The Feasability of Modeling Hypothetical Reasoning by Formal Logics.

Including an Overview of Adaptive Logics for Singular Fact Abduction

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1 Modeling Hypothetical Reasoning by means of Formal Logics

To an outsider, the claim that hypothetical or abductive reasoning[1] can be modeled by means of formal logics might sound as outlandish as claiming that computers have the same cognitive and creative abilities as humans. After all, abductive reasoning, the process of forming explanatory hypotheses for puzzling observations, is often considered to be the hallmark of creative ingenuity, leading to our rich and wide diversity of ideas, innovations and scientific theories. It just seems impossible that this richness can be reconstructed or created by just using formal tools, which are by nature abstracted from the specific semantic content. This argument, in short the "creativity excludes logic" argument, is the main reason why even the field itself is sharply divided between believers and non-believers.

This argument, however, is a straw man. Nobody would argue for the claim that hypothetical reasoning can be modeled by means of formal logics along these lines. What is argued for in this paper and the various sources it cites is the more modest claim that certain aspects and forms of hypothetical reasoning can be modeled with the aid of formal systems that are specifically suited for this task.

There are three important ways in which this modest claim differs from the straw man that is attacked by the "creativity excludes logic" argument. First, it not implied that the logics that are used are classical or deductive logics. Second, abductive reasoning is not a monolithic concept: it does not consist of a single method or procedure, but consists of many different patterns; formal logics are only used to capture one specific and precisely defined pattern at a time. Third, the relation between formal logics and abductive reasoning is not one of agent and activity (i.e. formal logics do not display themselves abductive reasoning like humans do) but one of model and target: formal logics are used by (human) agents to model and – to a certain extent – to simulate certain aspects of human abductive reasoning. The semantic content that is lacking in abductive reasoning is provided by these agents.

In the remainder of this introduction, I will go into further detail on the type of logics that are suitable for modelling abductive reasoning and introduce in general terms the framework that is used for the logics in this paper.

To those who are still a bit suspicious how abductive reasoning patterns can be modeled using formal logics, I want to stress that I do not mean that any of these patterns is a valid inference in classical logic or any other (non-trivial) deductive logic. To model defeasible reasoning steps such as hypothesis

[1] In this paper, I use the notion of hypothetical reasoning always and only to indicate the reasoning *towards* (explanatory) hypotheses, i.e. the act of *suggesting hypotheses* for certain observations or puzzling facts. This patterns is since Peirce commonly known as *abduction*. This kind of reasoning has to be distinguished from reasoning that *starts from* certain (possibly counterfactual) hypotheses, a meaning in which Rescher (1964) used the notion of hypothetical reasoning. To avoid confusion, I will generally use the notion of abductive reasoning.

formation, one has to use non-monotonic logics: logics for which an extension of a premise set does not always yield a consequence set that is a superset of the original consequence set. Or, put more simply, logics according to which new information may lead us to revoke old conclusions.

It is important to note that my purpose in using logics is not the classical purpose of the discipline of logic. Classically, the discipline of logic studies the correct way to infer further knowledge from already known facts. The correct way should guarantee the truth of the new facts, under the supposition that the old facts are true. Accordingly, this has motivated the search for the right (deductive) logic (whether it be Classical Logic or another one such as Intuitionistic Logic). My purpose, however, is to model or explicate human reasoning patterns. As these patterns are fallible, leading to conclusions that are not necessarily true even if the premises are assumed to be true, it should be possible to revoke previously derived results; hence, my use of non-monotonic logics. Also, because there are many patterns of human reasoning, I naturally conceive of a plenitude of logics in order to describe them.

Let me explain this a bit more formally. A logic can be considered as a function from the power set of the sentences of a language to itself. So, given a language L and the set $\mathcal{W}$ of its well-formed formulas:

$$\mathbf{L} : \wp(\mathcal{W}) \rightarrow \wp(\mathcal{W})$$

Hence, a logic determines for every set of sentences (or premise set) Γ which sentences can be inferred from it ($Cn_{\mathbf{L}}(\Gamma) =_{df} \mathbf{L}(\Gamma)$). Therefore, as a reasoning pattern is nothing more than the inference of some statements given some initial statements, in principle, a logic can be devised to model any reasoning pattern in science.[2] If this pattern can be formally described, description by a formal logic is in principle possible.

Deductive logics, such as Classical Logic (**CL**), have the property of monotonicity, i.e. for all premise sets Γ and Γ':

$$Cn_{\mathbf{L}}(\Gamma) \subseteq Cn_{\mathbf{L}}(\Gamma \cup \Gamma')$$

Most patterns of human reasoning, however, do not meet this criterion. For instance, if an agent infers a hypothesis, she is well aware that it might need to be revoked on closer consideration of the available background knowledge or in light of new information.

Although non-monotonic reasoning has typically received less attention in the field of logic than monotonic reasoning, various frameworks for defeasible reasoning and non-monotonic logics are available such as default logic, adaptive logics and belief revision.[3] In this article, I overview the progress that has

[2] In reality, scientific and human reasoning include not only sentences or propositions, but also direct observations, sketches and various other symbolic representations. Yet for the purpose of modeling particular reasoning patterns, we can generally represent those sources by suitable propositions.

[3] For an general overview of the variation in approaches, see Koons (2014).

been made on modelling abduction within the adaptive logics framework, a framework created by Batens over the past three decades.[4] This framework for devising non-monotonic logics has some advantages that suit very well the project of modelling abductive reasoning patterns.

First, the focus in the adaptive logics program is, in contrast with other approaches to non-monotonic reasoning, on proof theory. For these logics, a dynamic proof style has been defined in order to mimic to a certain extent actual human reasoning patterns. More in particular, these dynamic proofs display the two forms of revoking previously derived results that can also be found in human reasoning: revoking old conclusions on closer consideration of the available evidence (internal dynamics) and revoking them in light of new information (external dynamics).[5]

Second, over the years, a solid meta-theory has been built for this framework, which guarantees that if an adaptive logic is created according to certain standards (the so-called "standard format"), many important metatheoretical properties are generically proven. This creates an opportunity for projects such as this to focus almost exclusively on the application of these formal methods without having to worry too much about proving their meta-theoretical characteristics.

Finally, as the framework is presented as a unified framework for non-monotonic logics, it has been applied in many different contexts. Over the years, adaptive logics have been devised for, apart from abduction, paraconsistent reasoning, induction, argumentation, deontic reasoning, etc.[6]

2 Advantages and Drawbacks

Explicating patterns of hypothesis formation by means of formal logics has a clear advantage: by reducing patterns to their formal and structural essence, an insight into the pattern's precise conditions and applications is gained that is hard to achieve purely by studying different cases.

Another great advantage of the formal explication of human reasoning patterns is that it allows for the possibility to provide artificially intelligent agents (which in general lack the human capacity for context awareness unless it is

[4] For an extensive overview and thorough formal introduction, see Straßer (2013) or Batens (2007).

[5] One should not be misled, however, by this idea of dynamic proofs in thinking that the consequence set of adaptive logics for a certain premise set depends on the proof. Adaptive logics are proper proof-invariant logics that assign for each premise set Γ exactly one consequence set $Cn_{\mathbf{L}}(\Gamma)$.

[6] Most applications of the adaptive logics framework have been studied at the Centre for Logic and Philosophy of Science (Ghent University). At the Centre's preprints list (`http://logica.ugent.be/centrum/writings/pubs.php`), references can be found to many papers in various contexts. The reference works mentioned earlier, Straßer (2013) and Batens (n.d.), also give a good overview of the various applications.

explicitly provided) with formal patterns to simulate human reasoning. In the case of hypothesis formation, this possibility has presently already found applications in the AI subfields of abduction (diagnosis), planning and machine learning.[7]

The method of explicating patterns of hypothesis formation by means of formal adaptive logics also has certain drawbacks, however.

