An Exact Truthmaker Semantics for Explicit Permission and Obligation

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Abstract

We develop an exact truthmaker semantics for explicit permission and obligation. The idea is that with every singular act, we associate a *sphere of permissions* and a *sphere of requirements*: the acts that are rendered permissible and the acts that are rendered required by the act. We propose the following clauses for explicit permissions and obligations:

- a singular act is an exact truthmaker of $P\varphi$ iff every exact truthmaker of φ is in the sphere of permissibility of the act, and
- a singular act is an exact truthmaker of $O\varphi$ iff some exact truthmaker of φ is in the sphere of requirements of the act.

We show that this semantics is *hyperintensional*, and that it can deal with many of the so-called *paradoxes of deontic logic* in a natural way. Finally, we give a sound and complete axiomatization of the semantics.

1 Introduction

The aim of this paper is to develop an exact truthmaker semantics for explicit permission and obligation.

The basic idea of exact truthmaker semantics is that we can give the semantic content of a statement by saying what precisely in the world makes the statement true: by giving its exact truthmakers. Intuitively, an exact truthmaker of a statement is a state (of affairs) such that whenever the state obtains it is directly and wholly responsible for the truth of the statement. In particular, an exact truthmaker of a statement will not contain as a part any other state that is not wholly responsible for the truth of the statement. So, for example, the state of the pen being black is an exact truthmaker of the statement "the pen is black." But the complex state of the pen being black and full of ink is *not* an exact truthmaker of the statement, since it contains as a part the state of the pen being full of ink, which is irrelevant to the truth of "the pen is black." This idea traces back to a paper by Bas van Fraassen [11]. But in recent work, Fine uses it to give truth-conditions for: counterfactual conditionals [3], metaphysical ground [4], permission[5], and partial content and analytic equivalence [6].[1]

It turns out that the framework of exact truthmaker semantics has a natural action-theoretic interpretation: we can take an exact truthmakers of a sentence to be a concrete singular act, such that the performance of the act is is directly and wholly responsible for the truth of the sentence. For example, on this interpretation, President Obama's act of refilling the pen on Monday morning at 7 a.m. would be an exact truthmaker of the statement "Obama refills the pen." In contrast, Obama's act of refilling the pen and spilling his coffee would *not* be an exact truthmaker of the statement, because it has as a part the irrelevant act of Obama spilling his coffee. In this paper, we will use this interpretation to provide a natural semantics for explicit permission and obligation: permissions and obligations, which are the direct result of normative acts.

Once we interpreted the exact truthmaker framework in this way, there is a natural way to obtain truth-conditions for explicit permissions and obligations. For this purpose, let's assume that we're given a set of normatively admissible and a set of normatively required acts. Then we can say:

- a statement of the form $P\varphi$ is true iff every act that is an exact truthmaker of φ is admissible,[2] and
- a statement of the form $O\varphi$ is true iff some act that is required is an exact truthmaker of φ.

But this only gives us the *truth-conditions* for explicit permissions and obligations, and not their exact truthmakers. And from the perspective of exact truthmaker semantics, this means that these clauses don't give us the *content* of explicit permissions and obligations. To make things worse, the clauses cannot be applied to *iterated* permissions and obligations, where a permission or obligation occurs in the context of another permission or obligation. To see this, consider a statement of the form $OP\varphi$, for example. According to the above truth-conditions, we get:

- a statement of the form $OP\varphi$ is true iff some act that is required *is an exact truthmaker of* $P\varphi$.

But since we don't know what an exact truthmaker of $P\varphi$ is, we can't ascertain the truth-value of $OP\varphi$. In this paper, we shall propose recursive clauses for the exact truthmakers of explicit permissions and obligations, which can deal with these issues.

We propose that with every act there is associated a set of acts that, as a result of the act being performed are admissible, and a set of acts that, as a result of the act being performed are required: we associate with every act a *sphere of permissions* and a *sphere of requirements*. For example, if we consider John's act of checking in at the airport. This act permits him to proceed to his gate, but it obligates him to keep his luggage with him at all times. Thus, the act of John going to the gate is in the sphere of permissions of him checking in, and the act of John keeping his luggage with him is in the act's sphere of

[1] Note that Fine only gives *truth-conditions* for the concepts in question and not their exact truthmakers.

[2] Such a clause is essentially proposed by Fine [5].

obligations. We can then give the following clauses for the exact truthmakers of explicit permissions and obligations:

- an act is an exact truthmaker of $P\varphi$ iff every exact verifier of φ is in the sphere of permission of the act, and
- an act is an exact truthmaker of $O\varphi$ iff some exact verifier of φ is in the sphere of requirements of the act.

In the following, we shall develop this informal idea in formal detail.

