of Ω_t are of the form $\Box_n((A_1 \wedge ... \wedge A_n) \supset B) \wedge ... \wedge \neg(A_1 \wedge ... \wedge A_n)$. However, B is not allowed to occur in $A_1 \wedge ... \wedge A_n$ (check the definition of Ω_t above)! As a consequence, it is impossible for $A_1 \wedge ... \wedge A_n$ to yield B by itself. The same reasoning also applies to the elements of Ω_p.

Finally, as I have shown above, the abductive explanations obtained by $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ never contain irrelevant parts (which is a weak kind of minimality criterium). Although Aliseda–Llera didn't state this as a necessary condition for consistent and explanatory abduction in [1], it is easily verified that it should be a necessary condition (and in a lot of traditional logic–based approaches, it also is).

Practical vs. Theoretical Abductive Explanation

The logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ explicate different kinds of abductive explanation, viz. practical abductive explanation and theoretical abductive explanation respectively. As stated at the beginning of this section, in case a puzzling phenomenon has multiple possible explanations, practical abduction only yields the disjunction of these explanations, while theoretical abduction yields all explanations separately.[14] As a consequence, the former is more cautious than the latter, for practical abduction won't enable one to act on a single possible explanation in case there are multiple. This is appropriate for contexts in which it is important that no possible explanations are overlooked, for example when trying to diagnose the disease causing a patient's symptoms — in case there is more than one possibility, acting on a single one would be foolish, for this could leave the patient uncured. On the other hand, in some contexts one might want to derive all possible explanations, for example in case one wishes to compare the predictions yielded by various scientific explanations. On the basis of this comparison, one may then decide which explanation should be favored.

Example. To illustrate the different kinds of abductive explanation explicated by the logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ respectively, consider the example below, based on the premise set $\Gamma = \{\Box_n(p \supset q), \Box_n(r \supset q), \Box_n\neg(p \wedge r), q\}$. For the premise set Γ, the logic $\mathbf{AbL^p}$ should enable one to derive the disjunction $p \vee r$, while the logic $\mathbf{AbL^t}$ should enable one to derive both p and r separately. As a matter of fact, this is exactly what happens.

1	$\Box_n(p \supset q)$	–;PREM	$\emptyset$	$\emptyset$
2	$\Box_n(r \supset q)$	–;PREM	$\emptyset$	$\emptyset$
3	q	–;PREM	$\emptyset$	$\emptyset$
4	p	1,3;RC	$\{\Box_e q, \langle p, q\rangle^p\}$	$\{\Box_e q, \Box_n q, \langle p, q\rangle^t\}$
5	r	2,3;RC	$\{\Box_e q, \langle r, q\rangle^p\}$	$\{\Box_e q, \Box_n q, \langle r, q\rangle^t\}$

At stage 5 of the proof, p and r have been derived on an unmarked line in both the $\mathbf{AbL^p}$– and the $\mathbf{AbL^t}$–proof. Both proofs now proceed differently. Hence, I will consider them separately, starting with the $\mathbf{AbL^p}$–proof.

...	...	...	...		–
4	p	1,3;RC	$\{\Box_e q, \langle p, q\rangle^p\}$	✓	–
5	r	2,3;RC	$\{\Box_e q, \langle r, q\rangle^p\}$	✓	–
6	$\langle p, q\rangle^p \vee \langle r \wedge \neg p, q\rangle^p$	1–3;RC	$\{\Box_e q\}$		–
7	$\langle r, q\rangle^p \vee \langle p \wedge \neg r, q\rangle^p$	1–3;RC	$\{\Box_e q\}$		–
8	$p \vee r$	1–3;RC	$\{\Box_e q, \langle p \vee r, q\rangle^p\}$		–

At stage 10 of the $\mathbf{AbL^p}$–proof, two minimal *Dab*–consequences of the premise set Γ have been derived, viz. $\langle p, q\rangle^p \vee \langle r \wedge \neg p, q\rangle^p$ on line 8 and $\langle r, q\rangle^p \vee \langle p \wedge \neg r, q\rangle^p$ on line 9. As a consequence, lines 6 and 7 are marked, which implies that neither p nor r is considered as derivable anymore. However, the disjunction of p and r is considered as derivable, for the formula $p \vee r$ occurs on an unmarked line of the proof (line 10 to be precise). Moreover, it

[14] The distinction between both kinds of abduction was introduced by Meheus&Batens [10, p. 224].

is easily verified that line 10 will remain unmarked in any extension of the proof. Hence, the formula $p \vee r$ is a final abductive $\mathbf{AbL^p}$–consequence of Γ.