First, formal logics are expressed in terms of a formal language, in which not all elements of human reasoning processes can be represented. This leads inevitably to certain losses. A very obvious example is that in general only propositions can be represented in logics. That means that all observations, figures or other symbolic representations must be reduced to descriptions of them. A more important example in the case of abduction is the implication relation. The adaptive logics framework I use is, certainly for ampliative logics such as those for abduction or induction, largely built around the use of a classical material implication (mostly to keep things sufficiently simple).[8] As a result of this, all relations between a hypothesis and the observations that led to their formation (their triggers) are modeled by material implications. It is clear that this is a strong reduction of the actual richness of such relations. Hypotheses do not have to imply their triggers: they can also just be correlated with them or be probabilistically likely; or the relation can be much more specific, as in the case of an explanatory or causal relation.

Second, if one sets out to model actual historical human reasoning processes by means of dynamic logical proofs (as the adaptive framework allows us to do), one quickly finds that it is no easy task to boil down those actual processes to the micro structure of their individual reasoning steps. As human agents often combine individual steps and seldom take note of each individual step, this type of models always contains an aspect of simulation.

Human reasoning also does not proceed linearly step by step as proofs do: it contains circular motions, off-topic deviations and irrational connections that cannot be captured by formal logics. Therefore, models of such reasoning processes are always to a great extent idealized.

Natural languages are also immensely more complex than any formal language can aspire to be. Therefore, models of human reasoning are unavoidably simplifications. Furthermore, as formal logics state everything explicitly, any modeler of human reasoning has to simplify deliberately the actual cases, only to achieve a certain degree of comprehensibility.

Altogether, it is clear that formal models of human reasoning processes are, in fact, only models: they contain abstractions, simulations, simplifications and idealizations. And although these techniques are the key characteristics of mod-

[7] For an overview, see Paul (2000).

[8] This is an issue relevant beyond the field of adaptive logics. Paul (2000, p. 36) has claimed that most approaches to abduction use a material implication that is implicitly interpreted as some kind of explanatory or causal relation.

els, such as those used in science, it is not always easy to evade the criticism that formal logics can only handle toy examples.

Third, certain patterns of creative hypothesis formation, i.e. those that introduce the hypothetical existence of new concepts, cannot be modeled by first-order logics. They require the use of second-order logics, and this is a possibility of which, at present, the adaptive logics framework is not capable.

Fourth, as we are purely concerned with hypothesis formation and not with hypothesis selection, formal methods will generate sets of possible hypotheses that grow exponentially in relation to the growth of the agent's background knowledge. It is clear that this also poses a limit to the application of these methods to real world problems.

Finally, one might question the normativity of this project (and more generally of the adaptive logics program). By aiming to describe actual human reasoning processes, this branch of logics appears to put a descriptive ideal first, which contrasts sharply with the strongly normative ideals in the field of logic in general. The standard answer to this question is that adaptive logics attempt to provide both: on the one hand, they aim to describe actual reasoning patterns; on the other, once these patterns are identified, they aim to prescribe how these patters should be rationally applied. Yet this does not answer how the trade-off between these two goals of description and normativity should be conceived: is it better to have a large set of logics that is able to describe virtually any pattern actually found in human reasoning, or should we keep this set trimmed and qualify most actual human reasoning as failing to accord with the highest normative standards? Therefore, it remains a legitimate criticism that the goals of description and prescription cannot be so easily joined: how their trade-off should be dealt with needs further theoretical underpinning.

3 Four types of Hypothetical Reasoning

The quest to characterize abduction under a single schema was abandoned around 1980. The main reasons were that such attempts (e.g. Hanson's (1958; 1961) proposal to call abduction "the logic of discovery") often did not provide much detailed guidance for actual discovery processes, and that even these general attempts always captured only a part of the discovery process (e.g. *Inference to the Best Explanation* (Harman, 1965) describes only the selection of hypotheses, not their formation).

Around the same time, research from different fields such as philosophy of science based on historical cases, artificial intelligence and cognitive science resulted in a new consensus that there is a plenitude of patterns, heuristics and methods of discovery, which are open to normative guidance, yet this guidance might be content-, subject-, or context-dependent (Nickles, 1980; Simon, 1973).

Various authors in the literature on abduction have tried to provide classifications of various patterns of abduction (Thagard, 1988; Schurz, 2008a; Hoffmann, 2010). Although these attempts differ slightly, some general patterns clearly stand out. Below, I give my personal interpretation of these major general patterns. The main reason I deviate from the previous classifications is that I want to simplify the rather prolific classifications, yet provide a sufficient basis for formal modeling. The reasons why I think this is possible is that I neither attempt to give a fully exhaustive list nor a list the elements of which are mutually exclusive. I only want to give a simple list as a basis that covers most instances of abductive reasoning and can serve as the basis of formal modeling.

Before I give my personal classification of these major patterns found in abductive reasoning, it is important to note that abductive inferences form explanatory hypotheses for observed facts using the agent's background beliefs (or knowledge). Therefore, these patterns have the structure of the inference of a hypothesis (HYP) from some observed facts (OBS) and some of the agent's background beliefs (or knowledge) (BBK).

In line with the Fregean tradition, I consider *factual statements* as statements of a *concept* with regard to one or more *objects* (or a logical combination of such statements). For instance, the statement "There was a civil war in France in 1789" can be analyzed as the concept "civil war" with regard to "France in 1789". A *fact* is a true factual statement. As such, concepts can also be considered as the *class* of all objects (or tuples of objects) for which the concept with regard to that object (or tuple of objects) is a fact. An *observed fact* is a factual statement describing an agent's observation that she considers to be true. This can be broadly conceived to include also, for instance, a graph or a table of measurements in an article. Together, the observed facts form the *trigger* for the agent.

In my semi-formal description of these patterns, I express that p should be considered as a hypothesis by using a formulation of the form "It might be that p"; beliefs and observed facts can be expressed simply by stating their content. Concepts are denoted by uppercase letters, objects by lowercase letters. Addition of a subscript denotes a finite list of objects or concepts (including, unless stated otherwise, the possibility of a single object or concept).

1. ***Abduction of a Singular Fact***

   (OBS) F with regard to x_i
   (BBK) E with regard to some objects explains F with regard to those objects

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   (HYP) It might be that E with regard to x_i

   Some examples of this pattern, which has also been called "simple abduction" (Thagard, 1988), "factual abduction" (Schurz, 2008a) and "selective fact abduction" (Hoffmann, 2010), are:

   - the inference that the hominid who has been dubbed Lucy might have been bipedal, from observing the particular structure of her pelvis and knee bones and knowledge about how the structure of pelvis and knee bones relates to the locomotion of animals.

- the inference that two particles might have opposite electric charges, from observing their attraction and knowledge of the Coulomb force.

2. ***Abduction of a Generalization***

(OBS)	F with regard to all observed objects of class D
(BBK)	E with regard to some objects explains F with regard to those objects
(HYP)	It might be that E with regard to all objects of class D

Some examples of this pattern, which has also been called "rule abduction" (Thagard, 1988), "law abduction" (Schurz, 2008a) and "selective law abduction" (Hoffmann, 2010), are:

- the inference that all hominids of the last three million years might have been bipedal, from observing the similar structure of the pelvis and knee bones of all observed hominid skeletons dated to be younger than three million years and knowledge about how the structure of pelvis and knee bones relates to the locomotion of animals.
- the inference that all emitted radiation from a particular chemical element might be electrically neutral, from observing in all experiments conducted so far that radiation emitted by this element continues in a straight path in an external magnetic field perpendicular to the stream of radiation and knowledge of the Lorentz force and Newton's second law.