2 The Semantics

To develop our semantics, we assume that we're given a non-empty set A of *atomic singular acts*. These acts correspond to the concrete atomic acts an agent might perform, like Obama's concrete action of refilling the pen, for example. We then say that a *complex singular act* (over A) is a set of atomic acts:

$$X \text{ is a complex singular act iff } X \subseteq A.$$

Complex acts are "aggregates" of atomic acts, which we think of as being performed together, like the concrete act of Obama refilling the pen *and* spilling the coffee. [3] We denote the set of complex acts (over A) by $\mathbf{A}$, i.e. $\mathbf{A} = \wp(A)$. A *generic action* over A is a set of acts over A:

$$\mathcal{X} \text{ is a generic action iff } \mathcal{X} \subseteq \mathbf{A}.$$

A generic action is a collection of complex acts, which we think of as the different ways of performing the generic action. For example, there are various concrete ways in which Obama can refill the pen, e.g. he may refill it with blue ink, black ink, green ink etc. All these concrete acts are realizations of the same generic action of refilling the pen. A similar phenomenon can be found in metaphysics: various (concrete) objects can be concrete instances of one and the same (abstract) type. Obviously, the same holds for singular acts and generic actions: there are numerous (concrete) ways in which Obama can refill the pen, all of which are instances of the (abstract) type *Obama-refills-the-pen*. Hence and in line with the usual terminology, we will occasionally use 'action token' to talk about a singular act (atomic or complex), and 'action type' to talk about a generic action.

We denote the set of generic actions over A by $\mathbf{T}$, i.e. $\mathbf{T} = \wp(\mathbf{A})$. Finally, we assume that some subset $Ex \subseteq A$ of atomic singular acts are *executed*. We say that a complex singular act $X \in \mathbf{A}$ is executed iff *all* the members of X are executed:

[3] Two short comments are in order here. First, whenever we talk about concrete singular acts (atomic or complex), we do not presuppose that they are actually executed. "concrete singular acts" rather means "(possible) concrete singular acts", and we will introduce executed singular acts later.

Second, how to distinguish between atomic and complex singular acts certainly is an interesting philosophical question. Here we do not deal with this question though, and rather assume that this distinction is useful. However, nothing hinges on that. To get the theory off the ground, all we need is that we can individuate concrete acts $a_1 \ldots, a_n$ to construct the set $A = \{a_1, \ldots, a_n\}$ of concrete (atomic) acts. Anyone who deems the distinction between atomic and complex singular acts to be meaningless, may just take the singletons of A to be "complex" generic acts, which, in a sense, eliminates the distinction.

$$X \text{ is executed iff } X \subseteq Ex.$$

We denote the sets of executed complex singular acts by $\mathbf{Ex}$, i.e. $\mathbf{Ex} = \wp(Ex)$. And a generic action $\mathcal{X} \in \mathbf{T}$ is realized iff *some* member $X \in \mathcal{X}$ is executed:

$$\mathcal{X} \text{ is realized iff } \mathcal{X} \cap \mathbf{Ex} \neq \emptyset$$

Thus, we can think of a generic action as a disjunctive list of conjunctive complex acts. To realize a generic action means to execute (at least) one such complex act. We will call a structure of the form (A, Ex) an *action frame*. Structures of this form are the action theoretic backdrop to our semantics.

If (A, Ex) is an action frame, then we'll assume that we're given for every singular act $x \in A$, both a *sphere of permissions* $Ok_x \subseteq \mathbf{A}$ and a *sphere of obligations* $Req_x \subseteq \mathbf{A}$. Intuitively, the members of Ok_x for a (singular) act $x \in A$ are exactly those (complex) acts that are rendered normatively admissible by x: it is a normative consequence of x being executed that all members of Ok_x are admissible. Similarly, the members of Req_x are the acts that are rendered required by the performance of x: it is a normative consequence of x being executed that all members of Req_x are required.[4]

Let us consider an example.[5] Suppose that Johannes executes the following, concrete act: he buys a day ticket on March 7, 2016 at 8am for the public transport in Munich (a_1). This renders quite a number of other concrete acts admissible: He may take the U3 at 8:04am and go to Moosach (a_2). He may take the U6 at 8:08am to go to Marienplatz (a_3). Since Johannes bought a day ticket, he is also entitled to take the S3 after work at 7pm from Marienplatz to go to Haidhausen (a_4). And so on. In our formal framework, this is expressed by $Ok_{a1} = \{\{a_2\}, \{a_3\}, \{a_4\}, \ldots\}$.

