On the Implementation of Concept Structures in Fuzzy Logic

Stephan van der Waart van Gulik (Stephan.vanderWaartvanGulik@UGent.be)
Center for Logic and Philosophy of Science, Ghent University
Blandijnberg 2, 9000 Ghent, Belgium

Abstract

A procedure is presented which can modify a large number of fuzzy logics in such a way that the result integrates a logically meaningful representation of the family resemblance structure of fuzzy concepts. The most important aspect of this modification is the implementation of so-called 'concept matrices'. The interpretation and construction of these new formal objects is based upon Fintan Costello's 'Diagnostic Evidence Model' (2000), a contemporary cognitive scientific model of concept structure and concept combination. As a result, it becomes possible to formalize, explain and simulate new logical aspects of cognitive fuzziness such as meaning transformations by means of non-scalar hedges, and interpretational and inferential operations over non-intersective concept combinations.

Introduction

The disciplines of fuzzy logic (Hájek, 1998; Zadeh, 1965) and fuzzy concept theory (Rosch, 1973) often referred to each other, c.f. (Rosch & Mervis, 1975) and (Zadeh, 1982). However, the similarities between the formal machinery in fuzzy logic and models of fuzzy concepts in cognitive science are, generally speaking, rather superficial, and indeed often problematic (Osherson & Smith, 1981). Principles like graded extensions and connectives which function over a set of ordered truth-degrees in the truth-functional semantics of contemporary fuzzy logics such as Petr Hájek's $BL\forall$ (1998), can only be interpreted as indirect references to the fuzzy structure of concepts. There are no principles or objects integrated in these formalisms which simulate for instance the well known family resemblance structure of fuzzy concepts. However, a modification procedure can be defined which allows us to enrich a large number of fuzzy logics in such a way that the result integrates a logically meaningful formal representation of family resemblance structure. The most important aspect of this modification is the integration of so-called 'concept matrices' (CM). The interpretation and construction of these new formal objects is based upon Fintan Costello's Diagnostic Evidence Model (*DEM*)(2000), a contemporary cognitive scientific model for the modeling of concept structure and concept combination. As a result it becomes possible to formalize, explain and simulate new logical aspects of cognitive fuzziness. In what follows, I will first discuss this modification procedure. Given that this procedure can be applied to a large class of FL, I will use 'FL' as the generic name for the basic fuzzy logic of choice, and 'FL_c' for the corresponding modified result. Next, I will give two examples of new aspects of cognitive fuzziness which can be explained by means of FL_c.

Implementing Concept Structures in Fuzzy Logic

Since Rosch's seminal work in prototype theory, it is a classic idea in cognitive science that many fuzzy concepts have a family resemblance structure, cf. (1975) in particular. It has become quite common to interpret a fuzzy concept C as a conglomerate of associated concepts $\{F_1,..., F_n\}$ of which some are indicative for the presence of C only to a certain degree. As it is also natural in cognitive linguistics and lexical semantic models to understand concepts as the mental representations of meaning or 'intension', we can interpret the meaning of most lexical terms denoting fuzzy concepts as a conglomerate of associated concepts. In correspondence to this interpretation, the cognitive linguist George Lakoff (1973) defines fuzzy concepts as sets of 'meaning components' (themselves again being conceived as concepts).

As already mentioned in the introduction, the most important aspect of the modification of a FL into its FL_c-variant, is the integration of CM's. Formally speaking, a CM incorporates ordered sets of meaning components and relates each of these n-tuples to a unique element of a specific subset of predicates of the FL_h-language schema. These predicates are named 'complex (unary) predicates' (thereby functioning in FL_c as lexical items denoting complex fuzzy concepts). Semantically, these CM's are used to calculate the continuous membership values of instances for the extensions of complex predicates. Proof-theoretically, CM's are treated as part of a special premise called a 'matrix-set' which can be consulted during any inferential action involving formulas using complex predicates.

As already mentioned, the extra-logical construction of CM's is based upon Costello's *DEM* (2000). The model resolves around the notion 'diagnosticity'. Informally speaking, the diagnosticity of a concept D for a concept C is the output of a function which quantifies the extend to which D is indicative for the presence of C. I have chosen *DEM* as the model for the basis of the

CM's as DEM also predicts very well the dynamics of different types of concept combinations which are relevant in example 2 under. Of course, for the basic set-up of FL_c, there are also other valid options like standard 'cue-validity'. After having defined the set of meaning components $\{F_1,..., F_n\}$ of each complex concept C by means of this function, the components are ordered in function of their respective diagnosticity values for C. The result is an n-tuple of meaning components forming a row in a CM which is linked uniquely to C.

Apart from the construction demands above, the CM's used in FL_c need to meet some extra conditions in order to keep the semantics of FL_c recursive and realistic. For example, there cannot be any auto-definition for complex predicates. In other words, there cannot be any complex predicate π_i for which some (again possibly complex) meaning component denoted by a predicate π_j, ultimately comprises the meaning component denoted by π_i.

Example 1: Non-Scalar Hedges

A first example of how FL_c can be used to explicate new aspects of cognitive fuzziness deals with the logical usage and interpretation of so-called 'non-scalar hedges'. Examples of non-scalar hedged sentences are *"*Technically speaking*, it's a bird"* or *"*Loosely speaking*, it's a game"*. This type of hedge can only operate over complex predicates and is used to narrow down, loosen, or even shift the concept of a predicate. In other words, non-scalar hedges transform meaning. In (1973), Lakoff constructs a innovating theory concerning this type of hedges. For this, he mainly uses linguistic analysis of non-scalar hedged sentences. Lakoff explains what needs to be assumed with respect to fuzzy concepts if their corresponding predicates are equipped with non-scalar hedges. Basically, a fuzzy concept should be conceived as 'an ordered set of sets of meaning components'. Given this general theory, Lakoff also presents a formal semantic account of the non-scalar hedges *technically*, *strictly speaking*, and *loosely speaking*.