3. ***Existential Abduction***, or the abduction of the existence of unknown objects from a particular class

(OBS)	F with regard to x_i
(BBK)	the existence of objects y_i of class E would explain F with regard to x_i
(HYP)	It might be that there exist objects y_i of class E

Some examples of this pattern, which was already called "existential abduction" by Thagard (1988), and has also been called "first-order existential abduction" (Schurz, 2008a) and "selective type abduction" (Hoffmann, 2010), are:

- the inference that a hominid of the genus *Australopithecus* might have lived in this area, from observing a set of vulcanized foot imprints and the belief that these foot imprints are of an *Australopithecus*.

- the inference that there might be other charged particles in the chamber, from observing deflections in the path of a charged particle in a chamber without external electric or magnetic fields and knowledge of the Coulomb and Lorentz forces and Newton's second law.

4. ***Conceptual Abduction***, or the abduction of a new concept

(OBS) F_i with regard to multiple x_j individually
(BBK) No known concept explains why F_i for all x_j

(HYP) It might be that there is a similarity between the x_j, which can be labeled with a new concept E, that explains why F_i with regard to all the various x_j individually

Some examples of this pattern, which largely coincides with the various types of "second order abduction" Schurz (2008a) suggests,[9] and several of the types of "creative abduction" conceived by Hoffmann (2010), are:

- the inference that there might be a new species of hominids, from observing various hominid fossils that are similar in many ways and believing that these fossils cannot be classified in the current taxonomy of hominids.
- the inference that there might exist a new type of interaction, from observing similar interactive behavior between certain types of particles in similar experiments and believing that this behavior cannot be explained by the already known interactions, properties of the involved particles and properties of the experimental setup.

Using the terminology of Magnani (2001) and following the distinction of Schurz (2008a), the first two patterns, abduction of a singular fact and abduction of a generalization, can be considered as instances of *selective abduction*, as the agent selects an appropriate hypothesis in her background knowledge, while the latter two, existential abduction and conceptual abduction, can be called *creative abduction*, as the agent creates a new hypothetical concept or object.[10]

As stated before, my list is not exhaustive. Further patterns have been identified, such as the abduction of a new perspective (Hoffmann, 2010), e.g. to suggest that a problem might have a geometrical solution instead of an algebraic one; "analogical abduction" (Thagard, 1988), e.g. explaining similar properties

[9] It was Schurz who pointed out that this pattern is rational and useful for science only if the observation concerns several objects each individually having the same or similar properties, so that some form of conceptual unification is obtained. Otherwise, for each fact it could be suggested that there exists an *ad hoc* power that explains (only) this single fact.

[10] Hoffmann (2010) would dispute this distinction, as he sees the third pattern (existential abduction) in the first place as the selection of an already known type (e.g. the genus *Australopithecus*), and not so much as the creation of a new token (someone of this genus of which his/her existence is now hypothesized).

of water and light, by hypothesizing that light could also be wave-like; or "theoretical model abduction" (Schurz, 2008a), i.e. explaining some observation by suggesting suitable initial conditions given some governing principles or laws. Some have even considered "visual abduction", the inference from the observation itself to a statement describing this observation, as a separate pattern (Thagard and Shelley, 1997). For some of these patterns (or instances of them), it is possible to argue that they are a special case of one of the patterns above. For instance, the suggestion of the wave nature of light can also be seen as an instance of conceptual abduction, in which the (mathematical) concept 'wave behavior' is contructed to explain the similar properties of water and light; yet it is true that the analogical nature of this inference makes it a special subpattern with interesting properties in itself.[11]

Perhaps more important to note is that these patterns are not mutually exclusive given a particular instance of abductive reasoning. For instance, the inference that leads to the explanation of why a particular piece of iron is rusted can be described both as singular fact abduction (this piece of iron underwent a reaction with oxygen) or as existential abduction (there were oxygen atoms present with which this piece of iron reacted). But in essence it describes the same explanation for the same explanandum. Also, combinations occur. For instance, if a new particle is hypothesized as an explanation for an experimental anomaly,[12] then we have both an instance of existential abduction (there is a not yet observed particle that causes the observed phenomenon) and an instance of conceptual abduction (these hypothesized particles are of a new kind).[13] Yet in the mind of the scientist, this process of hypothesis formation might have occurred in a single reasoning step.

We should not, however, be too worried about these issues, if we remember that these patterns are categories for linguistic descriptions of actual reasoning processes. Any actual instance of hypothesis formation can be described in several ways by means of natural language, and some of these expressions can be formally analyzed in more than one way. Therefore, I do not think that we should focus too much on the exact classification of particular instances of hypothesis formation. Yet this does not render meaningless the project of explicating various patterns of hypothesis formation. The goal of this project is to provide normative guidance for future hypothesis formation. If particular problems or observations can be looked at from different perspectives and, therefore, expressed in various ways, it is only beneficial for an agent to have multiple patterns of hypothesis formation at her disposal.

[11] This is also how Schurz (2008a) presents it; in his classification, analogical abduction is one of the types of second order existential abduction he conceives of.

[12] See, for instance, Wolfgang Pauli's suggestion of the neutrino in the case of the anomalous β spectrum (Gauderis, 2013b).

[13] I think this particular combination coincides with Hoffmann's (2010) pattern of "creative fact abduction".

4 Abductive Reasoning and Adaptive logics

Let us turn now to the various attempts to model abductive reasoning by means of adaptive logics. Let me start by pointing out the main characteristics why the framework of adaptive logics is fit for this job.

First, adaptive logics allow for a direct implementation of defeasible reasoning steps (*in casu* applications of *Affirming the Consequent*). This makes it possible to construct logical proofs that nicely integrate defeasible (in this case ampliative) and deductive inferences. This corresponds to natural reasoning processes.

Second, the formal apparatus of an adaptive logic instructs exactly which formulas would falsify a (defeasible) reasoning step. As these formulas are assumed to be false (so long as one cannot derive them), they are called *abnormalities* in the adaptive logic literature. So, if one or a combination of these abnormalities is derived in a proof, it instructs in a formal way which defeasible steps cannot be maintained. This possibility of defeating previous reasoning steps mirrors nicely the dynamics found in actual human reasoning.

Third, for all adaptive logics in standard format, such as the presented logics $\mathbf{LA^r_s}$ and $\mathbf{MLA^s_s}$, there are generic proofs for most of the important metatheoretical properties such as soundness and completeness (Batens, 2007).

So far, most research effort has been focused on modeling singular fact abduction, which already proves to be, even it appears to be the easiest case, a rich and fruitful point of departure. This is not exclusive to the adaptive logics framework: in general, very little logics have been proposed for other forms besides singular abduction. Some of the few exceptions are Thagard (1988) and Gauderis and Van De Putte (2012). This last logic, which is constructed within the adaptive logics framework, suffers, however, from some complications (see Beirlaen and Aliseda (2014, appendix B) and Gauderis (2013c, p. 140)). Therefore, I shall limit myself in this overview to the various attempts to model singular fact abduction within the framework of adaptive logics.

The history of research into singular fact abduction within the adaptive logics community dates back to the early 2000s and can be traced through the articles Meheus, Verhoeven, Van Dyck and Provijn (2002), Batens, Meheus, Provijn and Verhoeven (2003), Meheus and Batens (2006), Meheus and Provijn (2007), Meheus (2007), Lycke (2009) and Lycke (2012). Besides presenting early logics for singular fact abduction, this research has also shown that there actually exist two types of singular fact abduction (see Section 5). In recent years, for each of these two types of abduction an adaptive logic in standard format (see Section 6) has been developped: $\mathbf{LA^r_s}$ for practical abduction (Meheus, 2011) and $\mathbf{MLA^s_s}$ for theoretical abduction Gauderis (2013a). These will be the two logics that will be presented and explained in this article (see Sections 7 and 8). It further needs to be noted that recent research has even pushed further by considering abduction from inconsistent theories (Provijn, 2012), adaptations for use in AI (Gauderis, 2011; improved version in Gauderis, 2013c, Ch. 5) and a first logic for propositional singular fact abduction (Beirlaen and Aliseda, 2014).