For a complex act $X \in \mathbf{A}$, we define the set Ok_X to be $\bigcup_{x \in X} Ok_x$ and Req_X to be $\bigcup_{x \in X} Req_x$. Thus, intuitively the members of Ok_X for an act $X \in \mathbf{A}$ are the acts that are rendered admissible by the performance of all the members of X and the members of Req_X are the acts that are rendered required by the performance of all the members of X. We call a structure of the form $(A, Ex, (Ok_x)_{x \in A}, (Req_x)_{x \in A})$, where (A, Ex) is an action frame and $((Ok_x)_{x \in A}, (Req_x)_{x \in A})$ are spheres of permissions and obligations for every act $x \in A$ a *deontic action frame*. Thus, a deontic action frame consists of a basic action theoretic structure together with a normative framework on top, which determines the normative consequences of actions.

Following von Wright [12], we take formulas of our language to represent action types. More formally, if $(A, Ex, (Ok_x)_{x \in A}, (Req_x)_{x \in A})$ is a deontic action frame, then we assign to every atomic formula p an action type $V(p) \in \mathbf{T}$, where we think of the members of $V(p)$ as all concrete actions that exactly realize what's expressed by p under V. We furthermore assign to every atomic formula p an action type $F(p) \in \mathbf{T}$, where we think of the members of $F(p)$ as all those actions that exactly prevent what's expressed by p under F. For example, a (possibly complex) concrete action of Obama signing a particular document with the pen is a member of $F(Obama - refills - the - pen)$.[6] We extend

[4] Note that not all acts have to be *normatively significant*, i.e. Ok_x and Req_x can also be empty.

[5] For reasons of simplicity, but without loss of generality, we take all the singular acts in the example to be atomic.

[6] He we assume that an exact realization of writing with the pen prevents Obama from refilling it at the same time, i.e. that it is not possible to do both.

the verifier and falsifiers to arbitrary propositional formulas by a simultaneous recursion on the construction of formulas using van Fraassen's clauses [11]:

- $V(\neg\varphi) = F(\varphi)$
- $F(\neg\varphi) = V(\varphi)$
- $V(\varphi \vee \psi) = V(\varphi) \cup V(\psi)$
- $F(\varphi \vee \psi) = \{X \cup Y \mid X \in F(\varphi), Y \in F(\psi)\}$
- $V(\varphi \wedge \psi) = \{X \cup Y \mid X \in V(\varphi), Y \in V(\psi)\}$
- $F(\varphi \wedge \psi) = F(\varphi) \cup F(\psi)$

If $\mathcal{F} = (A, Ex, (Ok_x)_{x\in A}, (Req_x)_{x\in A})$ is a deontic action frame and V and F are truthmaker assignments of the sort just described, then $(\mathcal{F}, V, F)$ is a *deontic action model*.

Since the underlying action frame tells us which actions are executed, we can define what it means for a formula to be true (false) under an interpretation of the sort just described: it is true iff the action type it expresses (prevents what it expresses) is executed. More precisely, if $\mathcal{M} = (\mathcal{F}, V, F)$ is a deontic action model, then:

- $\mathcal{M} \models \varphi$ iff $V(\varphi)$ is realized, i.e. $V(\varphi) \cap \mathbf{Ex} \neq \emptyset$

φ is true in a model $\mathcal{M} = (\mathcal{F}, V, F)$ iff there is an exact realization of φ that is executed according to the deontic action frame $\mathcal{F} = (A, Ex, (Ok_x)_{x\in A}, (Req_x)_{x\in A})$. This simply means that for at least one exact realization of φ, all atomic acts that constitute an exact realization of φ are in Ex.

- $\mathcal{M} \dashv \varphi$ iff $F(\varphi)$ is realized, i.e. $F(\varphi) \cap \mathbf{Ex} \neq \emptyset$

φ is false in a model $\mathcal{M} = (\mathcal{F}, V, F)$ iff there is an executed act according to the deontic action frame $\mathcal{F} = (A, Ex, (Ok_x)_{x\in A}, (Req_x)_{x\in A})$, such that an execution of φ is prevented. This simply means that for at least one exact falsifier of φ, all atomic acts that constitute such an exact falsifier of φ are in Ex.

As usual, validity ($\models$) is defined as truth in all deontic action models. We can then show the following lemma:

Lemma 2.1. *If $\mathcal{M}$ is deontic action model, then:*

i) a) $\mathcal{M} \models \neg\varphi$ iff $\mathcal{M} \dashv \varphi$
b) $\mathcal{M} \dashv \neg\varphi$ iff $\mathcal{M} \models \varphi$

ii) a) $\mathcal{M} \models \varphi \wedge \psi$ iff $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$
b) $\mathcal{M} \dashv \varphi \wedge \psi$ iff $\mathcal{M} \dashv \varphi$ or $\mathcal{M} \dashv \psi$

iii) a) $\mathcal{M} \models \varphi \vee \psi$ iff $\mathcal{M} \models \varphi$ or $\mathcal{M} \models \psi$
b) $\mathcal{M} \dashv \varphi \vee \psi$ iff $\mathcal{M} \dashv \varphi$ and $\mathcal{M} \dashv \psi$

