Capturing dynamic conceptual frames

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Abstract: The main focus of this paper is to develop an adaptive formal apparatus capable of capturing arguments conducted within a conceptual framework. I first explain one of the most recent theories of concepts developed by cognitivists, in which a crucial part is played by the notion of a *dynamic frame*. Next, I describe how a dynamic frame may be captured by a finite set of formulas and how a formalized adaptive framework for reasoning within a dynamic frame can be developed.

Key words: adaptive logic, dynamic frames, conceptual framework, paraconsistency.

1 Introduction

On the *classical theory of concepts* to each concept there corresponds a set of necessary and sufficient conditions for falling under that concept, a set of conditions that can be discovered by conceptual analysis. Arguably, the classical view is not an adequate picture of how concepts work in human cognition. It is not my purpose here to argue against the classical theory, but I will explain in a few words why one might want to reject it.[1]

First, the alleged definitional conditions of classical concepts are quite intractable: an average human being is usually unable to produce upon request a correct analysis (in the classical sense) of concepts that they use, and even philosophers — people who tend to spend an unusual amount of time on conceptual analysis — "have failed to provide a single generally agreed analysis of any important concept" (Andersen et al. 2006: 6).

Quite independently, cognitivists gathered psychological evidence to the effect that the structure of human concepts is graded — objects can fall under a concept to a higher or lower degree (Rosch 1973a, 1975a,b, 1978, 1983) — a possibility that can't be easily accounted for by the classical theory, according to which an object either satisfies the necessary and sufficient conditions for falling under a concept or it doesn't and that's the end of the story.

Also, a different account of how (at least certain) concepts work has been put forward by Wittgenstein (1953) and has gained some popularity since then. On this view, there are at least some concepts whose instances have no common features and bear a mere *family resemblance* to each other. In such a case any classical conceptual analysis of a concept is impossible because there are no necessary or sufficient conditions for falling under it.

[1] As far as the criticism of the classical theory is involved, the *locus classicus* is (Quine 1951). See however a more recent attack launched from a slightly different position (Fodor et al. 1999).

A few alternatives to the classical theory of concepts have been developed. For instance, the *feature list theory*, according to which a concept, instead of being associated with a set of necessary and sufficient conditions, is rather explicated in terms of a list of features that subjects typically produce for the category (see e.g. Glass and Holyoak 1975).[2] Another example is the *prototype theory of categorization.* On this view, a category is constructed around a quintessential example or a typical member and other members fall under that category to the degree to which they share attributes with that prototype (Rosch 1973b, 1983).[3]

One of the major and most recent accounts of concepts, inspired by the work of Rosch, employs the notion of a *dynamic conceptual frame.*[4] One of the most well-known formulations of the theory has been provided in (Barsalou 1987; Barsalou and Hale 1993; Barsalou 1993; Barsalou and Yeh 2006). Motivated by the work of Kuhn (esp. Kuhn 1974), certain applications to the history of science have been put forward and it has been argued that dynamic frames are a useful tool for accounting for scientific revolutions and conceptual frame incommensurability (Andersen et al. 2006).

Although very interesting, the cognitivists' treatment of dynamic frames is fairly informal and the logical aspects of the issue have not been investigated. The present work is intended as a step towards filling the gap between interesting but informal insights of cognitive researchers and the mainstream methodology of formal logicians. I will first explain at length what dynamic frames are. Having done that, I delimit a rather simple and yet interesting class of dynamic frames and develop a language which can be used to describe such frames by finite sets of formulas. Then I introduce a convenient way of capturing a reasoning led within a conceptual framework. The basic idea is that such a reasoning can be modeled as an adaptive framework which takes negations of the formulas that describe the frame to be abnormalities.

A few remarks about the structure of this paper are due. Section 2 describes the notion of a dynamic frame and specifies the class of frames that I will be interested in. Section 3 describes how a finite set of formulas that captures a given frame is to be constructed. Section 4 provides us with a first stab at the notion of reasoning within a conceptual frame and points towards the need for adaptivity in this context. Section 5 is meant as a fairly accessible explanation of what adaptive logics are. Section 6 explains how an adaptive approach can be developed to accommodate the notion of reasoning within a conceptual frame.

I hope the reader can forgive me the length of this paper, in the light of the fact that a strong emphasis is put on the accessibility of the material. Especially, accessibility considerations

[2]This theory is criticized by Barsalou (1992b: 25-28), who presents what he considers to be evidence against the feature list approach. For a logician it is hard to assess this criticism, mostly because Barsalou doesn't provide any methodological comments and it is sometimes unclear why exactly certain facts should count as evidence against or for the feature list theory. Getting into details lies beyond the scope of this paper, though.

[3]See also (MacLaur 1991) for a more recent survey of the prototype theory, and (Thagard 1992: 13-19, 24-36) for a more historical survey of main existing theories of concepts.

[4]The psychological evidence for the adequacy of this theory is surveyed by Andersen et al. (2006: 47-52).

motivated sections 3 and 5. Section 3 is needed because it explains the notion of a dynamic frame to non-cognitivists (and to those cognitivists who aren't familiar with the notion), and section 5 is needed because it describes adaptive logics to non-logicians (and to those logicians who aren't familiar with this approach).

2 Frames and their constituents

A frame developed for a single concept only is called a *partial* frame. In this paper I will be interested in partial frames only, for at least two reasons: (i) reasoning within partial frames is complex enough to provide an interesting subject of study, and (ii) in a sense to be specified, more complex frames result from superimposition of partial frames, and hence if we are to have a workable formal theory of reasoning within dynamic frames, partial frames seem to be a good point of departure.

A partial dynamic *frame* (developed for a single concept A only) is composed of two layers of concepts: *attributes* and *values*. Every object that falls under A is supposed to have all the attributes. Objects having a certain attribute are divided according to what values of those attributes they instantiate. Take a fairly primitive example. The concept BIRD can be considered in a frame where it has only two subordinate attributes: BEAK and FOOT, each having two values: ROUND, POINTED, and WEBBED, CLAWED respectively.

Some combinations of values (from among each of the attributes one value is chosen) are considered to be *activated*. This, roughly speaking, means that objects that instantiate values from that combination are taken to constitute a separate *taxonomical unit*. For instance, in the exemplary frame there may seem to exist just two activation patterns: {POINTED, CLAWED} and {ROUND, WEBBED}, giving raise to the taxonomical units LAND BIRD and WATER BIRD respectively. In this sense, a (partial) dynamic frame specifies a taxonomy of the concept under consideration. The concept BIRD in the frame under discussion is divided exhaustively and exclusively into two separate taxonomical units.

What are the *nodes* of a conceptual frame? Barsalou (1993: 10) suggests that "frames are large collections of perceptual symbols," where perceptual symbols are supposed to be "aspects of experience stored in memory via selective attention that function symbolically" (Barsalou 1993: 5). Considering the fact that clarifying these notions might be quite difficult, I will just stick to the notions that I want to end up with anyway: in those contexts which we are presently interested in we might just treat the root concept as well as the attributes and values just like predicates whose reference is taken for granted.[5]

The notions of *attributes* and *values* haven't been very clearly defined by the cognitivists.[6]

[5] This does not mean that there is no important distinction between concepts, meanings and predicates, as Barsalou (1993) insists. My decision is mainly pragmatic.

[6] For instance, Barsalou says:

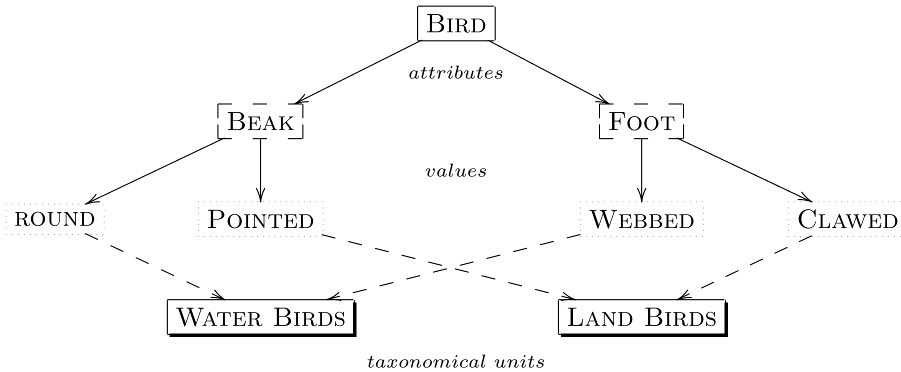

Figure 1: A partial dynamic frame for the concept Bird.

However, when we consider a frame constructed for a single frame, the idea seems fairly simple. Any object that falls under the root concept is supposed to have one of the values for each of the attributes. Attributes are just aspects in which objects that fall under the root concept are classified and values are various relevant features that an object can have with respect to those aspects. So one of the main constituents of a dynamic frame is a *tree-like structure*. This, however, does not exhaust the components of the frame.