5 The Problem of Multiple Explanatory Hypotheses

The early research into logics for abduction has shown that two types of abduction logics can actually be constructed, depending on how the logic deals with multiple explanatory hypotheses for a single observation.

To explain this problem, consider the following example. Suppose we have to form hypotheses for the puzzling fact Pa while our background knowledge contains both $(\forall x)(Qx \supset Px)$ and $(\forall x)(Rx \supset Px)$. There are two ways in which one can proceed. First, we can construct a logic in which we can derive only the disjunction $(Qa \vee Ra)$ and not the individual hypotheses Qa and Ra. This first way, called *practical abduction*[14] and modeled by the logic $\mathbf{LA^r_s}$ (Meheus, 2011, see Section 7), is suitable for modeling situations in which one has to *act* on the basis of the conclusions before having the chance to find out which hypothesis actually is the case. A good example is how people react to unexpected behavior. If someone suddenly starts to shout, people will typically react in a hesitant way, taking into account that either they themselves are somehow at fault or that the shouting person is just frustrated or crazy and acting inappropriately.

Second, someone with a theoretical perspective (for instance, a scientist or a detective) is interested in finding out which of the various hypotheses is the actual explanation. Therefore it is important that she can *abduce* the individual hypotheses Qa and Ra in order to examine them further one by one. Early work on these kind of logics has been done in Lycke (2009, 2012) and another solution of Lycke (personal communication). Yet these logics have a quite complex proof theory. This is because, on the one hand, one has to be able to derive Qa and Ra separately, but on the other, one has to prevent the derivation of their conjunction $(Qa \wedge Ra)$, because it seems counterintuitive to take the conjunction of two possible hypotheses as an explanation: for instance, if the street is wet, it would be weird to suggest that it has rained and that the fire department also just held an exercise. Moreover, if the two possible hypotheses are actually incompatible, it would lead to explosion in a classical context.

The logic $\mathbf{MLA^s_s}$ (Gauderis, 2013a) presented in this overview article (Section 8) solves this problem by adding modalities to the language and deriving the hypotheses $\Diamond Qa$ and $\Diamond Ra$ instead of Qa and Ra. By conceiving of hypotheses as logical possibilities, the conjunction problem is automatically solved because $\Diamond Qa \wedge \Diamond Ra$ does not imply $\Diamond(Qa \wedge Ra)$ in any standard modal logic. This approach also nicely coincides with the common idea that hypotheses are pos-

[14] According to the definition suggested in Meheus and Batens (2006, pp. 224–225) and first used in Lycke (2009).

sibilities. These features make the logic $\mathbf{MLA^s_s}$ very suitable for the modeling of actual theoretical abductive reasoning processes.

6 The Standard Format of Adaptive Logics

Before I present the logics for abduction $\mathbf{LA^s_r}$ and $\mathbf{MLA^s_s}$, I must first provide the reader with the necessary background about the adaptive logics framework, and, more in particular, with the nuts and bolts of its standard format. This will of course be a limited introduction, and I refer the reader to e.g. Straßer (2013) or Batens (2007) for a thorough introduction.

Definition An *adaptive logic in standard format* is defined by a triple:

(i) A *lower limit logic* (henceforth **LLL**): a reflexive, transitive, monotonic and compact logic that has a characteristic semantics.[15]
(ii) A *set of abnormalities* Ω: a set of **LLL**-*contingent* formulas (formulas that are not theorems of **LLL**) characterized by a logical form, or a union of such sets.
(iii) An adaptive *strategy*.

The lower limit logic **LLL** specifies the stable part of the adaptive logic. Its rules are unconditionally valid in the adaptive logic, and anything that follows from the premises by **LLL** will never be revoked. Apart from that, it is also possible in an adaptive logic to derive defeasible consequences. These are obtained by assuming that the elements of the set of abnormalities are "as much as possible" false. The adaptive strategy is needed to specify "as much as possible". This will become clearer further on.

Dynamic Proof Theory As stated before, a key advantage of adaptive logics is their *dynamic proof theory* which models human reasoning. This dynamics is possible because a *line* in an adaptive proof has – along with a line number, a formula and a justification – a fourth element, i.e. the *condition*. A condition is a finite subset of the set of abnormalities and specifies which abnormalities need to be assumed to be false for the formula on that line to be derivable.

The inference rules in an adaptive logic reduce to three generic rules. Where Γ is the set of premises, Θ a finite subset of the set of abnormalities Ω and

[15] Strictly speaking, the standard format for adaptive logics requires that a lower limit logic contains, in addition to the **LLL**-operators, also the operators of **CL** (Classical Logic). However, these operators have merely a technical role (in the generic meta-theory for adaptive logics) and are not used in the applications presented here. Therefore, given the introductory nature of this section, I will not go into further detail. In the logics presented in this dissertation, the condition is implicitly assumed to be satisfied.

$Dab(\Theta)$ the (classical) disjunction of the abnormalities in Θ, and where

$$A \qquad \Delta$$

abbreviates that A occurs in the proof on the condition Δ, the inference rules are given by the generic rules:

PREM If $A \in \Gamma$:

$$\begin{array}{ll} \vdots & \vdots \\ \hline A & \emptyset \end{array}$$

RU If $A_1, ..., A_n \vdash_{\mathbf{LLL}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC If $A_1, ..., A_n \vdash_{\mathbf{LLL}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

The premise rule PREM states that a premise may be introduced at any line of a proof on the empty condition. The unconditional inference rule RU states that, if $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ and $A_1, \ldots, A_n$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_n$, we may add B on the condition $\Delta_1 \cup \ldots \cup \Delta_n$. The strength of an adaptive logic comes from the third rule, the conditional inference rule RC, which works analogously to RU, but introduces new conditions. So, it allows one to take defeasible steps based on the assumption that the abnormalities are false.[16] Several examples of how these rules are employed in the following sections.

The only thing we still need is a criterion that defines when we consider a line of the proof to be defeated. At first sight, it seems straightforward to mark[17] lines of which one of the elements of the condition is *unconditionally*[18] derived from the premises. But this strategy, called the *simple strategy*, usually has a serious flaw. If it is possible to derive unconditionally a disjunction of abnormalities $Dab(\Delta)$ that is *minimal*, i.e. if there is no $\Delta' \subset \Delta$ such that

[16] The rule also makes clear that any adaptive proof can be transformed into a Fitch-style proof in the **LLL** by writing down for each line the disjunction of the formula and all of the abnormalities in the condition.

[17] Defeated lines in a proof are marked instead of deleted, because, in general, it is possible that they may later become unmarked in an extension of the proof.

[18] *Unconditionally* derived is to be understood as derived on the empty condition.

$Dab(\Delta')$ can be unconditionally derived, the simple strategy would ignore this information. This is problematic, however, because at least one of the disjuncts of the ignored disjunction has to be true. Therefore, we can use the simple strategy only in cases where

$$\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta) \text{ only if there is an } A \in \Delta \text{ such that } \Gamma \vdash_{\mathbf{LLL}} A$$

with $Dab(\Delta)$ any disjunction of abnormalities out of Ω. This condition will be met for the logic $\mathbf{MLA^s_s}$ (Section 8); this logic will, hence, employ the simple strategy.