We might want to put conditions on the verifiers and falsifiers of formulas. If V and F are verifier and falsifier assignments in a deontic action frame, we say that:

- (V, F) is complete iff for all p, $V(p) \cap \mathbf{Ex} \neq \emptyset$ or $F(p) \cap \mathbf{Ex} \neq \emptyset$

  (i.e. p is either realized or prevented)

- (V, F) is consistent iff for no p, $V(p) \cap \mathbf{Ex} \neq \emptyset$ and $F(p) \cap \mathbf{Ex} \neq \emptyset$

  (i.e. p is not realized and prevented)

- (V, F) is classical iff for all p, either $V(p) \cap \mathbf{Ex} \neq \emptyset$ or $F(p) \cap \mathbf{Ex} \neq \emptyset$

  (i.e. p is either realized or prevented, but not both)

It it easily shown, that these conditions extend to all formulas:

Lemma 2.2. *If $\mathcal{M} = (\mathcal{F}, V, F)$ is a deontic action model, then for all φ without P or O:*

i) if (V, F) is complete, then for all φ, $\mathcal{M} \vDash \varphi$ or $\mathcal{M} \dashv \varphi$

ii) if (V, F) is consistent, then for all φ, not both $\mathcal{M} \vDash \varphi$ and $\mathcal{M} \dashv \varphi$

iii) if (V, F) is classical, then for all φ, either $\mathcal{M} \vDash \varphi$ or $\mathcal{M} \dashv \varphi$

In particular, this means that by imposing conditions on the assignments, we can ensure that we obtain a certain background logic:[7]

Lemma 2.3. *For all Γ and φ without P or O,*

i) $\Gamma \vDash_{\mathbf{FDE}} \varphi$ iff for all deontic action models $\mathcal{M}$, if $\mathcal{M} \vDash \Gamma$, then $\mathcal{M} \vDash \varphi$

ii) $\Gamma \vDash_{\mathbf{K3}} \varphi$ iff for all deontic action models $\mathcal{M}$ such that (V, F) is consistent, if $\mathcal{M} \vDash \Gamma$, then $\mathcal{M} \vDash \varphi$

iii) $\Gamma \vDash_{\mathbf{LP}} \varphi$ iff for all deontic action models $\mathcal{M}$ such that (V, F) is complete, if $\mathcal{M} \vDash \Gamma$, then $\mathcal{M} \vDash \varphi$

iv) $\Gamma \vDash_{\mathbf{CL}} \varphi$ iff for all deontic action models $\mathcal{M}$ such that (V, F) is classical, if $\mathcal{M} \vDash \Gamma$, then $\mathcal{M} \vDash \varphi$

Proof. This follows from the previous two lemmas. □

We could therefore, in principle, use different background logics, but in the following we shall restrict ourselves to classical logic: we shall assume that all verifier and falsifier assignments are classical. We shall call a deontic action model $(A, Ex, (Ok_x)_{x\in A}, (Req_x)_{x\in A}, V, F)$ classical iff (V, F) is classical.

It is now high time to introduce our clauses for the verifiers and falsifiers of permissions and obligations. The case for the verifiers is relatively straightforward. If $(A, Ex, (Ok_x)_{x\in A}, (Req_x)_{x\in A}, V, F)$ is a classical deontic action model, we say that:

- $V(P\varphi) = \{X \mid V(\varphi) \subseteq Ok_X\}$

  A complex act exactly realizes that φ is permitted iff the execution of that act renders all exact realizations of φ admissible.

[7] Here we assume that the reader is familiar with the many valued semantics for the logic of first-degree entailment (**FDE**), strong Kleene logic (**K3**), the logic of paradox (**LP**), and (of course) classical logic (**CL**). For the details of these semantics, see e.g. [9, §§ 7–8].

- $V(O\varphi) = \{X \mid V(\varphi) \cap Req_X \neq \emptyset\}$

  A complex act exactly realizes that φ is obligatory iff the execution of that act renders at least one exact realizations of φ required.

In other words, a complex act is a verifier of an explicit permission $P\varphi$ iff every verifier of φ is in the sphere of permissions of the act, and an act is a verifier of an explicit obligation $P\varphi$ iff some verifier of φ is in the sphere of obligations of the act.