Another important constituent of a dynamic frame are *activation patterns*. These decide which combinations of values for the attributes that occur in the frame actually occur together and constitute a separate taxonomical unit. The notion of taxonomical unit isn't really defined by the cognitivists, but the basic intuition is that a taxonomical unit is a group of objects that satisfy certain selection of values not excluded by the conceptual frame and constitutes (for some purposes) a separate group falling under the root concept. For instance, in the frame from fig. 1, the combination {round beak,webbed foot} constitutes a taxonomical unit of water birds and the combination {pointed beak,clawed foot} constitutes the taxonomical unit of land birds, and the distinction between these two groups is introduced for instance because there are certain useful generalizations that apply to all land birds but not to all water birds, and so on. On the other hand, the above frame does not admit an activation pattern where an object has a pointed beak but webbed feet, or a round beak and clawed feet.

For any object that falls under the root concept and for any attribute in the frame this

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> A fundamental task for frame theorists is to provide satisfactory definitions for *attribute* and *value*. I define an attribute as a concept that describes an aspect of at least some category members[7] . . . The definition of *value* follows from the definition of *attribute*: Values are subordinate concepts of an attribute. Because values are subordinate concepts, they inherit information from their respective attribute concepts. (Barsalou 1992b: 30-31)

object has to instantiate exactly one value for that attribute (it has to have at least one, because otherwise the attribute wouldn't be relevant for the classification and it has to have at most one because for any attribute its values are exclusive).[8] Also, the taxonomical units that arise from the activation patterns are taken to be a division of the domain of objects that fall under the root concept: no object should belong to two taxonomical units (see Andersen et al. 2006: 56) and every object should belong to a taxonomical unit (see Andersen et al. 2006: 27).

Yet another component of a dynamic frame are the so-called *structural invariants*, that is, dependencies between the nodes of a frame.[9] Those seem to be of two kinds: (i) between the attributes and (ii) between values.[10]

Consider first the dependencies between the attributes that are accepted together with the whole conceptual framework. For instance, if we extend the above example by adding an attribute NECK having two values: LONG, SHORT, we could count among our structural invariants that "anything with a beak must also have a neck, but not everything with a foot also has a beak" (Andersen et al. 2006: 44). This, however, doesn't seem very useful, at least as long as partial frames are involved. Considering that any object falling under the root concept has to have some values for all attributes it just seems that for every bird, if it has feet, it has a beak. Indeed, the dependency might seem to fail when we consider a frame within a larger frame (where there actually are things that can have feet without having a beak). However, as long as we're considering a single partial dynamic frame, the relation between attributes seems fairly trivial: all objects that fall under the root concept have the attributes one way or another. Hence, I will mostly ignore structural invariants pertaining to attributes in my considerations.

On the other hand, the dependencies between values in a partial dynamic frame are more interesting. For instance, it may seem that birds with webbed feet have round beaks and birds with clawed feet have pointed beak. Andersen et al. (2006: 44) suggest that "these patterns may be understood as physical constraints imposed by the nature," whereas Hoyningen-Huene (1993: 112-118) calls them "knowledge of regularities." Barsalou (1992b: 37) on the other hand mentions:

(a) constraints that hold for conceptual reasons,

(b) constraints that are empirical truths, and

[8] "... all of the attribute nodes are activated for every subordinate concept. However, value nodes appear in mutually exclusive clusters." (Andersen et al. 2006: 44)

[9] "Attributes in a frame are not independent slots but are often related correlationally and conceptually." (Barsalou 1992b: 35)

[10] There seem to be no interesting dependencies across the layers, besides those already captured by the tree-like structure (a value entails its superordinate attribute, the attribute entails the disjunction of its values, etc.). For (a) an attribute cannot exclude a value (it wouldn't be a value anymore) and (b) it cannot entail a specific value (otherwise there wouldn't be multiple values available for that attribute and hence the attribute would be redundant for the classification purposes).

(c) constraints that represent just statistical patterns or personal preferences.[11]

For my purpose I will ignore those constraints that aren't taken to hold universally for all objects falling under the root concept. The rationale is that if something is just a statistical pattern it is not a part of our conceptual framework, but rather just an empirical belief that we happen to hold, which does not enforce activation patterns anyway. I do, however, agree with Barsalou that the constraints don't have to be true — for a dynamic frame user it is enough that they take those constraints to be true.

As for constraints that are either logical or causal, I will ignore the distinction between them. For the purpose of this paper, the distinction is irrelevant.[12] Also, there is no a priori reason to exclude causal stories from conceptual frames. Sometimes our causal stories are so entrenched in our conceptual framework that they actually determine our taxonomy and it is not even clear that they are purely empirical truths. For instance, consider a frame that I might entertain when I choose whether I should go to work by bike or just walk there (fig. 2).

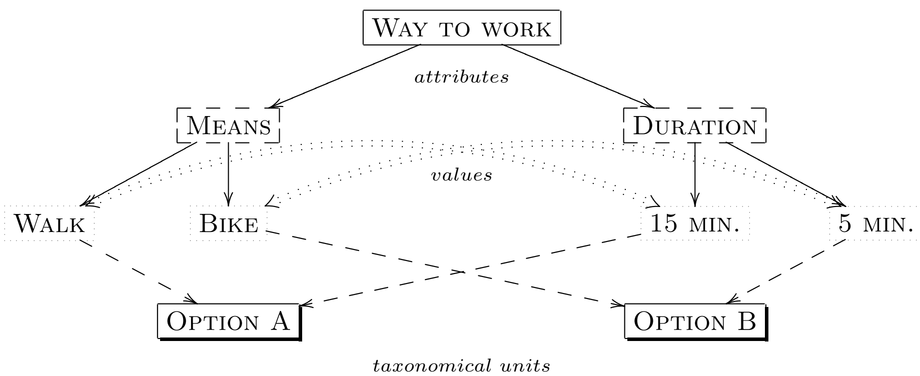

Figure 2: A partial dynamic frame for my WAY TO WORK.

The dotted arrows represent constraints that hold *ceteris paribus*. That is, I assume that the sole purpose of the trip is to get to work as soon as I can, that the route is the same no matter what means of getting there I choose, and that I will be equally successful in getting there in both cases etc. If I choose to walk, I will get there in 15 minutes, and if my trip takes 15 minutes this means I'm walking (and similarly for the other condition). Now, it is unclear whether the constraints hold for purely conceptual reasons (my concept of travel at a distance in this world is that speed has a certain correlation with time) or for causal reasons (they

[11] "...constraints need neither be logical nor empirical truths...attribute constraints often represent statistical patterns or personal preferences..." (Barsalou 1992b: 37)

[12] It might be relevant when we want to revise a conceptual frame, though. We might be more willing to give up a causal story rather than a logical point.

depend on some contingent truths of physics). What matters for our purpose, however, is that they are entrenched deep enough to exclude certain activation patterns (the possibilities that I walk to work in 5 minutes or that I bike there in 15 minutes under the circumstances).

In the case of fig. 2 the list of available activation patterns is:

{Walk, 15 min.}, {Bike, 5 min.}

and it so happens that all other activation patterns are excluded by the constraints that we have. Is it always the case that the constraints determine the activation patterns? Barsalou himself says only that "through the representation of increasingly specific subordinates, taxonomies emerge in frames" (Barsalou 1992b: 51), but he doesn't say explicitly how they emerge or that they are determined by the constraints. Andersen et al. (2006: 59-60), on the other hand, seem to suggest that the constraints determine the class of activation patterns:

> ... the frame determines which potential concepts are possible at the subordinate level but that constraints exclude some of these ... the value constraints also determine that only specific activation patterns are possible.

This, however, doesn't seem very obvious. For instance, on a fairly common view on the nature of constraints called the Theory-Theory view (Murphy and Medin 1999), the values in a conceptual framework are not connected just in virtue of co-occurrence. Rather there either is a logical or causal story to tell, when one wants to explain why certain values are connected or exclude each other.[13] But if that's the case, if conceptual constraints on values really determined the activation patterns, it would mean that a conceptual frame user always has a theory at hand that explains why the taxonomy is the way it is. But this is highly unlikely: I can believe that a certain taxonomy of birds is adequate and have some explanation as to why certain combinations of values exclude each other without actually believing a theory that completely explains why there are those taxonomical units of birds that there are. For this reason I will just assume that structural invariants pertaining to values only inform us about the impossibility or necessity of co-occurrence of certain combinations of values without actually determining the whole taxonomy.

Some other decisions that I make in order to approach dynamic frames formally pertain to:

(a) the *membership* relation, because I assume that if a taxonomical unit is determined by a certain activation pattern an object has to instantiate all the values from that activation pattern to fit into this unit (be a member of it);

[13] "Features in categories are not correlated by virtue of random combinations. Rather, correlations arise from logical and biological necessity ... It is no accident that animals with wings often fly or that objects with walls tend to have roofs. Even less obvious correlations ... usually have clear explanation." (Murphy and Medin 1999: 439)

(b) the "*grounded-ness*" of the whole frame, for I assume that neither are the attributes or values roots of further dynamic frames, nor are the nodes a frame tree taxonomical units resulting from other frames. Also,

(c) I provide only a very brief explanation of how the frame theory is supposed to explain the graded structure of human concepts, without providing a formal framework that would allow to deal with this issue in detail.

(d) I will assume that any partial frame is *finitary*: both the number of attributes and the number of values for each attribute are finite.