The majority of logics, however, does not meet this criterion and for those logics, more advanced strategies have been developed. The best-known of these are *reliability* and *minimal abnormality*. The logic $\mathbf{LA^r_s}$ uses the reliability strategy. This strategy, which will be explained and illustrated below, orders to mark any line of which one of the elements is unconditionally derived as a disjunct from a minimal disjunction of abnormalities.[19]

7 $\mathbf{LA^r_s}$: a Logic for Practical Singular Fact Abduction

In this section, I will introduce the reader to the logic $\mathbf{LA^r_s}$ (Meheus, 2011) in an informal manner. This will allow the reader to gain a better understanding of the framework of adaptive logics and the functioning of its dynamic proof theory. In the next section, I will do the same for the logic $\mathbf{MLA^s_s}$, and, finally, in section 9, I will give the formal definitions for both logics.

In order to model abductive reasoning processes of singular facts, the logic $\mathbf{LA^r_s}$ (as will the logic $\mathbf{MLA^s_s}$) contains, in addition to deductive inference steps, defeasible reasoning steps based on an argumentation schema known as *Affirming the Consequent* (combined with Universal Instantiation):

$$(\forall\alpha)(A(\alpha) \supset B(\alpha)), B(\beta)/A(\beta)$$

The choice for a predicate logic is motivated by the fact that a material implication is used to model the relation between *explanans* and *explanandum*. As it is well known that $B \vdash_{\mathbf{CL}} A \supset B$, a propositional logic would allow us to derive anything as a hypothesis. In the predicative case, the use of the universal

[19] At this point, I have introduced all elements to explain the naming of the two logics that will be presented in this paper: as might be expected $\mathbf{LA}$ and $\mathbf{MLA}$ stand for "Logic for Abduction" and "Modal Logic for Abduction" and the superscripts $^{\mathbf{r}}$ and $^{\mathbf{s}}$ stand for the adaptive strategies *reliability* and *simple strategy*. The subscript $_{\mathbf{s}}$ originally denoted that the logic was formulated in the standard format for adaptive logics, but in Gauderis (2013c), I argued that it is more useful to interpret this $_{\mathbf{s}}$ as that they are logics for *singular fact abduction*. After all, most adaptive logics are nowadays formulated in the standard format anyhow, and this allows to contrast these logics with the logic $\mathbf{LA^r_\forall}$ which is a logic for *abduction of generalizations* (Gauderis and Van De Putte, 2012; Gauderis, 2013c).

quantifier can avoid this.[20] For a propositional logic for abduction that solves this problem in another way, see Beirlaen and Aliseda (2014).

Let me first overview the list of desiderata for this logic. This is important because in specifying the set of abnormalities and the strategy, we have to check whether they allow us to model practical abductive reasoning according to our expectations. Apart from the fact that by means of this logic we should be able to derive hypotheses according to the schema of *Affirming the Consequent*, we have to make sure that we cannot derive – as a side effect – random hypotheses which are not related to the explanandum. Finally, as I pointed out in the introduction, it is a nice feature of adaptive logics that they enable us to integrate defeasible and deductive steps.

Lower Limit Logic The lower limit logic of $\mathbf{LA^r_s}$ is classical first order logic $\mathbf{CL}$. This means that the deductive inferences of this logic are the reasoning steps modeled by classical logic. Also, as this logic is an extension of classical logic, any classical consequence of a premise set will also be a consequence of the premise set according to this logic.

Set of abnormalities Ω If we take (here and in further definitions) the meta variables A and B to represent (well-formed) formulas, α a variable and β a constant of the language in which the logic is defined $\mathcal{L}$, we can define the set of abnormalities of the logics $\mathbf{LA^r_s}$ as:

$$\begin{aligned}\Omega \quad = \quad & \{(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \mid \\ & \text{No predicate that occurs in } B \text{ occurs in } A\}\end{aligned}$$

The first line is the logical form of the abnormality; the second line in the definition is to prevent self-explanatory hypotheses. To understand the functioning of this logical form, consider the following example starting from the premise set $\{Qa, \forall x(Px \supset Qx)\}, \forall x(Px \supset Rx)$:

1	$\forall x(Px \supset Qx)$	-;PREM	$\emptyset$
2	Qa	-;PREM	$\emptyset$
3	$Pa \vee \neg Pa$	-; RU	$\emptyset$
4	$Pa \vee (\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))$	1,2,3;RU	$\emptyset$
5	Pa	4;RC	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$

From this premise set we would like to be able to form the hypothesis Pa. This is possible, because this premise set allows us to derive the statement on line 4, and this disjunction has the form that allows us to derive conditionally the hypothesis Pa by applying the rule RC. From this hypothesis we can now reason further deductively by applying e.g. *modus ponens* (note that the result of this inference has also a non-empty condition):

[20] For example, compare $\vdash_{\mathbf{CL}} B(\beta) \supset (A(\beta) \supset B(\beta))$ with $\nvdash_{\mathbf{CL}} B(\beta) \supset (\forall\alpha)(A(\alpha) \supset B(\alpha))$.

⋮	⋮	⋮	⋮
5	Pa	4;RC	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$
6	$\forall x(Px \supset Rx)$	-; PREM	$\emptyset$
7	Ra	5,6;RU	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$

Suppose now that we come to know that $\neg Pa$ is the case and add this premise to the premise set and continue the proof.[21]

⋮	⋮	⋮	⋮	
5	Pa	4;RC	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$	$\checkmark^{9}$
6	$\forall x(Px \supset Rx)$	-; PREM	$\emptyset$	
7	Ra	5,6;RU	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$	$\checkmark^{9}$
8	$\neg Pa$	-;PREM	$\emptyset$	
9	$\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)$	1,2,6; RU	$\emptyset$	

This new premise makes it possible to derive unconditionally on line 9 the condition of the hypothesis Pa. At this point, it is clear that we should not trust anymore the hypothesis formed on line 5, which we indicate by marking this line with a checkmark, indicating that we lost our confidence in this formula once we wrote down line 9. As the formula Ra is arrived at by reasoning futher upon the hypothesis Pa, it has (at least) the same condition, and is, hence, at this point also marked.

In summary, each time we defeasibly derive a hypothesis, we have to state explicitly the condition the (suspected) truth of which would defeat the hypothesis. Therefore, we can assume the hypothesis to be true as long as we can assume the condition to be false; but as soon as we have evidence that the condition might be true, we should withdraw the hypothesis.

Reliability Strategy In the previous example, we withdraw the hypothesis because its condition was explicitly derived. However, have a look at the following example proof from the premise set $\{Qa, Ra, \forall x(Px \supset Qx), \forall x(\neg Px \supset Rx)\}$:

1	$\forall x(Px \supset Qx)$	-;PREM	$\emptyset$
2	$\forall x(\neg Px \supset Rx)$	-;PREM	$\emptyset$
3	Qa	-;PREM	$\emptyset$
4	Ra	-;PREM	$\emptyset$
5	Pa	1,3;RC	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$
6	$\neg Pa$	2,4;RC	$\{\forall x(\neg Px \supset Rx) \wedge (Ra \wedge Pa)\}$

[21] Strictly speaking, this is not what is actually done. What I actually do is start a new proof with another premise set (the extended set). But it is easily seen that I can start this new proof with exactly the same lines as the old proof. This way, it looks as if I extended the old proof. This qualification needs to be considered each time I speak about "adding premises and continuing a proof", a phrase I will continue to use because it nicely mirrors how human beings deal with incoming information: they do no start over their reasoning but incorporate the new information at the point where they have arrived.