But when it comes to the falsifiers of explicit permissions and obligations, the issue becomes a bit more complicated. Intuitively, what makes an explicit permission or obligation false is that no corresponding normative acts have been executed. But what act is that makes this the case then? We propose that if indeed no corresponding normative act has been executed, then it is the totality of the executed acts that jointly makes it the case that something is not explicitly permitted or obligated:

- $F(P\varphi) = \begin{cases} \{Ex\} & if\ V(P\varphi) \cap \mathbf{Ex} = \emptyset \\ \emptyset & otherwise \end{cases}$

- $F(O\varphi) = \begin{cases} \{Ex\} & if\ V(O\varphi) \cap \mathbf{Ex} = \emptyset \\ \emptyset & otherwise \end{cases}$

Remember that we confined our semantics to classical deontic action models. The classicality of deontic action models and the definition of $F(P\varphi)$ and $O(P\varphi)$ result in a very natural reading of what prevents a permission (an obligation) to hold in that model. On the one hand, classicality implies completeness: given a classical deontic action model $\mathcal{M}$, every φ is either realized or prevented (given the set of executed singular acts $\mathbf{Ex}$ of the model $\mathcal{M}$). As a consequence, either $P\varphi$ is realized or $P\varphi$ is prevented in a classical deontic action model $\mathcal{M}$. Now suppose that there is no executed act that allows φ, i.e. $V(P\varphi) \cap \mathbf{Ex} = \emptyset$. Since the model is maximal, there is no further executable act, and the totality of all executed atomic acts (Ex) is responsible for $P\varphi$ ($O\varphi$) being prevented. On the other hand, classicality also implies consistency: if there is an executed act that allows φ, i.e. $V(P\varphi) \cap \mathbf{Ex} \neq \emptyset$, then there cannot be an act that exactly prevents it from being permitted, i.e. $F(P\varphi) = \emptyset$ (same for $O\varphi$).

We shall conclude this section with an observation about how our semantics relates to the truth-conditions that we sketched in the introduction to this paper. Remember that we said that once we've identified what states are admissible and required, natural truth conditions for P and O are as follows:

- a statement of the form $P\varphi$ is true iff every act that is an exact truthmaker of φ is admissible, and

- a statement of the form $O\varphi$ is true iff some act that is required is an exact truthmaker of φ.

Indeed, in our semantics above, we can recover these truth-conditions in the following lemma:

Lemma 2.4. *If $\mathcal{M}$ is classical deontic action model, then:*

i) $\mathcal{M} \models \neg\varphi$ *iff* $\mathcal{M} \not\models \varphi$

ii) $\mathcal{M} \models \varphi \wedge \psi$ *iff* $\mathcal{M} \models \varphi$ *and* $\mathcal{M} \models \psi$

iii) $\mathcal{M} \models \varphi \vee \psi$ *iff* $\mathcal{M} \models \varphi$ *or* $\mathcal{M} \models \psi$

iv) $\mathcal{M} \models P\varphi$ *iff* $V(\varphi) \subseteq \bigcup_{x \in Ex} Ok_x$

v) $\mathcal{M} \models O\varphi$ *iff* $V(\varphi) \cap \bigcup_{x \in Ex} Req_x \neq \emptyset$

In other words, in a given classical deontic action model, we can identify the admissible acts in the model with the acts that are rendered admissible by the executed acts ($\bigcup_{x \in Ex} Ok_x$) and the required acts with the acts rendered required by the executed acts ($\bigcup_{x \in Ex} Req_x$).

3 The Paradoxes

In this section, we shall show that our semantics deals in a natural way with some well-known paradoxes of deontic logic.

3.1 The Paradox of Free Choice Permission

Suppose Johannes issues the following permission "Albert, you may have tiramisu or zabaglione for dessert." Albert (naturally) concludes that he is free to choose: that he may have zabaglione, and that he may have tiramisu. In everyday discourse, the permission of a disjunction seems to imply the permission of both disjuncts (cf. [8]):

$$(FCP) \qquad P(\varphi \vee \psi) \rightarrow P\varphi \wedge P\psi$$

Put differently, permitting Albert to have tiramisu or zabaglione, but not permitting him to have tiramisu seems to be inconsistent. It is well-known that FCP is recipe for disaster: already very weak principles, if augmented with FCP, lead to unacceptable consequences. Take, for instance, the rule RE, that warrants substitution of logically equivalent formulas:

$$(RE) \qquad \frac{\vdash \varphi \leftrightarrow \psi}{\vdash P\varphi \leftrightarrow P\psi}$$

According to classical logic, we have $\vdash \varphi \leftrightarrow (\varphi \wedge \psi) \vee (\varphi \wedge \neg\psi)$. This equivalence and RE+FCP already leads to a disastrous result, i.e. if φ is permitted, then φ together with any ψ is permitted, in formal terms:

$$(IC) \qquad P\varphi \rightarrow P(\varphi \wedge \psi)$$

is a theorem of CL+FCP+RE, and it seems to be completely unacceptable as a theorem of any useful deontic logic. This suggests that it is generally very hard to find a logic which contains FCP but also avoids problematic consequences like IC. As Sven Ove Hansson puts it: "It [i.e. the derivation of IC] indicates that the free choice permission postulate may be faulty in itself, even if not combined with other deontic principles such as those of SDL."[7, p.208] This is *the problem of free choice permission.*