(e) I will avoid getting into a discussion pertaining to the question whether the framework provides a good explanation of the phenomenon of *family resemblance*.[14]

Even though the *membership* in a certain taxonomical unit requires falling under the root concept and possessing all the values that the activation pattern for that unit contains, it sometimes seems that we qualify an object to a certain unit even before we check whether it actually has all the values included in its activation pattern. This, however, can be disregarded for the formalization purposes, for it can be counted as an epistemic claim about classifying objects with insufficient evidence, not as a claim about what makes an object a member of a taxonomical unit. I will still assume that an object has to have all the values from the activation pattern in order to be a member of a taxonomical unit, even if we sometimes classify an object without sufficient empirical evidence for the claim that it instantiates all the required properties, knowing only that it has some of the values that belong to an activation pattern (and, ideally, that those values belong to only one activation pattern).

On the face of it, it may seem that we haven't really gotten that far from the classical view. One might ask, aren't the attributes just the necessary conditions for an object to fall under the root concept? Actually, there are some important differences here. First, on the classical theory there is an objective, subject-independent answer to the question what the necessary conditions are. A dynamic frame, on the other hand, describes the way a certain person thinks about the root concept — for all that matters, other language users might use different attributes and values in their frames for the same root concept and as long as they agree on identification and classification of external objects those differences won't matter. Also, on the dynamic frame view, there is no ultimate level of analysis and there are no primitives, frames are in this sense called *ungrounded*. "For any attribute, structural invariant, or constraint, people can always construct further attributes, structural invariants, and constraints that capture variability across instances . . . What was once a simple, unitary primitive becomes analyzed and elaborated, such that it becomes a complex concept." (Barsalou 1992b: 41)

[14]Claims that it does can be for instance found in (Andersen et al. 2006: 11). The basic idea is that a root concept is a family resemblance concept if no two activation patterns in its dynamic frame have a value in common.

For example, the division of the attribute Foot into values Webbed and Clawed can be analyzed in terms of the frame given in fig. 3.

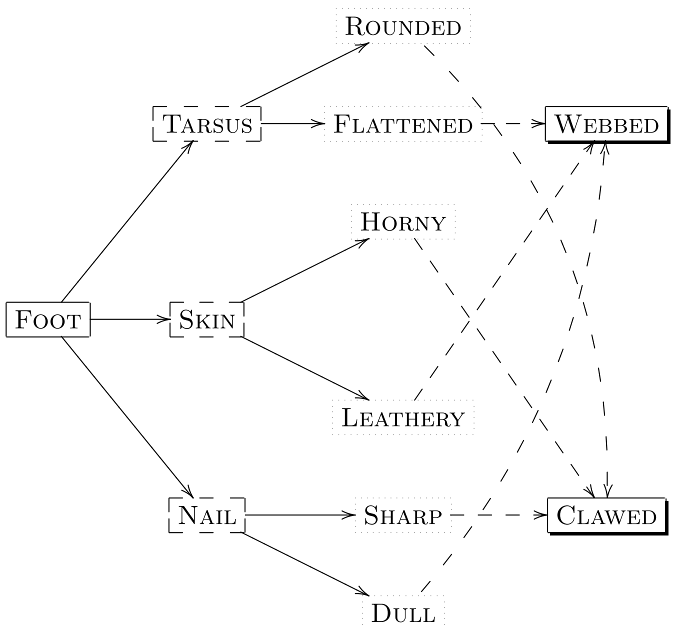

Figure 3: A partial dynamic frame for the concept Foot.

The concept Bird, on the other hand, may be considered as a taxonomical unit resulting from the frame presented in fig. 4.[15] Hence the slogan: "Human conceptual knowledge appears to be frames all the way down."(Barsalou 1992b: 40)

Clearly, partial frames can be superimposed in the manner described above, thus resulting in larger structures. I, however, will just consider partial and grounded dynamic frames in which the interpretation of the attributes and values is taken for granted and the root concept is not considered as a taxonomical unit resulting from another conceptual frame. Besides the fact that this decision facilitates the formalization that will follow, there are other pragmatic factors to be considered. In many everyday contexts, when we consider whether we should classify an object as belonging to a such-and-such taxonomical unit, or when we want to infer something using the claim that it belongs to a certain taxonomical unit, we actually reason within a partial and grounded conceptual frame. For example, when we ask ourselves whether a given bird is a land bird or a water bird we usually do not even think about all those attributes and values that the frame in fig. 4 is concerned with: they are irrelevant for the purpose at hand.

Arguably, dynamic conceptual frames can also model *typicality* and the *graded structure* of a concept. The basic idea is that in one's experience, objects instantiating a specific activation

[15]Yes, the frame is a huge simplification and it doesn't get some things right.

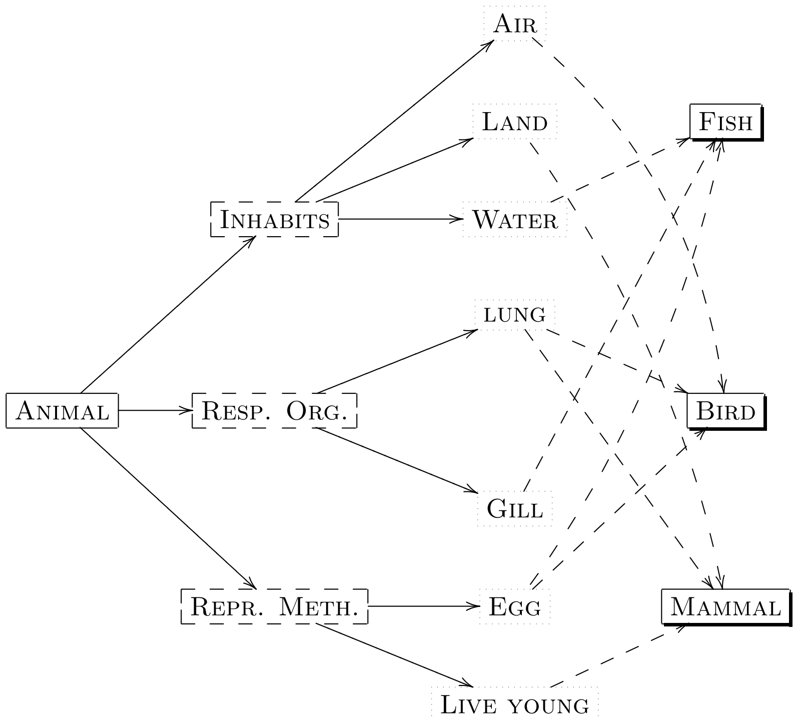

Figure 4: A partial dynamic frame for the concept ANIMAL.

pattern may have occurred more often than objects instantiating other activation patterns. Those objects are taken to be more typical representatives of the root concept (see Barsalou 1992b: 47). This does not mean that there is a unique taxonomical unit which is universally typical for the root concept: the choice is fairly subjective and will vary with the user's experience.[16] In a similar way, an object can be considered more or less a typical instance of the root concept, depending on how often the frame user has encountered objects instantiating the activation pattern of its taxonomical unit in their previous experience.

Andersen et al. (2006: 14) insist that "there is no restriction on the number of attributes linked to a major concept, or values linked to each attribute." To the contrary, I will assume that both the numbers of attributes and the number of values have to be finite. To start with, the frames that we usually come up with are finite. Also, if we had an infinite number of attributes, in order to classify an object to one of the taxonomical units we would have to consider it under infinitely many fairly independent aspects, which we, humans, usually don't do. Given that the number of attributes is finite, the only way the number of values

[16] "For Westerners a small bird with a sharp beak, a short neck, and a medium-sized body, like a blackbird, starling, or an American robin, is a good example of the concept. Those with longer legs, necks or beaks are less good examples. For Asians, however, the best examples of 'bird' are likely to resemble ducks, geese, or swans." (Barsalou 1992a: 176)

can be infinite is if there would be infinitely many values subordinate to one of the attributes. This however, would mean that we either would have infinitely many taxonomically redundant values or that there are infinitely many taxonomical units in our frame. But again, we rather tend to avoid situations like these.[17]

3 Expressing dynamic frames

In the previous section I provided philosophical motivations and intuitively explained the main concepts at play. Now it's time to approach those concepts more formally. Ultimately, I will want to capture formally the notion of reasoning within a conceptual framework. The basic idea here is that if we can express a frame using a finite set Γ of sentences of a fairly manageable language, we can model reasoning within a conceptual framework as reasoning with Γ as the set of background premises. In order to do that, I have to provide a method of constructing such a set first. This will be the subject of the present section.

Suppose we have a frame with the tree-like structure given in fig. 5.

The way the frame is represented is slightly different, but it should be self-explanatory. R is the root concept, $A_1, A_2, \ldots A_i$ are its attributes, and each attribute A_k has n_k values falling under it: $V^k_1, V^k_2, \ldots, V^k_{n_k}$. The language used for describing the frame is a fairly straightforward variant of a first-order language. Its alphabet consists of:

(i) a finite assembly of **node letters**:

$$R, A_1, A_2, \ldots, A_i, V^1_1, V^1_2, \ldots, V^1_{n_1}, V^2_1, V^2_2, \ldots, V^2_{n_2}, V^i_1, V^i_2, \ldots, V^i_{n_i}$$

where i is a natural number determined by the number of the attributes of the original frame.