There is clearly something fishy about this situation. As the conditions on line 5 and 6 are not derivable from this premise set, logical explosion would allow us to derive anything from this premise set, if we were to use the simple strategy. Still, it is quite obvious that at least one of those two conditions have to be false, as the disjunction of these two conditions is a theorem of the lower limit logic. Yet, as we do not know from these premises which disjunct is true and which one is false, the most reliable thing to do is to mark both lines:

⋮	⋮	⋮	⋮	
5	Pa	1,3;RC	$\{\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)\}$	✓[7]
6	$\neg Pa$	2,4;RC	$\{\forall x(\neg Px \supset Rx) \wedge (Ra \wedge Pa)\}$	✓[7]
7	$(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)) \vee$ $(\forall x(\neg Px \supset Rx) \wedge (Ra \wedge Pa))$	1-4;RU	$\emptyset$	

This marking strategy is called the *reliability strategy* and it orders us to mark lines for which an element of the condition has been unconditionally derived as a disjunct of a minimal disjunction of abnormalities (or in short, a minimal *Dab*-formula). It is important to note that (1) the disjunction should only hold disjuncts that have the form of an abnormality (otherwise, a defeating disjunction could be constructed for every hypothesis) and (2) that this disjunction should be minimal (as disjunctions can always be extended by applications of the addition rule). To clarify this last point: suppose we were able to derive the condition of line 5 by itself, then the disjunction on line 7 would not be minimal anymore and there would be no reason anymore to mark line 6.

Practical Abduction The logic $\mathbf{LA^r_s}$ is a logic for practical abduction. This means that it solves the problem of multiple explanatory hypotheses by only allowing the disjunction of the various hypotheses to be derived. Consider the following example from the premise set $\{Ra, \forall x(Px \supset Rx), \forall x(Qx \supset Rx)\}$:

1	$\forall x(Px \supset Rx)$	-;PREM	$\emptyset$	
2	$\forall x(Qx \supset Rx)$	-;PREM	$\emptyset$	
3	Ra	-;PREM	$\emptyset$	
4	Pa	1,3;RC	$\{\forall x(Px \supset Rx) \wedge (Ra \wedge \neg Pa)\}$	✓[6]
5	Qa	1,2;RC	$\{\forall x(Qx \supset Rx) \wedge (Ra \wedge \neg Qa)\}$	✓[7]
6	$(\forall x(Px \supset Rx) \wedge (Ra \wedge \neg Pa)) \vee$ $(\forall x((Qx \wedge \neg Px) \supset Rx) \wedge$ $(Ra \wedge \neg(Qa \wedge \neg Pa)))$	1-3;RC	$\emptyset$	
7	$(\forall x(Qx \supset Rx) \wedge (Ra \wedge \neg Qa)) \vee$ $(\forall x((Px \wedge \neg Qx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \wedge \neg Qa)))$	1-3;RC	$\emptyset$	
8	$\forall x((Px \vee Qx) \supset Rx)$	1,2;RU	$\emptyset$	
9	$Pa \vee Qa$	3,8;RC	$\{\forall x((Px \vee Qx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \vee Qa))\}$	

Because of the fact that the minimal *Dab*-formulas on line 6 and 7 could be derived from the premises, the individual hypotheses Pa and Qa have to be withdrawn, yet the condition of their disjunction on line 9 is not part of a minimal *Dab*-formula from these premises.

Avoiding Random Hypotheses Another important feature of a logic for abduction is that it prevents to allow to derive random hypotheses. The three most common ways to introduce random hypotheses is (1) by deriving an explanation for a tautology, e.g. deriving Xa from the theorems $Pa \vee \neg Pa$ and

$\forall x(Xx \supset (Px \vee \neg Px)))$; (2) by deriving contradictions as explanations, which leads to logical explosion, e.g. deriving $Xa \wedge \neg Xa$ from Pa and the theorem $\forall x((Xx \wedge \neg Xx) \supset Px)$; or (3) by deriving hypotheses that are not the most parsimonious ones, e.g. deriving $Pa \wedge Xa$ from Qa and $\forall x(Px \supset Qx)$ (and its consequence $\forall x((Px \wedge Xx) \supset Qx))$. The logic $\mathbf{LA^r_s}$ prevent these three ways by similar mechanisms as the mechanism to block individual hypotheses illustrated above. Elaborate examples for each of these three ways can be found in Meheus (2011).

8 $\mathbf{MLA^s_s}$: a Logic for Theoretical Singular Fact Abduction

In this section, I will introduce the reader to the logic $\mathbf{MLA^s_s}$ (Gauderis, 2013a) in a similar informal manner. The formal definitions can also be found in Section 9. Analogously, this logic also models deductive steps combined with applications of *Affirming the Consequent* (combined with Universal Instantiation), yet it treats the problem of multiple explanatory hypotheses now in a different way: it allows to derive these hypotheses individually, yet to avoid logical explosion caused by mutually exclusive hypotheses, it treats them as modal possibilities (see Section 5).

The list of desiderata for this logic is very analogous as the one for the logic $\mathbf{LA^r_s}$, except for treating the problem of multiple explanatory hypotheses in a different manner. Specific for this logic (as this logic is aimed at modeling the reasoning of e.g. scientists or detectives (Gauderis, 2012)) is the desideratum that it handles contradictory hypotheses, predictions and counterevidence in a natural way.

Formal Language Schema As this logic is a modal logic, the language of this logic is an extension of the language of classical logic $\mathbf{CL}$. Let us denote the standard predicative language of classical logic with $\mathcal{L}$. I will further use $\mathcal{C}$, $\mathcal{V}$, $\mathcal{F}$ and $\mathcal{W}$ to refer respectively to the sets of individual constants, individual variables, all (well-formed) formulas of $\mathcal{L}$ and the closed (well-formed) formulas of $\mathcal{L}$.

$\mathcal{L}_M$, the language of the logic $\mathbf{MLA^s_s}$, is $\mathcal{L}$ extended with the modal operator $\Box$. $\mathcal{W}_M$, the set of closed formulas of $\mathcal{L}_M$ is the smallest set that satisfies the following conditions:

1. if $A \in \mathcal{W}$, then $A, \Box A \in \mathcal{W}_M$
2. if $A \in \mathcal{W}_M$, then $\neg A \in \mathcal{W}_M$
3. if $A, B \in \mathcal{W}_M$, then $A \wedge B, A \vee B, A \supset B, A \equiv B \in \mathcal{W}_M$

It is important to notice that there are no occurrences of modal operators within the scope of another modal operator or a quantifier. I further define the set $\mathcal{W}_\Gamma$, the subset of $\mathcal{W}_M$ the elements of which can act as premises in the logic, as:

$$\mathcal{W}_\Gamma = \{\Box A \mid A \in \mathcal{W}\}$$

It is easily seen that $\mathcal{W}_\Gamma \subset \mathcal{W}_M$.

Lower Limit Logic The **LLL** of $\mathbf{MLA}^{\mathbf{s}}_{\mathbf{s}}$ is the predicative version of **D**, restricted to the language schema $\mathcal{W}_M$. **D** is characterized by a full axiomatization of predicate **CL** together with two axioms, an inference rule and a definition:

K $\Box(A \supset B) \supset (\Box A \supset \Box B)$
D $\Box A \supset \neg\Box\neg A$
NEC if $\vdash A$, then $\vdash \Box A$
$\Diamond_{df}$ $\Diamond A =_{df} \neg\Box\neg A$

This logic is one of the weakest normal modal logics that exist and is obtained by adding the **D**-axiom to the axiomatization of the better-known minimal normal modal logic **K**.

The semantics for this logic can be expressed by a standard possible world Kripke semantics where the accessibility relation R between possible worlds is *serial*, i.e. for every world w in the model, there is at least one world w' in the model such that Rww'.