It probably doesn't come as a surprise that FCP is highly controversial and regarded to be implausible by most deontic logicians. Given certain interpretations of permission, FCP turns out to be valid though. Take, for instance, the open reading of permission (cf. [2],[1]) where $P\varphi$ is interpreted as "every way to ensure φ is admissible".[8] Now, given that $\varphi \to \varphi \vee \psi$ is a theorem (cf. [7]), this interpretation validates FCP. However, this reading (intuitively and formally) also validates IC: Since every way to ensure $\varphi \wedge \psi$ is a way to ensure φ, the permission of φ implies the permission of $\varphi \wedge \psi$. However, accepting the (intuitively) unacceptable consequence IC in order to make sense of the (intuitively) acceptable principle FCP is far from an ideal solution to the problem of free choice permission. This approach just replaces one evil with another.

In our opinion, the semantics developed in the previous section offers a *real* solution to the problem of free choice permission. First, note that according to our reading of permission, FCP turns out to be valid. Suppose that $\varphi \vee \psi$ is permitted. This means that there is an executed singular act that renders all exact realizations of $\varphi \vee \psi$ admissible. Every exact realization of $\varphi \vee \psi$ is an exact realization of φ or an exact realization of ψ. This implies that every exact realization of φ is rendered admissible, and that every exact realization of ψ is rendered admissible. And this means that both, φ and ψ are permitted. In this respect, our semantics is similar to the open reading of permission. In more formal terms:

Lemma 3.1. $\vDash P(\varphi \vee \psi) \to P\varphi \wedge P\psi$.

Proof. Let $\mathcal{M} = (\mathcal{F}, V, F)$ be deontic action model and suppose $\mathcal{M} \vDash P(\varphi \vee \psi)$ i.e. (by Lemma 2.4) $V(\varphi \vee \psi) \subseteq \bigcup_{x \in Ex} Ok_x$. Hence, $V(\varphi) \cup V(\psi) \subseteq \bigcup_{x \in Ex} Ok_x$, by the construction of exact realizations of disjunctive generic actions. Basic set theory now gives us $V(\varphi) \subseteq \bigcup_{x \in Ex} Ok_x$ and $V(\psi) \subseteq \bigcup_{x \in Ex} Ok_x$, which according to Lemma 2.4 means that $\mathcal{M} \vDash P\varphi \wedge P\psi$. □

But how do we now avoid the seemingly unavoidable consequence IC? The reason for this is quite simple: in our semantics RE is not a sound rule. The semantics we developed in the previous section is hyperintensional: logical (even necessary) equivalences may not generally be substituted for one another. In order to see why RE is not a plausible rule in exact truthmaker semantics, take the problematic equivalence statement $\varphi \leftrightarrow (\varphi \wedge \psi) \vee (\varphi \wedge \neg\psi)$ again. Although classically equivalent, φ and $(\varphi \wedge \psi) \vee (\varphi \wedge \neg\psi)$ may have completely different exact realizations. An exact realization of $(\varphi \wedge \psi)$ must consist of an exact realization of both φ and ψ, and exact realization of $(\varphi \wedge \neg\psi)$ of an exact realization of φ and an exact prevention of ψ. An exact realization of φ does not have to be either, just take an exact realization of φ that is neither an exact realization of ψ nor an exact prevention of ψ. This idea shows us how to find a countermodel for IC:

Lemma 3.2. $\nvDash P\varphi \to P(\varphi \wedge \psi)$.

Proof. Let $\mathcal{F} = (A, Ex, (Ok_x)_{x \in A}, (Req_x)_{x \in A})$ be a deontic action frame with $A = \{a_1, a_2\}$, $Ex = \{a_1\}$, $Ok_{a_1} = \{\{a_1\}\}$. Let $\mathcal{M} = (\mathcal{F}, V, F)$ based on $\mathcal{F}$ s.t. $V(\varphi) = \{\{a_1\}\}$ and $V(\psi) = \{\{a_2\}\}$. This gives us $\mathcal{M} \vDash P\varphi$ (since $V(\varphi) \subseteq Ok_x$ s.t. $x \in Ex$). We also have $V(\varphi \wedge \psi) = \{\{a_1, a_2\}\}$, but since there is no x with $\{\{a_1, a_2\}\} \nsubseteq Ok_x$ and $x \in Ex$, we get $\mathcal{M} \nvDash P(\varphi \wedge \psi)$. □

[8]Or "every execution of φ leads to an Ok-state", depending on your preferred framework, cf. [2]

The lesson to be drawn from this is that material equivalence does not adequately express identity of exact realizations. This is how we solve the problem of free choice permission in our semantics.