(ii) a finite number **type letters** (that will be used to represent taxonomical units):

$$T_1, T_2, \ldots, T_k$$

(k is finite but it has to be large enough to perform the task the type letters are intended for, read on for details.)

(iii) **classical connectives and brackets**:[18]

$$\neg, \wedge, \vee, \rightarrow, \equiv, (,)$$

(iv) a countable reservoir of **individual variables**:

$$x_1, x_2, \ldots (\text{ also abbreviated as } x, y, z, \ldots)$$

[17] As an exception someone may suggest the case where, for instance, we have SIZE as an attribute and it's measured in terms of infinitely many numbers. But even then (at least for taxonomical purposes) we divide the whole range of sizes into finitely many groups.

[18] For abbreviations, the ordering of the binding strength is fairly standard: $\neg, \wedge, \vee, \rightarrow, \equiv$. Hence $p \wedge \neg q \vee r \rightarrow q \equiv r \vee p \wedge s$ abbreviates $(((p \wedge \neg q) \vee r) \rightarrow q) \equiv (r \vee (p \wedge s))$.

	attributes	values
R	A_1	V_1^1
		V_2^1
		$\vdots$
		$V_{n_1}^1$
	A_2	V_1^2
		V_2^2
		$\vdots$
		$V_{n_2}^2$
	$\vdots$	$\vdots$
	A_i	V_1^i
		V_2^i
		$\vdots$
		$V_{n_i}^i$

Figure 5: The tree structure of a frame to be expressed.

(v) **quantifiers** binding individual variables:

$$\forall, \exists$$

Both node and type letters are taken to be one-place predicates. The formation rules are standard formation rules for a first order language.

It will be convenient to use generalized conjunctions and disjunctions. $\bigvee_i^n P(x)$ stands for:

$$P_1^n(x) \vee P_2^n(x) \vee \cdots \vee P_i^n(x)$$

and $\bigwedge_i^n P(x)$ abbreviates:

$$P_1^n(x) \wedge P_2^n(x) \wedge \cdots \wedge P_i^n(x)$$

where n is an optional superscript, P a predicate and x an individual variable. Also, if Γ is a finite set of formulas, $\bigvee \Gamma$ ($\bigwedge \Gamma$) is just a disjunction (conjunction) of all elements of Γ.

To express the tree structure we first have to say that all objects that fall under the root concept possess all its attributes. That is, we need n formulas of the form:

$$\forall_x (R(x) \to A_k(x)) \tag{1}$$

Let's call the set of formulas falling under this schema $\mathbb{R}$.

Next, for any $1 \leq k \leq i$ we need the formula:

$$\forall_x [R(x) \to (A_k(x) \to \bigvee_{n_k}^{k} V^k_{n_k}(x))] \tag{2}$$

which says that any object that has an attribute A_k instantiates at least one value for that attribute. Let's call the set of all needed instances of (2) $\mathbb{A}$.

We also need to say that values falling under each of the attributes are exclusive, that is, that no object can simultaneously possess two different values for one and the same attribute. If we were allowed to quantify over indices, we could just express this claim by:

$$\forall_k \forall_{m,\ l}\ (m \neq l \wedge m \leq n_k \wedge l \leq n_k \to \forall_x (A_k(x) \to \neg (V^k_m(x) \wedge V^k_l(x))))$$

Since, however, we want to keep the language as simple as possible, we have to use a slightly different way around this issue. Take any $1 \leq k \leq i$ and consider the values $V^k_1, V^k_2, \ldots, V^k_{n_k}$. For each k, we extend our set of formulas that describe the frame by all formulas of the form:

$$\neg \exists_x (A_k(x) \wedge V^k_m(x) \wedge V^k_l(x)) \tag{3}$$

where $m \neq l, 1 \leq m \leq n_k, 1 \leq l \leq n_k$. Let's call the set of formulas falling under this schema $\mathbb{V}$.

The next item on our list is *value constraints*. It might seem that it is enough to say that a value constraint is of the form:

$$\forall_x \phi(x)$$

where x is the only variable that occurs (free or bound) in $\phi(x)$ and $\phi(x)$ is a truth-functional combination of atomic formulas constructed from V-predicates and its arguments only. This, however, would be too wide: for instance, it would turn out that formulas entailing instances of (3) would qualify as value constraints. Besides, determining a more specific form of value constraints might be useful when we discuss possible ways of revising a dynamic frame.[19] More importantly, if the definition of constraints is such that there are finitely many possible constraints for a frame, this will impact the effective decidability of the consistency of arbitrary sets of constraints and of the set of formulas representing a frame itself. Hence, I will try to specify the form of value constraints in more detail.

The intuition here is that the constraints relate values of different attributes to each other, thus excluding or enforcing certain combinations of values. We want to be able to say that the

[19] Alas, revisions of a conceptual frame lie beyond the scope of this (already long) paper.

presence (or lack) of certain combinations of values (or a certain value) for some attributes (or for a single attribute) entails the presence (or lack) of a certain value (or certain combinations of values) for another attribute (other attributes). A few definitions first.

A set of predicates Π is a **choice set** of the tree if and only if:

(a) Every predicate in Π is a V^l_m, that is, Π is a set of value predicates,

(b) For every k there is an m such that V^k_m is in Π,

(c) For no k there are m, l, $m \neq l$ such that V^k_m and V^k_l are both in Π.

The set of all choice sets of the tree will be called γ. Similarly, a set of predicates Π is a **partial choice set** of the tree if it satisfies the conditions (a) and (c) (condition (b) is dropped).

If a set of predicates Π is non-empty and finite, and $P_1, P_2, \ldots, P_k$ are all the members of Π, by $\bigwedge \Pi(x)$ ($\bigvee \Pi(x)$) I abbreviate $P_1(x) \wedge P_2(x) \wedge \cdots \wedge P_k(x)$ ($P_1(x) \vee P_2(x) \vee \cdots \vee P_k(x)$). I allow the degenerate case when $k = 1$. In this case both $\bigwedge \Pi$ and $\bigvee \Pi$ are the same as the sole member of Π.

A set of (possibly complex) predicates Π with members of the form:

$$\phi(\_\_)$$

is a **constraint set** iff:

(a) Every $\phi(\_\_) \in \Pi$ is either an atomic predicate or a negated atomic predicate.

(b) The set Π^+ obtained from Π by removing all negation symbols is a partial choice set.

Now, the general form of a value constraint is:

$$\forall_x (\bigwedge \Pi_1(x) \rightarrow \bigvee \Pi_2(x)) \tag{4}$$

where Π_1, Π_2 are constraint sets and $\Pi^+_1 \cap \Pi^+_2 = \emptyset$ (the sets are exclusive because we don't want to have constraints that values put on themselves, in a way). Formulas of this form will be called C-formulas.

Remarks. Observe the following:

(a) Given the definition of C-formulas, for any dynamic frame there are finitely many possible different value constraints.

(b) We don't need formulas of the form $\phi \vee \psi \rightarrow \chi$ because they are equivalent to $\{\phi \rightarrow \chi, \psi \rightarrow \chi\}$.

(c) We don't need formulas of the form $\phi \rightarrow \psi \wedge \chi$ because they are equivalent to $\{\phi \rightarrow \psi, \phi \rightarrow \chi\}$.[20]

[20] I say that a formula is equivalent to a set of formulas when all formulas from that set can be classically derived from that formula and the other way round (possibly with background premises if they're explicitly mentioned).

(d) We don't need (well, we even don't want) to have formulas with $\bigwedge \Pi(x)$ in the antecedent where Π contains V_l^k, V_m^k, where $m \neq l$.

(e) It might seem we also need formulas of the form (4) where Π_2 contains both V_m^k and V_l^k ($m \neq l$) in order to express the fact that a certain value combination restricts a selection of values for A_k, but doesn't determine the value for this attribute completely. For instance, we may want to state:

$$\forall_x (V_1^1(x) \rightarrow V_1^2(x) \vee V_2^2(x)) \tag{5}$$

There is, however, a way around it. Let $V_1^2, V_2^2, V_3^2, V_4^2, \ldots, V_m^2$ be all value predicates for A_k. Then, because we've already included all needed formulas of the form (2) and (3), (5) is equivalent to the set of formulas:

$$\begin{array}{rcl} \forall_x (V_1^1(x) & \rightarrow & \neg V_3^2(x)) \\ \forall_x (V_1^1(x) & \rightarrow & \neg V_4^2(x)) \\ & \vdots & \\ \forall_x (V_1^1(x) & \rightarrow & \neg V_m^2(x)) \end{array}$$

which are all C-formulas.

(f) If one feels more comfortable with a restricted version of the constraint, (4) should be replaced by a restricted formula:

$$\forall_x [R(x) \rightarrow (\bigwedge \Pi_1(x) \rightarrow \bigvee \Pi_2(x))]$$

The set describing the frame is now supposed to be extended by a consistent set of C-formulas of one's choice (notice that the consistency of such a set is effectively decidable: every such set is finite and the language is monadic first-order). Let's call this set .