Though Lakoff's ideas are very powerful and inspiring, it is problematic that his formal semantic account of these hedges remains only an onset. No complete semantics, nor any actual pure logical analysis of valid reasoning with non-scalar hedges is developed. However, when some small extra modification of the CM's is carried through and some extra semantic definitions are integrated in the semantics of FL_c, it is possible to develop a variation named FL_h, in which these hedges and their semantics can be easily implemented. As mentioned above, Lakoff explains that, in the context of non-scalar hedges, every complex fuzzy concept needs to be conceived as an ordered set of sets of meaning components. More specifically, Lakoff defines 3 different sets of meaning components which are needed for the semantics of *technically*, *strictly speaking*, and *loosely speaking*. Translated in terms of Costello's DEM, these sets consist out of respectively those meaning components with definitional or absolute diagnosticity, medium diagnosticity and relatively low diagnosticity. Recall that the CM's defined above already consist of ordered sets of meaning components corresponding to a unique complex predicate. Also recall that each meaning component's place in an ordering depends on its the level of diagnosticity. In order to get the 3 sets needed according to Lakoff, a k-means cluster algorithm can be applied to the respective diagnosticity values of the initial ordered set. Next, Lakoff's semantic definitions of *technically*, *strictly speaking*, and *loosely speaking* using these newly obtained sets of ordered sets in the CM's can be integrated easily in the semantics.

Developing a complete formal logical account of Lakoff's theory such as FL_h brings along many advantages. For example, FL_h makes it possible to generate several interesting theorems concerning the inferential relation between hedged formulas and their non-hedged variant. These type of theorems, only possible in a fully developed logic, turn out to confirm our linguistic intuitions concerning *technically*, *strictly speaking*, and *loosely speaking* and the way in which they transform meaning. Finally, it is important to realize that CM's are critical for the development of FL_h. Consequentially, it is safe to say that the implementation of a formal variant of family resemblance structure in a FL has enabled us to get more precise insights in the logical dynamics of (a set of typical) non-scalar hedges.

Example 2: Non-intersective Concept Combination

Daniel Osherson and Edward Smith (1981) have presented a series of problems that arise when simulating concept combinations suing prototype theory formalized by means of fuzzy set theory. One of these problems is a sort of 'extension shift' (also historically referred to by some authors as the 'guppy-effect'). Consider the following example. Let the concepts *Apple*, *Striped*, and *Striped-Apple* be denoted respectively by the predicates A, S and SA. Let μ_π be the function characterizing the extension of a predicate π. Now imagine an apple a in front of you which is perfectly striped. Of course, in this case, a is more a typical *Striped-Apple* than it is an *Apple*, as a typical *Apple* is not *Striped*. Consequentially, (1) $\mu_{SA}(a) > \mu_A(a)$ should hold. Given for instance the standard fuzzy operators defined by Zadeh (1965), it is clear that also (2) $\mu_{SA} = min(\mu_A(a), \mu_S(a))$ necessarily holds. From (2), $\mu_{SA}(a) \leq \mu_A(a)$ follows, thereby clearly contradicting (1). Osherson and Smith conclude that, because of this contradiction, formalizations of prototype theory using fuzzy set theory are not compatible with strong intuitions concerning the combinations of concepts. In the following decades, many researchers accepted Osherson and Smith's conclusion and, generally speaking, FL was no longer considered an option for the formalization of fuzzy common sense reasoning.

However, the already used contemporary DEM is a powerful model. By means of an alternative diagnosticity function using a different contrast class than in the case of single concepts, DEM also predicts very well the cognitive dynamics active in different types of non-intersective concept combinations like property concept combinations and hybrid combinations. In property concept combinations a meaning component of one concept also holds for the other (e.g. 'cactus fish'). In the case of a hybrid concept combinations, each constitutive concept's ordered set of meaning components is modified by the semantic influence of the other (e.g.'pet fish'). By now, it should not come as a surprise that it is possible to develop an extra pair of CM's simulating the meaning component sets of these two types of non-intersective combinations for all possible combinations of complex concepts present in the language of the logic under consideration. More specifically, in order to extend FL_c into a logic which can deal with these types of non-intersective concept combinations in an intuitively clear and logically valid manner, only two things have to be done. First the two extra CM's are integrated in a completely similar way as was done for the initial CM's of FL_c. Next, the semantics of FL_c is extended in such a way that when interpreting a specific type of combination the correct type of meaning components are consulted. The result is generically named FL_{nicc}.

A great advantage of FL_{nicc} is that, despite the fact that FL_{nicc} deals with predicates denoting non-intersective concept combinations, contradictions of the type described by Osherson and Smith are excluded. Moreover, it is even possible to generate many nuanced inference rules for formulas using complex predicates denoting property and hybrid combinations combinations. The scope and precision of these inference rules is also larger compared to other known, contemporary strategies, e.g. the supervaluationist strategy suggested Hans Kamp and Barbara Partee (1995) which uses a recalibration function which only deals with property combinations of a very specific kind. As a result, we can safely conclude that, also in this case, the implementation of a rich concept structure by means of CM's is an interesting and promising technique both for logic and cognitive science.

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