3.2 The Good Samaritan Paradox

Another, paradox of deontic logic that has a natural solution in our semantics is Prior's Good Samaritan paradox [10]. This paradox arises in systems where obligation is closed under logical consequence, i.e. systems which validate the following rule:

$$(CL) \qquad \frac{\vdash \varphi \to \psi}{\vdash O\varphi \to O\psi}$$

This rule is validated by many systems of deontic logic, such as the system SDL of standard deontic logic, but it leads to counterintuitive results in certain cases. Consider the case of Smith who has been robbed. Intuitively, it is obligatory that Jones helps smith. Thus, it is obligatory that John helps Smith who has been robbed. According to Prior, we can formalize this by the formula $O(p \wedge q)$, where p stands for *John helps Smith* and q stands for *Smith has been robbed*. But since in classical logic we have $\vdash p \wedge q \to p$, it follows by CL that Oq, which means that its obligatory that Smith has been robbed and is absurd.

Prior's concrete example may be more or less convincing, but there are many examples of the same logical structure and lead to the same result: CL is intuitively flawed. For example, it is intuitively obligatory for the nurse to give his patient the medicine A and medicine B, if together they heal him, but medicine A alone might kill the patient, so it is not obligatory for the nurse to give his patient medicine A. Intuitively, the problem is that certain acts, such as the nurse giving the patient medicine A and the nurse giving him medicine B, are only required in conjunction and not by themselves. And in our semantics, we can faithfully represent this intuitive claim.

To see this, let's model this situation in our semantics. Consider an action frame (A, Ex) with two atomic acts $A = \{a, b, c\}$ and one executed action $\{a\}$. Intuitively, a is the act of the doctor telling the nurse that he should give medicine A and B to the patient, b is the act of the nurse giving medicine A to the patient, and c is the act of the nurse giving medicine B to the patient. Since a the act of the doctor telling the nurse that he should give medicine A and B to the patient, we can plausibly assume that $Req_a = Req_{\{a\}} = \{\{b, c\}\}$, and for simplicity we can assume that the spheres of permissions and obligations for all the other acts are empty. Let $\mathcal{M}$ be the corresponding deontic action frame. Now let p stand for *the nurse gives the patient medicine* A and q for *the nurse gives the patient medicine* B, we will have $V(p) = \{\{b\}\}$, $V(q) = \{\{c\}\}$, and thus $V(p \wedge q) = \{\{b, c\}\}$. Moreover, we'll have $V(P(p \wedge q)) = \{\{a\}\}$ and hence that $\mathcal{M} \vDash P(p \wedge q)$. But we'll neither have $\mathcal{M} \vDash Pp$ nor $\mathcal{M} \vDash Pq$, exactly as we want. More generally, this model shows that CL is not sound with respect to our semantics:

Lemma 3.3. $\nvDash O(\varphi \wedge \psi) \to O\varphi$

In this consists our solution to the Good Samaritan paradox.

4 Axioms

In this section, we give a sound and complete axiomatization of our semantics. However, we shall use a slightly non-standard technique to obtain such an axiomatization, which is nevertheless adequate to the hyperintensional spirit of our semantics.

In a recent paper, Fine sketches how to obtain an axiomatization of sameness of exact truthmakers according to van Fraassen's clauses, which we've used in our above semantics [6]. The axiomatization consists of the following axioms and rules:

$$\varphi \leftrightharpoons \varphi$$
$$\varphi \leftrightharpoons \neg\neg\varphi \qquad \varphi \wedge (\psi \vee \theta) \leftrightharpoons (\varphi \wedge \psi) \vee (\varphi \vee \theta)$$
$$\varphi \vee \varphi \leftrightharpoons \varphi \qquad \varphi \wedge \varphi \leftrightharpoons \varphi$$
$$\varphi \vee \psi \leftrightharpoons \psi \vee \varphi \qquad \varphi \wedge \psi \leftrightharpoons \psi \wedge \varphi$$
$$\varphi \vee (\psi \vee \theta) \leftrightharpoons (\varphi \vee \psi) \vee \theta \qquad \varphi \wedge (\psi \wedge \theta) \leftrightharpoons (\varphi \wedge \psi) \wedge \theta$$
$$\neg(\varphi \vee \varphi) \leftrightharpoons \neg\varphi \wedge \neg\psi \qquad \neg(\varphi \wedge \varphi) \leftrightharpoons \neg\varphi \vee \neg\psi$$

$$(\textit{Replacement}) \qquad \theta(\varphi), \varphi \leftrightharpoons \psi / \theta(\psi)$$