I now have to explain how the activation patters are to be described and how the taxonomical types are to be introduced. The set of **activation patterns** is a subset α of γ. It has to satisfy the following condition:

$$\neg \exists_{\Pi \in \alpha} [\mathbb{R} \cup \mathbb{A} \cup \mathbb{V} \cup \mathbb{C} \models \neg \exists_x \bigwedge \Pi(x)] \tag{6}$$

That is, the activation patterns cannot contain patterns already excluded by the tree structure together with the value constraints.

However, it doesn't have to be the case that:

$$\forall_{\Pi \in \gamma} [\mathbb{R} \cup \mathbb{A} \cup \mathbb{V} \cup \mathbb{C} \not\models \neg \exists_x \bigwedge \Pi(x) \rightarrow \Pi \in \alpha]$$

which means, a choice set doesn't have to be an activation pattern just because it's not excluded by the structure and value constraints. Clearly, α will be finite.

Now take $\alpha^\neg$ to be the set of all those choice sets that are not in α. Suppose $\Pi_1', \Pi_2', \ldots, \Pi_n'$ are all the members of $\alpha^\neg$. We need to say that no object that falls under the root concept falls under one of the Π_i''s. That is, we need n formulas of the form:

$$\neg \exists_x [R(x) \wedge \bigwedge \Pi_i'(x)] \tag{7}$$

The set of all needed formulas falling under (7) will be called $\mathbb{P}$.

Remark. Some cognitivists insist also that the frame embraces what they call "ontological knowledge," that is, the claim that every activation pattern is instantiated. I'm not sure whether this is what we need. For instance, we still think that the taxonomical unit 'tyrannosaurus rex' makes sense, even though, to the best of my knowledge, there are no exemplars alive walking around. Of course, one could say that quantification is not restricted to the present moment etc. Just to avoid issues like that, I'll say: nothing logically relevant for the further discussion hangs on which option you choose (although, if you incorporate ontological knowledge into your framework, it is hard to think of it as a purely *conceptual* framework and that's why I don't do that). If you feel like using the requirement, it will require us to include n formulas of the form:

$$\exists_x \bigwedge \Pi_i(x) \tag{8}$$

for each $1 \leq i \leq n$.

Given that α has n members: $\Pi_1, \ldots, \Pi_n$, we will need n type letters $T_1, \ldots, T_n$ in our language.[21] We introduce n definitions of the form:

$$\forall_x [T_i(x) \equiv \bigwedge \Pi_i(x)] \tag{9}$$

for $1 \leq i \leq n$. Call the set of all formulas of this form $\mathbb{T}$.[22]

This ends our description of the frame. The set of formulas $\mathbb{F}$ that expresses the frame is now defined by:

$$\mathbb{F} = \mathbb{R} \cup \mathbb{A} \cup \mathbb{V} \cup \mathbb{C} \cup \mathbb{P} \cup \mathbb{T} \tag{10}$$

So, a partial dynamic frame can be expressed by a finite set of first-order formulas.

4 Reasoning with frames - the need for adaptivity

A first stab at capturing the notion of reasoning from the set of premises Γ within a dynamic frame represented by $\mathbb{F}$ might be: Γ proves (or entails) a formula ϕ within the framework $\mathbb{F}$ iff $\Gamma \cup \mathbb{F}$ (classically) proves ϕ. A proof of ϕ from Γ within $\mathbb{F}$, on this view, would be just a proof of ϕ whose ultimate premises all belong to $\Gamma \cup \mathbb{F}$. I start with an unproblematic example in which this strategy is employed. Next, I give an example that raises some difficulties and I argue that a different strategy might be needed if we want also to capture the idea of reasoning within a frame even when faced with an anomaly.

Suppose we are reasoning using classical logic within the frame given in fig. 4 (p. 10). Suppose, further, that we introduced one additional constant a for an object, and we know that a is an animal, has lungs and reproduces by laying eggs. We also know that for any

[21] Recall the remark about a finite but sufficiently large set of type letters.

[22] Or we can take the conditional version, if we prefer it:

$$\forall_x [R(x) \rightarrow (T_i(x) \equiv \bigwedge \Pi_i(x))]$$

Given the sane presumption that we'd rather not use the same type predicates for different things depending on whether they fall under the root or not, (9) is enough.

object, this object either isn't a bird or has wings. The first thing we infer is that according to our frame, anything that is an animal, has lungs and reproduces by laying eggs, is a bird, and then we apply universal instantiation to infer that a either isn't a bird or has wings. Then, by disjunctive syllogism, we infer that a has wings.

If we represent the frame by a set of formulas $\mathbb{F}$ according to the instructions already given, this reasoning can be easily emulated in the formalism as a classical proof of $\textsc{Wings}(a)$ from the set:

$$\mathbf{Prem_1} = \mathbb{F} \cup \{\textsc{Animal}(a), \textsc{Lungs}(a), \textsc{Egg}(a), \forall_x(\neg\textsc{Bird}(x) \vee \textsc{Wings}(x))\}$$

First we use appropriate formulas from $\mathbb{F}$ together with our knowledge that $\textsc{Animal}(x)$, $\textsc{Lungs}(a)$ and $\textsc{Egg}(a)$ to infer that a instantiates the activation pattern $\{\textsc{Air}, \textsc{Lungs}, \textsc{Egg}\}$. Then we use an appropriate $\mathbb{T}$-formula to infer $\textsc{Bird}(a)$ and employ:

$$\forall_x(\neg\textsc{Bird}(x) \vee \textsc{Wings}(x))$$

together with universal instantiation and the disjunctive syllogism to infer finally that $\textsc{Wings}(a)$. So far, so good.

In our free time, however, we watch 'Free Willy,' a movie about an orca named Willy (w, for short).[23] Orcas turn out to reproduce by giving birth to live young and yet they live in water. No need to say, as far as our conceptual framework is involved, this movie will change our life.

Now, our premise set is:

$$\mathbf{Prem_2} = \mathbf{Prem_1} \cup \{\textsc{Animal}(w), \textsc{Water}(w), \textsc{Live young}(w)\}$$

Clearly, $\mathbf{Prem_2}$ is classically inconsistent (even though $\mathbf{Prem_2} \setminus \mathbb{F}$ isn't). Our frame tells us that no animal can inhabit the water environment and yet reproduce by giving birth to live young. Now the problem is, at least temporarily, that before we revise our conceptual framework we would like to be able to reason from our premises without being able to derive just anything. How can we do that?

One strategy to use when we hit an inconsistency is to go paraconsistent. That is, we can just replace classical logic with a weaker, non-explosive consequence operation with a fairly manageable proof theory (say, a monotonic logic that can be given a Hilbert-style axiomatization) and use it in our arguments based on $\mathbf{Prem_2}$. For instance, we may want to use **CLuN** as the weaker, non-explosive logic.[24]

[23] Interestingly, it seems that the first scientific description of orcas can be found in a treatise *Systema Naturae* published by Carolus Linnaeus in 1735 in the Netherlands.

[24] **CLuN**, roughly speaking, is what remains from the **CL**assical logic when we drop the consistency requirement and allow gl**U**ts with respect to **N**egation. Axiomatically, it is the full positive **CL** extended with the excluded middle. The negation in **CLuN** is obviously non-classical (it doesn't have quite a few classical prop-

The difficulty with this proposal is that intuitively speaking, the problem that we have pertains only to a "part" of our framework and even though we want to be careful when we reason about mammals or about Willy, we have no reason so far to believe that something's wrong with how we classify an object as a bird or with our inference about a which convinced us that a has wings. Alas, the unproblematic argument that led us to this conclusion relies on the disjunctive syllogism which happens to fail in **CLuN** and thus, our conclusion that a has wings is no longer legitimate if we think that **CLuN** is the logic we need to use. In this sense, if we decide to switch to **CLuN** we not only avoid the contradiction that we wanted to avoid, but we also lose those conclusions and arguments that we had no reasons to suspect.[25] A certain group of formal systems devised to deal with difficulties of this sort are **adaptive logics**. I will now describe the standard format of adaptive logics and then move on to explaining how an adaptive framework suitable for reasoning within a dynamic frame may be constructed.

5 A gentle introduction to adaptive logics

An adaptive logic adapts itself to the premises it is applied to: the correctness of some rules or steps depends on the choice of premises.[26] The basic idea is that while reasoning using an adaptive logic we "swing between" two simpler logics (called the 'lower limit logic,' **LLL**, and the 'upper limit logic,' **ULL**), **ULL** being a strengthening of **LLL**, so that when no problematic formula (details to follow) is derived from a set of premises, we apply **ULL**, and once some premises turn out to lead to difficulties, we restrict ourselves only to those conclusions that we can derive from the problematic premises using **LLL**, even though we may still apply **ULL** to those steps which don't rely on the normal behavior (= falsehood) of those abnormalities that we know follow from the premises. This approach results in adaptive proofs being doubly dynamic.