Intended Interpretation of the modal operators As indicated above, explanatory hypotheses – the results of abductive inferences – will be represented by formulas of the form $\Diamond A$ ($A \in \mathcal{W}$). Formulas of the form $\Box B$ are used to represent explananda, other observational data and relevant background knowledge. Otherwise, this information would not be able to revoke derived hypotheses.[22] The reason **D** is chosen instead of **K** is that it is assumed that the explananda and background information are together consistent. This assumption is modeled by the **D**-axiom.[23]

Set of Abnormalities Since the final form of the abnormalities is quite complex – although the idea behind it is straightforward – I will first consider two more basic proposals that are constitutive for the final form and show why they are insufficient. Obviously, only closed well-formed formulas can be an element of any set of abnormalities. This will not be explicitly stated each time.

[22] For instance, $\neg A$ and $\Diamond A$ are not contradictory, whereas $\Box\neg A$ and $\Diamond A$ are.
[23] For instance, the premise set $\{\Box\neg Pa, \Box(\forall x)Px\}$ is a set modeling an inconsistent set of background knowledge and observations. However, in the logic **K**, this set would not be considered inconsistent, because we cannot derive anything from this set by *Ex Falso Quodlibet*. To be able to do this, we need the **D**-axiom.

First proposal Ω_1 This first proposal is a modal version of the set of abnormalities of the logic $\mathbf{LA^r_s}$:

$$\Omega_1 = \{\Box(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \mid \text{No predicate that occurs in } B \text{ occurs in } A\}$$

Analogous to the logic $\mathbf{LA^r_s}$, this means that a derived hypothesis will be defeated if one shows explicitly that the hypothesis cannot be the case.

Simple Strategy For this logic we can use the *simple strategy*, which means, as stated before, that we have to mark lines for which one of the elements of the condition is unconditionally derived. We can easily see that the condition for use of the simple strategy, i.e.

$$\Gamma \vdash_{\mathbf{LLL}} Dab(\Delta) \text{ only if there is an } A \in \Delta \text{ such that } \Gamma \vdash_{\mathbf{LLL}} A,$$

is fulfilled here. Since all premises have the form $\Box A$, the only option to derive a disjunction of abnormalities would be to apply addition, i.e. to derive $(\Box A \vee \Box B)$ from $\Box A$ (or $\Box B$), because it is well-known that $\Box(A \vee B) \nvdash \Box A \vee \Box B$ in any standard modal logic.[24]

Contradictory hypotheses As a first example of the functioning of this logic, consider the following example starting from the premise set $\{\Box Qa, \Box Ra, \Box\forall x(Px \supset Qx), \Box\forall x(\neg Px \supset Rx)\}$. As the reader is by now probably accustomed with the functioning of the abnormalities, it is also already shown how this logic is able to handle contradictory hypotheses without causing explosion.

1	$\Box\forall x(Px \supset Qx)$	-;PREM	$\emptyset$
2	$\Box\forall x(\neg Px \supset Rx)$	-;PREM	$\emptyset$
3	$\Box Qa$	-;PREM	$\emptyset$
4	$\Box Ra$	-;PREM	$\emptyset$
5	$\Diamond Pa$	1,3;RC	$\{\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))\}$
6	$\Diamond\neg Pa$	2,4;RC	$\{\Box(\forall x(\neg Px \supset Rx) \wedge (Ra \wedge \neg\neg Pa))\}$
7	$\Diamond Pa \wedge \Diamond\neg Pa$	5,6;RU	$\{\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa)),$ $\Box(\forall x(\neg Px \supset Rx) \wedge (Ra \wedge \neg\neg Pa))\}$

$\Diamond Pa$ and $\Diamond\neg Pa$ are both derivable hypotheses because the conditions on lines 5-7 are not unconditionally derivable from the premise set. It is also interesting to note that, because of the properties of the lower limit $\mathbf{D}$, it is not possible to derive from these premises that $\Diamond(Pa \wedge \neg Pa)$. The conjunction of two hypotheses is never considered as a hypothesis itself, unless there is further background information that links the two hypotheses in some way.

[24] It is also possible to derive a disjunction from the premises by means of the $\mathbf{K}$-axiom. For instance, $\Box(A \supset B) \vdash \neg\Box A \vee \Box B$, but the first disjunct will always be equivalent to a possibility ($\Diamond\neg A$) and can, hence, not be an abnormality.

Predictions and Evidence To show that this logic handles predictions and (counter)evidence for these predictions in a natural way, let us extend the premise set with the additional implication $\Box\forall x(Px \supset Sx)$:

8	$\Box\forall x(Px \supset Sx)$	-;PREM	$\emptyset$
9	$\Diamond Sa$	5,8;RU	$\{\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))\}$

With this extra implication we can derive the prediction $\Diamond Sa$. As long as we have no further information about this prediction (for instance, by observation), it remains a hypothesis derived on the same condition as $\Diamond Pa$. If we would test this prediction, we would have two possibilities. On the one hand, if the prediction turns out to be false, the premise $\Box\neg Sa$ could be added to the premise set:

⋮	⋮	⋮	⋮	
5	$\Diamond Pa$	1,3;RC	$\{\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))\}$	$\checkmark^{12}$
⋮	⋮	⋮	⋮	
9	$\Diamond Sa$	5,8;RU	$\{\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))\}$	$\checkmark^{12}$
10	$\Box\neg Sa$	PREM	$\emptyset$	
11	$\Box\neg Pa$	8,10;RU	$\emptyset$	
12	$\Box(\forall x(Px \supset Qx) \wedge (Qa \wedge \neg Pa))$	1,3,11;RU	$\emptyset$	

In this case, we could subsequently derive $\Box\neg Pa$, which would falsify the hypothesis $\Diamond Pa$. On the other hand, if the prediction Sa turned out to be true, the premise $\Box Sa$ could have been added, but this extension of the premise set would not allow us to derive $\Box Pa$. Since true predictions only *corroborate* the hypothesis and do not *prove* it, while false predictions directly *falsify* the hypothesis, one can say that this logic handles predictions in a *Popperian* way.[25]

Contradictions One of the three ways a logic of abduction could generate random hypotheses as a side effect is by allowing for the abduction of contradictions. How this is possible and how the logic prevents this is illustrated in the following proof from the premise set $\{\Box Qa\}$:

1	$\Box Qa$	-;PREM	$\emptyset$	
2	$\Box\forall x((Xx \wedge \neg Xx) \supset Qx)$	-;RU	$\emptyset$	
3	$\Diamond(Xa \wedge \neg Xa)$	1,2;RC	$\{\Box(\forall x((Xx \wedge \neg Xx) \supset Qx) \wedge$ $(Qa \wedge \neg(Xa \wedge \neg Xa)))\}$	$\checkmark^{4}$
4	$\Box(\forall x((Xx \wedge \neg Xx) \supset Qx) \wedge$ $(Qa \wedge (\neg Xa \vee Xa)))$	1;RU	$\emptyset$	

[25] It needs to be remembered that $\mathbf{MLA}^{\mathbf{s}}_{\mathbf{s}}$ is a logic for modeling abduction and handling explanatory hypotheses, not a formal methodology of science. This logic has nothing to say about the confirmation of theories.

Tautologies Still, there are other ways to derive random hypotheses that are not prevented by the first proposal for the set of abnormalities Ω_1. For instance, Ω_1 does not prevent that random hypotheses can be derived from a tautology, as illustrated by the following example. As it is impossible in the following proof from the premise set $\emptyset$ to unconditionally derive the abnormality in the condition of line 3 from the premises, the formula of line 3, the random hypothesis $\Diamond Xa$, remains derived in every possible extension of the proof.