Let's denote derivability in this system by $\vdash_E$. Then we get the following theorem:

Theorem 4.1 (Fine)**.** *For all φ and ψ without P or O, we have: $\vdash_E \varphi \leftrightharpoons \psi$ iff for all deontic action models $(\mathcal{F}, V, F)$, we have $V(\varphi) = V(\psi)$.*

Our goal is to use this system to obtain an axiomatization for our semantics of permission and obligation. The first step along the way is to get a grip of the truthmakers of explicit permissions and obligations. We get this in the following lemma:

Lemma 4.2. *For all deontic action models $(\mathcal{F}, V, F)$, we have for all φ and ψ:*

i) $V(P(\varphi \vee \psi)) = V(P\varphi \wedge P\psi)$.

ii) $V(O(\varphi \vee \psi)) = V(O\varphi \vee O\psi)$.

Proof. Note that since on our semantics we have that $Ok_X = \bigcup_{x \in X} Ok_x$ and $Req_X = \bigcup_{x \in X} Req_x$, we get:

- $Ok_{\bigcup_i X_i} = \bigcup_i Ok_{X_i}$
- $Req_{\bigcup_i X_i} = \bigcup_i Req_{X_i}$

Using these identities, we get:

i) $\underbrace{\{X \mid V(\varphi) \cup V(\psi) \subseteq Ok_X\}}_{=V(P(\varphi\vee\psi))} = \underbrace{\{X \cup Y \mid V(\varphi) \subseteq Ok_X, V(\psi) \subseteq Ok_Y\}}_{=V(P\varphi\wedge P\psi)}$

ii) $\underbrace{\{X \mid (V(\varphi) \cup V(\psi)) \cap Req_X \neq \emptyset\}}_{=V(O(\varphi\vee\psi))} = \underbrace{\{X \mid V(\varphi) \cap Req_X \neq \emptyset\} \cup \{X \mid V(\psi) \cap Req_X \neq \emptyset\}}_{=V(O\varphi\vee O\psi)}$

□

It turns out that these two identities are enough to obtain a sound and complete axiomatization of our semantics. The system consists of the above axioms and rules plus all axioms (over the full language including P and O) and rules of classical propositional logic and:

$$P(\varphi \vee \psi) \fallingdotseq P\varphi \wedge P\psi$$

$$O(\varphi \vee \psi) \fallingdotseq O\varphi \vee O\psi$$

We shall denote derivability in this system by $\vdash_{EDL}$.

Theorem 4.3. *For all φ and Γ without $\fallingdotseq$, $\Gamma \vdash_{EDL} \varphi$ iff $\Gamma \vDash \varphi$.*

Let's conclude with a few sample derivations to show how the system works:

1. $\vdash_{EDL} P(\varphi \vee \psi) \leftrightarrow P\varphi \wedge P\psi$
   (a) $P(\varphi \vee \psi) \leftrightarrow P(\varphi \vee \psi)$ (Tautology)
   (b) $P(\varphi \vee \psi) \fallingdotseq P\varphi \wedge P\psi$ (Axiom)
   (c) $P(\varphi \vee \psi) \leftrightarrow P\varphi \wedge P\psi$ (a,b, Replacement)

2. $\vdash_{EDL} O\neg(\varphi \wedge \psi) \leftrightarrow O\neg\varphi \vee O\neg\psi$
   (a) $O\neg(\varphi \wedge \psi) \leftrightarrow O\neg(\varphi \wedge \psi)$ (Tautology)
   (b) $\neg(\varphi \wedge \psi) \fallingdotseq \neg\varphi \vee \neg\psi$ (Axiom)
   (c) $O\neg(\varphi \wedge \psi) \leftrightarrow O(\neg\varphi \vee \neg\psi)$ (a,b, Replacement)
   (d) $O(\neg\varphi \vee \neg\psi) \fallingdotseq O\neg\varphi \vee O\neg\psi$ (Axiom)
   (e) $\neg(\varphi \wedge \psi) \leftrightarrow O\neg\varphi \vee O\neg\psi$ (c,d, Replacement)

3. $P\neg\neg(\varphi \vee \psi) \vdash_{EDL} P\varphi$
   (a) $P\neg\neg(\varphi \vee \psi)$ (Assumption)
   (b) $\neg\neg(\varphi \vee \psi) \fallingdotseq \varphi \vee \psi$ (Axiom)
   (c) $P(\varphi \vee \psi)$ (a,b,Replacement)
   (d) $P(\varphi \vee \psi) \leftrightarrow P\varphi \wedge P\psi$ (1.)
   (e) $P\varphi \wedge P\psi$ (c,d, Logic)
   (f) $P\varphi$ (e, Logic)

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