They are **externally dynamic** because most of them are *nonmonotonic*: once our premise set is extended by *new* input, we might have to cancel some of our previous conclusions if the new information makes the premises unreliable (say, if it depends on the falsehood of some abnormalities which now the new set of premises can derive). They are also **internally dynamic** because even if we keep the premise set *the same*, it may turn out that a conclusion that we **ULL**-derived from it using certain premises is no longer reliable once we discover at

erties of negation). To obtain a version of **CLuN** that's nice to work with it is useful to extend the language of **CLuN** by introducing the classical negation as well, thus running **CLuN** on a language with two negations. For more details, see (Batens 1999).

[25] There are, of course, other variants of this claim, depending on what paraconsistent logic we talk about and on what types of inferences fail in it. The main point stands, though.

[26] Some basic papers about adaptive logics are (Batens 1995, 2004, 2007b). For more references, see the website of the Centre for Logic and Philosophy of Science at Ghent University, http://logica.ugent.be/centrum/writings/.

some later point that those premises also **LLL**-derive a problematic formula and hence those rules that are specific for **ULL** cannot be reliably applied in certain cases.

There is a general form that various adaptive logics instantiate, called the *standard format*. An adaptive logic **AL** is characterized by a choice of:

(a) a lower limit logic **LLL**,

(b) a set of abnormalities Ω, and

(c) a strategy.

LLL is the logic which is taken to hold unconditionally and whose rules are correct even in the case of problematic premises. **LLL** is monotonic and the language contains the classical connectives.[27] For instance, in an inconsistency-adaptive logic the **LLL** will be a non-explosive logic of some sort (say, **CLuN**).

Ω is a set of those formulas which are assumed to be false unless proven otherwise. Batens (2007a: 130-131) requires that Ω should be a set of formulas of a certain logical form, so that "it enables one to consider adaptive logics as formal logics."[28] For instance, in an inconsistency-adaptive logic one might take the abnormalities to be formulas of the form $\phi \wedge \neg\phi$ (or existential closures of those, if the language is first-order rather than purely sentential). **LLL** together with Ω determine **ULL**. The upper limit logic is just the lower limit logic plus the assumption that the elements of Ω are false. Let Γ be a set of formulas. If we define:

$$\Gamma^{\neg} = \{\neg\phi \mid \phi \in \Gamma\} \tag{11}$$

and we represent the provability relations of various logics by the subscripted '$\vdash$' symbol, we can put this point concisely:

$$\Gamma \vdash_{ULL} \phi \text{ iff } \Gamma \cup \Omega^{\neg} \vdash_{LLL} \phi \tag{12}$$

A strategy, basically, is a set of instructions that tell us when we have to cancel a conclusion that we obtained previously or when to remove the cancelation symbol from a line (technically, the activity is called *marking*). In order to be able to explain one of the simplest strategies, however, I have to say a few words about dynamic proofs.

A dynamic proof is a sequence of lines, which can be taken to consist of four main components: a *line number*, a *formula*, a *justification* for that formula, and a set of *conditions* upon which the formula is derived. Besides, each line can be marked (marks can come and go as

[27] Even though the logic doesn't have to be classical itself.

[28] When I will be constructing an adaptive framework for arguments given within a dynamic frame, I will drop this assumption and require only that Ω should be effectively decidable: there should be a decision procedure which for any formula in a finite number of steps gives an answer as to whether the formula belongs to Ω or not. I will elaborate on the rationale for this move once we get to that stage.

the proof progresses). If a line is at some point marked, it means that the formula in this line is not considered derived at that stage. The first three components are fairly self-explanatory. Conditions and marking require some more attention.

Recall that our task in a proof is not only to derive formulas from premises but also to recognize those steps that can't be trusted and to cancel those conclusions which rely on the normal behavior of abnormalities which we have reasons to believe not to behave normally. Now, what do we mean when we say that a step can't be trusted? At the first stab, one might try to say that a step is unreliable if it depends on the falsehood of an abnormality which as it turns out follows from the premises. This is quite close. There's one minor complication, though. A premise set may prove a disjunction of abnormalities without proving any of its disjuncts separately, thus, in a sense, implying that at least one of them has to behave erratically, but not telling us that there is a single disjunct that we can blame. To keep track of these things, in the *conditions* column we put sets of those abnormalities on whose normal behavior (=falsehood) the present step relies. And (on one of the simplest strategies) we mark a line as unreliable if it depends on (the normal behavior of) a set of abnormalities Δ, and some member of Δ is a disjunct in a (minimal)[29] disjunction of abnormalities **LLL**-derived from the premises. Let's take a look at dynamic proofs in a bit more detail.

There are three rules for **AL**-proving formulas from Γ. The first rule, PREM allows one to introduce any premise $\phi \in \Gamma$ with the empty set in the *conditions* column. That is, if $\phi \in \Gamma$, infer:

$$\phi \quad \text{PREM} \quad \emptyset$$

where the first, empty column normally contains an appropriate line number.

The second rule, RU says that if we have proven ϕ_1 on the assumption that Δ_1 behaves normally, ϕ_2 on the assumption that Δ_2 behaves normally, ..., and ϕ_n on the assumption that Δ_n behaves normally, and if ψ can be **LLL**-derived from $\phi_1, \dots, \phi_n$, we can introduce ψ as relying on the normal behavior of $\Delta_1 \cup \Delta_2 \cup \dots \cup \Delta_n$. That is, if:

$$\{\phi_1, \dots, \phi_n\} \vdash_{LLL} \psi$$

then from:

$$\begin{array}{ll} \phi_1 & \Delta_1 \\ \phi_2 & \Delta_2 \\ \vdots & \\ \phi_n & \Delta_n \end{array}$$

we can derive:

[29] This notion and the rationale for its introduction will be explained below.

$$\phi \quad \textsc{Ru} \quad \Delta_1 \cup \Delta_2 \cdots \cup \Delta_n$$

That is, we can add **LLL**-consequences relying on nothing more and nothing less than the union of those sets on which the premises depended.

The third rule, Rc is based on the following idea. If from Γ we can **LLL**-derive that either ψ is true or one of the abnormalities in a set Δ occurs, we can conclude that ψ **AL**-follows from Γ on the assumption that Δ behaves normally. If Δ is a finite set of abnormalities, let us call the classical disjunction of the members of Δ '$Dab(\Delta)$.' Just like before, assume we have proven ϕ_1 on Δ_1 (that's an obvious piece of jargon that I'll use instead of saying 'we have proven ϕ_1 on the assumption that the set of abnormalities Δ_1 behaves normally'), ϕ_2 on Δ_2, ..., and ϕ_n on Δ_n:

$$\begin{array}{cc} \phi_1 & \Delta_1 \\ \phi_2 & \Delta_2 \\ & \vdots \\ \phi_n & \Delta_n \end{array}$$

Rc now tells us that if for some finite $\Theta \subseteq \Omega$:

$$\{\phi_1, \ldots, \phi_n\} \vdash_{LLL} \psi \vee Dab(\Theta)$$

we can infer:

$$\phi \quad \textsc{Rc} \quad \Delta_1 \cup \Delta_2 \cdots \cup \Delta_n \cup \Theta$$

Now the most tricky part: *marking for the reliability strategy.*[30] A proof, as we conduct it, can be considered as proceeding in stages. Every application of a rule carries us to the next stage. A formula $Dab(\Delta)$ (i.e., a classical disjunction of abnormalities) is a **minimal *Dab* formula** of a proof at a stage s iff $Dab(\Delta)$ occurs in the proof at a line with the condition $\emptyset$ and for no $\Delta' \subset \Delta$ (that is, for no Δ' which is a proper subset of Δ) the proof contains at s a line with $Dab(\Delta')$ which has $\emptyset$ as the condition.[31]

The intuitive reason why we are interested in *minimal* Dab-formulas of a proof instead of just *any* proven Dab-formulas whatsoever is this. We want Dab-formulas to help us discover those abnormalities on whose normal behavior we can't rely. The fact that $Dab(\Delta)$ has been proven tells us only that at least one member of Δ has to be true if the premises are to be true (at least as long as **LLL** is truth-preserving). However, we want to assume that as many

[30]The reliability strategy is a fairly simple strategy of marking. There are others. For the sake of this paper, I will however stick to it.

[31]Containing a line at stage s doesn't mean that this line was introduced at stage s. It only means that when we take the proof as it is in stage s, this line is one of the lines of the proof at that stage.

abnormalities are false as possible and we take any abnormality to be false unless really forced to do otherwise. Thus, if we know that both $Dab(\Delta)$ and $Dab(\Delta')$ are **LLL**-derived from our premises, but also $\Delta' \subset \Delta$, we know that we don't really have to blame any member of $\Delta \setminus \Delta'$ for this fact. If we want to deal with as few abnormalities being **LLL**-derived as possible, it's enough to assume that it is the members of Δ' that we can't rely on.

Hence, we first define $U_s(\Gamma)$ to be the union of all Δ's that are constituents of those minimal *Dab*-formulas that have been derived so far from Γ at stage s. Then, we mark a line in a proof as canceled (or unreliable) if it depends on the normal behavior of Θ, and yet at least one member of Θ is a member of $U_s(\Gamma)$. That is, if a certain line relies on the falsehood of all abnormalities from a certain set, and yet at least one member of this set is among those abnormalities on whose normal behavior we can't rely, the formula in the line itself turns out to be unjustified.