Now, consider the $\mathbf{AbL^t}$–proof below. At first, this proof seems to proceed as the $\mathbf{AbL^p}$–proof above. However, because the logic $\mathbf{AbL^t}$ is based on the normal selections strategy instead of the reliability strategy, neither line 6 nor line 7 is marked (nor will these lines be marked in any extension of the proof). As a consequence, both p and r are final abductive $\mathbf{AbL^t}$–consequences of Γ.

...	...	...	–	...	
6	p	1,3;RC	–	$\{\Box_e q, \Box_n q, \langle p, q\rangle^t\}$	
7	r	2,3;RC	–	$\{\Box_e q, \Box_n q, \langle r, q\rangle^t\}$	
8	$\langle p, q\rangle^t \vee \langle r \wedge \neg p, q\rangle^t$	1–3;RC	–	$\{\Box_e q, \Box_n q, \Box_n(r \supset q), \Box_n(\neg p \supset q)\}$	✓
9	$\langle r, q\rangle^t \vee \langle p \wedge \neg r, q\rangle^t$	1–3;RC	–	$\{\Box_e q, \Box_n q, \Box_n(p \supset q), \Box_n(\neg r \supset q)\}$	✓
10	$p \wedge r$	6,7;RU	–	$\{\Box_e q, \Box_n q, \langle p, q\rangle^t, \langle r, q\rangle^t\}$	

To conclude, consider the formula on line 10 of the $\mathbf{AbL^t}$–proof above. This is the formula $p \wedge r$, viz. the conjunction of both possible explanations for q. As line 10 is unmarked at this stage of the proof, the formula $p \wedge r$ is a conditional consequence of the premise set Γ. As distinct possible explanations are usually considered as mutually exclusive, this clearly is absurd. However, different possible explanations for the same phenomenon do not have to be mutually exclusive, for one of these may yield the other(s) — in the example above, this would be the case if $\Box_n(r \supset p)$ would have been an element of the premise set Γ. In this case, the derivation of the conjunction of multiple possible explanations makes perfect sense. Nonetheless, in case the possible explanations are mutually exclusive, their conjunction should not be derivable. As is shown below, this is exactly what happens in $\mathbf{AbL^t}$–proofs. Given that $\Box_n \neg(p \wedge r) \in \Gamma$, p and r are mutually exclusive. Hence, the line on which their conjunction occurs will get marked eventually. For example, in case the proof is extended as follows.

...	...	...	–	...	
10	$p \wedge r$	6,7;RU	–	$\{\Box_e q, \Box_n q, \langle p, q\rangle^t, \langle r, q\rangle^t\}$	✓
11	$\Box_n \neg(p \wedge r)$	–;PREM	–	$\emptyset$	
12	$\langle p, q\rangle^t \vee \langle r, q\rangle^t$	1–3,11;RC	–	$\{\Box_e q, \Box_n q\}$	

5 Conclusion

The (prioritized) adaptive logics $\mathbf{AbL^p}$ and $\mathbf{AbL^t}$ provide a formal explication of practical and theoretical abductive explanation respectively. In contradistinction to the traditional logic–based approaches to abduction, these logics not only capture abductive explanation metatheoretically and/or semantically, but also proof theoretically, viz. as a combination of multiple defeasible inference rules. In general, this shows that logics for abduction based on the adaptive logics programme provide a more realistic explication of abductive explanation than most traditional logic–based approaches.

Further Research. In this paper, I only provided a formal explication of practical and theoretical abductive explanation. These are not the only kinds of abductive reasoning though, for a lot of other abduction processes have been characterized in the literature — for example, preferential abductive explanation, abductive explanation triggered by an anomaly (a formula contradicting the background theory),... — for an overview, see e.g. Aliseda–Llera [2]. The formal explication of these abduction processes is left as further research. As a consequence, the logics presented in this paper should be considered as the first step in the direction of a general formal approach to abductive reasoning based on the adaptive logics programme.

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