One last notion that we need: *final derivability.* Since we are sometimes allowed to cancel a line that we derived before, the fact that a line is derived does not mean that it actually **AL**-follows from the premises. Hence we also introduce the notion of final derivability. A formula ϕ is finally derived in a proof if it is derived in an unmarked line of that proof and also no way we can continue the proof will force us to finally mark that line (that is, any extension of the proof which would force us to mark the line can always be itself extended into a proof where this line is unmarked). There are two ways this can happen:

(a) ϕ follows from our premises by **LLL**, in which case it is derived on $\emptyset$ and then it is vacuously true that we cannot prove any abnormality on whose normal behavior it relies (just because it doesn't relies on the normal behavior of any abnormality), and

(b) For some non-empty set of abnormalities Θ, our derivation of ϕ depends on the normal behavior of all the members of Θ, but no member of Θ is a member of a minimal *Dab*-formula **LLL**-derivable from the premises.

Here's an example that might help to see how the machinery works. Let's take **LLL** to be **CLuN**. We can construct an adaptive logic where all substitution instances of $(\exists)(\phi \wedge \neg\phi)$ are the constituents of Ω,[32] and we quite naturally obtain **CL** (classical logic) as the upper limit logic. Take the adaptive logic thus obtained and consider the example where our premise set is:

$$\mathbf{Prem_3} = \{\neg p, p \vee q, r, \neg r, r \vee s\}$$

Before we get to our adaptive proof, observe three facts:

$$\{p \vee q, \neg p\} \vdash_{CLuN} q \vee (p \wedge \neg p) \tag{13}$$

[32] That is, if ϕ is purely propositional we just take $\phi \wedge \neg\phi$ and if it is first-order we take $\phi \wedge \neg\phi$ and bind all free variables by existential quantifiers.

$$\{p \vee q, \neg p, r, \neg r, r \vee s\} \nvdash_{CLuN} p \wedge \neg p \tag{14}$$

$$\{r, \neg r\} \vdash_{CLuN} r \wedge \neg r \tag{15}$$

In the proof we first write down the premises:

1.	$p \vee q$	PREM	$\emptyset$
2.	$\neg p$	PREM	$\emptyset$
3.	r	PREM	$\emptyset$
4.	$\neg r$	PREM	$\emptyset$
5.	$r \vee s$	PREM	$\emptyset$

Since $p \wedge \neg p$ is an abnormality and $q \vee (p \wedge \neg p)$ **LLL**-follows from lines 1 and 2, we can apply RC to lines 1 and 2 and conclude q, relying on the normal behavior of $p \wedge \neg p$:

6.	q	RC: 1, 2	$\{p \wedge \neg p\}$

Similarly, $s \vee (r \wedge \neg r)$ is **LLL**-derivable from lines 4 and 5, and hence:

7.	s	RC: 4, 5	$\{r \wedge \neg r\}$

However, we are faced with the following difficulty: $\mathbf{Prem_3}$ is clearly problematic. Our input data inform us that r doesn't behave normally. In fact, according to our premises, both r and $\neg r$ are true. But then, should we really conclude that s is true just because $r \vee s$ is true? The usual rationale behind the disjunctive syllogism ($\phi \vee \psi, \neg\phi \vdash \psi$) is this: $\phi \vee \psi$ tells us that at least one of the involved sentences is true; $\neg\phi$, however, tells us that ϕ is false and hence it cannot be true. Therefore, the only option is that it is ψ that is true and its truth accounts for the truth of $\phi \vee \psi$. On the paraconsistent approach, however, a formula and its negation can be both true: the claim that $\neg\phi$ is true doesn't entail the claim that ϕ is false. For this reason, when both r and $\neg r$ are true the disjunctive syllogism applied to $\neg r$ and $r \vee s$ might fail. For even if s is still (only) false, $r \vee s$ will come out true in virtue of r being true. Hence, our premise set **LLL**-proves an abnormality: $r \wedge \neg r$:

8.	$r \wedge \neg r$	RU: 3, 4	$\emptyset$

This also shows that $r \wedge \neg r$ is not an abnormality on whose falsehood we can rely: it is actually derivable from our premises. But this, according to our marking regulations requires us to cancel any conclusion that relied on the normal behavior of $r \wedge \neg r$, which (in our case) means that we have to withdraw line 8 (which I mark by putting '$\curlyvee$' besides a line):

7.	s	RC: 4, 5	$\{r \wedge \neg r\}$	$\curlyvee$
8.	$r \wedge \neg r$	RU: 3, 4	$\emptyset$	

So, as things stand, the only line with s as its formula has been canceled. It might be the case, however, that a formula actually is **AL**-derivable from a set of premises even though it is a formula of a marked line. The fact that ϕ is a formula in a marked line only means that the way it is introduced in that line is unreliable. This doesn't have to mean that ϕ is not **AL**-derivable from the premises in general. Is it the case with s? A moment of consideration should convince us that it isn't. From these premises, there is no other sensible way we could introduce s without relying on the normal behavior of $r \wedge \neg r$.

If we were simply reasoning in **CLuN** we also wouldn't be allowed to introduce line 7 either, because we wouldn't be allowed to use disjunctive syllogism in general. In our **AL**, however, we can still keep line 7 as long as the abnormality on which it depends is not **CLuN**-derived from our premises. That is, q is finally derivable if no possible extension of our proof derives on $\emptyset$ a minimal *Dab*-formula $Dab(\Delta)$ with $(p \wedge \neg p) \in \Delta$ such that for no $\Delta' \subseteq \Delta$, $(p \wedge \neg p) \notin \Delta'$ $Dab(\Delta')$ is **LLL**-derivable from the premises. Luckily, the question whether these formulas are **CLuN**-derivable from $\mathbf{Prem_3}$ is decidable (as long as we stay on the level of propositional logic). Hence we can consider q to be finally derived, even though s is not derivable.

6 Adaptivity and dynamic frames

A strategy of dealing with problems like those raised in section 4 seems to emerge now: take an inconsistency-adaptive logic of your choice and model reasoning from Γ within a frame $\mathbb{F}$ as reasoning by this logic from $\Gamma \cup \mathbb{F}$. This is the approach that I will *not* take.

First off, going inconsistency-adaptive would require some serious philosophical heavy lifting. I would have to choose a paraconsistent lower limit logic and this would require a justification. There are quite a few paraconsistent logics and justifying the claim that this one and not another is more suitable for the purpose of reasoning within a dynamic frame would require some effort. I'm not saying it's undoable: it's just an arduous and, as it will turn out, unnecessary task. We will be able to handle examples like those described in section 4 without using any paraconsistent logic, and hence, I will not employ the inconsistency-adaptive approach. If it ain't broken, don't fix it.

Secondly, blending Γ (in the intended interpretation: our empirical data and laws) and $\mathbb{F}$ together this way hides the fact that the epistemic status of our conceptual framework is quite different from the status of the members of Γ. It's better to keep them apart. For instance, there is an essential difference between the case when $\Gamma \cup \mathbb{F}$ is inconsistent even though Γ is consistent and the case when $\Gamma \cup \mathbb{F}$ is inconsistent because Γ is. We usually don't want to say that something's wrong with our conceptual framework because our *data* in Γ are inconsistent. Rather, before we go on to criticize and revise our framework we try to convince ourselves that we really have to do that: that it is *not* a problem with our data. This is also the reason why I will only need to consider cases where Γ is a consistent set, and pretty much the only way problems may arise is when we use Γ with the conceptual frame that we have.

To encode adaptive reasoning within a dynamic frame it'll be useful to use a close relative of an adaptive logic, which I will call an **adaptive framework**. The difference is that in the definition of an adaptive logic, Ω has to be determined by a (possibly restricted) logical form, whereas in an adaptive framework I only require that Ω be effectively decidable. That is, there should be a procedure which for any formula will tell us in a finite number of mechanical steps whether this formula belongs to Ω or not. There are two questions that come to mind: why do I drop Batens' requirement that Ω should be formally uniform? Why do I require that Ω should be effectively decidable?

Batens requires that the set of abnormalities Ω be formally uniform, if the system is to be counted as a formal logic at all, and I completely agree. This, however, doesn't mean that dropping the requirement can't give us a workable theory. Adaptive frameworks are not logics *sensu stricto*. As will be clear in a moment, I will use Ω to encode our conceptual framework. But our conceptual frameworks go beyond logic and the whole point of having them is that they allow us to treat certain predicates differently than certain other predicates.

However, Ω has to be effectively decidable, otherwise our definition of marking would go astray. Recall that the marking of a line depends on whether a certain minimal *Dab*-formula has been derived. If there were no procedure for finding out whether a formula is in Ω, there would be no procedure for finding if a formula is a *Dab* formula and it could happen that we actually derived a *Dab* formula but don't know that it is one. This is the situation that we want to avoid.[33]

So here is the trick. Take a (possibly) monadic first order language which contains all the predicates needed to construct $\mathbb{F}$. To model reasoning from consistent data within a dynamic frame expressed by $\mathbb{F}$, take **CL** to be the lower limit logic, use, say, the reliability strategy for marking, and take Ω to be $\mathbb{F}^{\neg}$, which is a set obtained from $\mathbb{F}$ by (i) preceding all formulas in $\mathbb{F}$ with a negation, and then (ii) deleting all double negations that occur in front of a formula. Here is where our lengthy construction of $\mathbb{F}$ pays off: since $\mathbb{F}$ is finite, $\mathbb{F}^{\neg}$ is an effectively decidable set, so it can serve as the set of abnormalities.

The intuition is that once we have a consistent set of data that we want to reason from, our conceptual framework allows us to go beyond what we can infer using **CL**, it strengthens our inferential powers. Given that we assume that the data set is consistent, we take our conceptual framework for granted unless proven otherwise. An interesting twist here is that the knowledge provided by a dynamic frame is not, in a sense, analytic. New empirical data may force us to reject our conceptual framework and rearrange our predicates so that they

[33]However, I have to note that it is possible to massage the adaptive frameworks into the standard format. On one hand, this requires a translation that uses modalities and the proofs are slightly less perspicuous. One the other hand, if we get a system in the standard format we get lots of metatheory practically for free (see Batens 2007b). The issue lies beyond the scope of this already long paper, though.

agree better with what we know (this is not to say that certain constituents of a dynamic frame cannot be more entrenched than others). The adaptive framework obtained from $\mathbb{F}$ will be called $\mathbf{AL}_{\mathbb{F}}$.

To see how this approach handles situations like those described in section 4, consider how we now can deal with that situation. First, I list those abnormalities in the adaptive framework generated by the frame from fig. 4 that we'll need for our proof. First, negating two formulas from $\mathbb{R}$ gives us:

$$\neg\forall_x(\textsc{Animal}(x) \rightarrow \textsc{Inhabits}(x)) \tag{16}$$

$$\neg\forall_x(\textsc{Animal}(x) \rightarrow \textsc{Resp. Org.}(x)) \tag{17}$$

Next, (the negations of) two of the $\mathbb{A}$-formulas will yield:

$$\neg\forall_x(\textsc{Inhabits}(x) \rightarrow \textsc{Air}(x) \vee \textsc{Land}(x) \vee \textsc{Water}(x)) \tag{18}$$

$$\neg\forall_x(\textsc{Resp. Org.}(x) \rightarrow \textsc{Lungs}(x) \vee \textsc{Gill}(x)) \tag{19}$$

Abnormalities resulting from four of our $\mathbb{P}$-formulas will be:

$$\exists_x(\textsc{Animal}(x) \wedge \textsc{Lungs}(x) \wedge \textsc{Egg}(x) \wedge \textsc{Land}(x)) \tag{20}$$

$$\exists_x(\textsc{Animal}(x) \wedge \textsc{Lungs}(x) \wedge \textsc{Egg}(x) \wedge \textsc{Water}(x)) \tag{21}$$

$$\exists_x(\textsc{Animal}(x) \wedge \textsc{Water}(x) \wedge \textsc{Live Young}(x) \wedge \textsc{Lungs}(x)) \tag{22}$$

$$\exists_x(\textsc{Animal}(x) \wedge \textsc{Water}(x) \wedge \textsc{Live Young}(x) \wedge \textsc{Gill}(x)) \tag{23}$$

And we need one abnormality resulting from one of our $\mathbb{T}$-formulas:

$$\neg\forall_x(\textsc{Animal}(x) \rightarrow (\textsc{Bird}(x) \equiv \textsc{Air}(x) \wedge \textsc{Lungs}(x) \wedge \textsc{Egg}(x))) \tag{24}$$

We start with writing down the premises:

1.	$\textsc{Animal}(w)$	Prem	$\emptyset$
2.	$\textsc{Water}(w)$	Prem	$\emptyset$
3.	$\textsc{Live Young}(w)$	Prem	$\emptyset$
4.	$\textsc{Animal}(a)$	Prem	$\emptyset$
5.	$\textsc{Lungs}(a)$	Prem	$\emptyset$
6.	$\textsc{Egg}(a)$	Prem	$\emptyset$
7.	$\forall_x(\neg\textsc{Bird}(x) \vee \textsc{Wings}(x))$	Prem	$\emptyset$

Now, let's focus on proving $\textsc{Wings}(a)$ first. Since the negation of (16) together with our premises **CL**-proves $\textsc{Inhabits}(a)$, we have:

$$8. \quad \textsc{Inhabits}(a) \quad \textsc{Rc}\text{: } 4 \quad \{(16)\}$$

which together with the negation of (18) **CL**-proves $\textsc{Air}(a) \vee \textsc{Land}(a) \vee \textsc{Water}(a)$, so:

$$9. \quad \textsc{Air}(a) \vee \textsc{Land}(a) \vee \textsc{Water}(a) \quad \textsc{Rc}\text{: } 4 \quad \{(16), (18)\}$$

But lines 4, 5, 6 and 9, if we assume the negations of (20) and (21) derive $\textsc{Air}(a)$:

$$10. \quad \textsc{Air}(a) \quad \textsc{Rc}\text{: } 4, 5, 6, 9 \quad \{(16), (18), (20), (21)\}$$

and thanks to a definition whose negation is (24) we obtain:

$$11. \quad \textsc{Bird}(a) \quad \textsc{Rc}\text{: } 4, 5, 6, 10 \quad \{(16), (18), (20), (21), (24)\}$$

We now apply universal instantiation to line 7:

$$12. \quad \neg\textsc{Bird}(a) \vee \textsc{Wings}(a) \quad \textsc{Ru}\text{: } 7 \quad \emptyset$$

And clearly, lines 11 and 12 **CL**-derive $\textsc{Wings}(a)$:

$$13. \quad \textsc{Wings}(a) \quad \textsc{Ru}\text{: } 11, 12 \quad \{(16), (18), (20), (21), (24)\}$$

Now comes the problematic part: it seems that we can equally well prove that Willy the orca has wings. First we rely on (17) to obtain $\textsc{Resp. Org.}(w)$, then we use (19) to get $\textsc{Lungs}(w) \vee \textsc{Gill}(w)$, which together with lines 1, 2, 3, thanks to (22) and (23) generate a contradiction, and hence, by our **LLL**, any conclusion whatsoever, $\textsc{Wings}(w)$ included:

14.	$\textsc{Resp. Org.}(w)$	$\textsc{Rc}$: 1	$\{(17)\}$
15.	$\textsc{Lungs}(w) \vee \textsc{Gill}(w)$	$\textsc{Rc}$: 14	$\{(17), (19)\}$
16.	$\textsc{Wings}(w)$	$\textsc{Rc}$: 1, 2, 3, 15	$\{(17), (19), (22), (23)\}$

So, is there any essential difference between lines 11 and 16? As it turns out, yes, there is. Line 11 relies on the normal behavior of $\{(16), (18), (20), (21), (24)\}$ and for any *Dab*-formula in which one of those sentences occur, which is provable from our premises, there is a *Dab*-formula provable from these premises which is a subformula of this formula in which none of those sentences occurs. On the other hand the disjunction of $\{(17), (19), (22), (23)\}$ is **CL**-provable from the premises. This, however, means that on the normal behavior of these abnormalities we can't rely and hence, we have to mark line 16 once we learn that this disjunction follows:

16.	$\textsc{Wings}(w)$	$\textsc{Rc}$: 1, 2, 3, 15	$\{(17), (19), (22), (23)\}$	$\curlyvee$
17.	$(17) \vee (19) \vee (22) \vee (23)$	$\textsc{Ru}$: 1, 2, 3	$\emptyset$	

Observe that our ability to 'isolate' the problematic parts of our framework and reason normally with those parts that don't raise any difficulties is highly sensitive to the way $\mathbb{F}$ is constructed. For instance, if instead of n $\mathbb{R}$-formulas, one for each A_i, we had only one formula of the form:

$$\forall_x (R(x) \to \bigwedge_n A_n(x))$$

we wouldn't be able to separate the two conclusions from the above example, in the sense that both would depend on the sole $\mathbb{R}$ formula which would occur in a minimal *Dab* formula derived from the premises, and therefore we would have to cancel them both. This fact indicates that logically equivalent descriptions of a frame don't have to be equally useful. The way we describe a frame is also important — it mirrors our convictions about what parts of the frame are 'separable' and immune to difficulties that other parts are susceptible to. Also, it is important to notice that the construction method I described isn't the only possible way a finite set of formulas capturing a frame may be constructed. Different constructions, mirroring different assumptions about the independence of various aspects of a frame are possible.

7 Summary

After briefly mentioning the motivations for our interest in dynamic frames: the rejection of the classical theory of concepts and the psychological evidence for the adequacy of the theory of dynamic frames, I explained the dynamic frame account of concepts, thus setting the background for the formal development. Next, I provided one of quite a few possible ways of representing a partial, grounded and finitary dynamic frame by means of a finite set of formulas of a monadic first-order language. Having done that, I intuitively explained why the first attempt to model reasoning within a frame as reasoning with the set of premises extended by the set that describes that frame might be unsatisfactory if we encounter an anomaly, and thus I provided motivation for developing an adaptive approach to arguments given in such circumstances. Then I spent some time explaining what adaptive logics are and how the machinery is supposed to work in general. Then, I defined an adaptive approach to dynamic frames and showed how it can handle the problematic examples elaborated on in this paper.

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