1	$\Box(Qa \vee \neg Qa)$	-;RU	$\emptyset$
2	$\Box\forall x(Xx \supset (Qx \vee \neg Qx))$	-;RU	$\emptyset$
3	$\Diamond Xa$	1,2;RC	$\{\Box(\forall x(Xx \supset (Qx \vee \neg Qx)) \wedge$ $((Qa \vee \neg Qa) \wedge \neg Xa))\}$

Therefore, let me adjust the set of abnormalities to obtain the second proposal Ω_2.

Second proposal Ω_2 No hypothesis can be abduced from a tautology if the abnormalities have the following form:

$$\begin{aligned}\Omega_2 \quad = \quad & \{\Box(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \\ & \vee \Box\forall\alpha B(\alpha) \mid \\ & \text{No predicate that occurs in } B \text{ occurs in } A\}\end{aligned}$$

It is clear that we can keep using the simple strategy with this new set of abnormalities. It is also easily seen that all of the advantages and examples described above still hold. Each time we can derive an abnormality of Ω_1, we can derive the corresponding abnormality of Ω_2 by a simple application of *addition*. Finally, the problem raised by the tautologies, as illustrated in the previous example, is solved in an elegant way, because the form of the abnormalities makes sure that the abnormality will always be a theorem in case the explanandum is a theorem. So, nothing can be abduced from tautologies.

Most parsimonious explanantia Still, there is third way to derive random hypotheses that cannot be prevented by Ω_2. Consider, for instance, the following proof from the premise set $\{\Box Ra, \Box\forall x(Px \supset Rx)\}$:

1	$\Box Ra$	-;PREM	$\emptyset$
2	$\Box\forall x(Px \supset Rx)$	-;PREM	$\emptyset$
3	$\Box\forall x((Px \wedge Xx) \supset Rx)$	2;RU	$\emptyset$
4	$\Diamond(Pa \wedge Xa)$	1,3;RC	$\{\Box(\forall x((Px \wedge Xx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \wedge Xa))) \vee \Box\forall xRx\}$
5	$\Diamond Xa$	4;RU	$\{\Box(\forall x((Px \wedge Xx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \wedge Xa))) \vee \Box\forall xRx\}$

The reason why we can derive the random hypothesis $\Diamond Xa$ is the absence of a mechanism to ensure that the abduced hypothesis is the most parsimonious one and not the result of *strengthening the antecedent* of an implication. Before defining the final and actual set of abnormalities that also prevents this way of generating random hypotheses, I have to introduce a new notation to keep things as perspicuous as possible.

Notation Suppose $A_{PCN}(\alpha)$ is the *prenex conjunctive normal* form of $A(\alpha)$. This is the equivalent form of $A(\alpha)$ where all quantifiers are first moved to the front of the expression and where, consequently, the remaining (quantifier-free) expression is written in conjunctive normal form, i.e. as a conjunction of disjunctions of literals.

$$A_{PCN}(\alpha) = (Q_1\gamma_1)\dots(Q_m\gamma_m)(A_1(\alpha) \wedge \dots \wedge A_n(\alpha))$$
$$\text{and} \vdash A_{PCN}(\alpha) \equiv A(\alpha)$$

with $m \geqslant 0$,$n \geqslant 1$,$Q_i \in \{\forall, \exists\}$ for $i \leqslant m$, $\gamma_i \in \mathcal{V}$ for $i \leqslant m$, $\alpha \in \mathcal{V}$ and $A_i(\alpha)$ disjunctions of literals in $\mathcal{F}$ for $i \leqslant n$.

Then, I can introduce the new notation $A_i^{-1}(\alpha)$ $(1 \leqslant i \leqslant n)$ so that I have a way to take out one of the conjuncts of a formula in PCN form. In cases where the conjunction consists of only one conjunct (and, obviously, no more parsimonious explanation is possible), the substitution with a random tautology will make sure that the condition for parsimony, added in the next set of abnormalities, is satisfied trivially.

$$\begin{aligned} \text{if } n > 1 \quad : \quad & A_i^{-1}(\alpha) =_{df} (Q_1\gamma_1)\dots(Q_m\gamma_m)(A_1(\alpha) \wedge \dots \wedge A_{i-1}(\alpha) \wedge \\ & \qquad A_{i+1}(\alpha) \wedge \dots \wedge A_n(\alpha)) \\ & \text{with } A_j \ (1 \leqslant j \leqslant n) \text{ the } j^{\text{th}} \text{ conjunct of } A_{PCN}(\alpha) \\ \text{if } n = 1 \quad : \quad & A_1^{-1}(\alpha) =_{df} \top \\ & \text{with } \top \text{ any tautology of } \mathbf{CL} \end{aligned}$$

Final proposal Ω With this notation I can write the logical form of the set of abnormalities Ω of the logic $\mathbf{MLA^s_s}$.

$$\begin{aligned} \Omega \quad = \quad & \{\Box(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \\ & \vee \Box\forall\alpha B(\alpha) \vee \bigvee_{i=1}^{n} \Box\forall\alpha(A_i^{-1}(\alpha) \supset B(\alpha)) \mid \\ & \text{No predicate that occurs in } B \text{ occurs in } A\} \end{aligned}$$

This form might look complex, but its functioning is quite straightforward. I have actually constructed the disjunction of the three reasons why we should refrain from considering $A(\beta)$ as a good explanatory hypothesis for the phenomenon $B(\beta)$, even if we have $(\forall\alpha)(A(\alpha) \supset B(\alpha))$. The disjunction will make sure that the hypothesis $A(\beta)$ is rejected as soon as one of the following is the case: (i) when $\neg A(\beta)$ is derived, (ii) when $B(\beta)$ is a tautology (and obviously, does not need an explanatory hypothesis) or (iii) when $A(\beta)$ has a redundant part and is therefore not an adequate explanatory hypothesis. For the same reasons as stated in the description of Ω_2, we can keep using the simple strategy and all of the advantages and examples described above will still hold.

Let us have a look at how this final set of abnormalities solves the previous problem. As we fully wrote out the condition, we can easily see that the third disjunct is actually a premise, and that, hence, the abnormality is unconditionally derivable.

1	$\Box Ra$	-;PREM	$\emptyset$	
2	$\Box\forall x(Px \supset Rx)$	-;PREM	$\emptyset$	
3	$\Box\forall x((Px \wedge Qx) \supset Rx)$	2;RU	$\emptyset$	
4	$\Diamond(Pa \wedge Qa)$	1,3;RC	$\{\Box(\forall x((Px \wedge Qx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \wedge Qa))) \vee \Box\forall x Rx$ $\vee\Box\forall x(Px \supset Rx)$ $\vee\Box\forall x(Qx \supset Rx)\}$	$\checkmark^5$
5	$\Box(\forall x((Px \wedge Qx) \supset Rx) \wedge$ $(Ra \wedge \neg(Pa \wedge Qa))) \vee \Box\forall x Rx$ $\vee\Box\forall x(Px \supset Rx)$ $\vee\Box\forall x(Qx \supset Rx)\}$	2; RU	$\emptyset$	

9 Formal Presentations of the Logics $\mathbf{LA^r_s}$ and $\mathbf{MLA^s_s}$

In this final section, I will define the logics $\mathbf{LA^r_s}$ and $\mathbf{MLA^s_s}$ in a formal and precise way.[26] Like any adaptive logic in standard format, the the logics $\mathbf{LA^r_s}$ and $\mathbf{MLA^s_s}$ are characterized by the triple of a lower limit logic, a set of abnormalities and an adaptive strategy.

For $\mathbf{LA^r_s}$, the lower limit logic is $\mathbf{CL}$, the strategy is the *reliability strategy* and the set of abnormalities $\Omega_{\mathbf{LA^r_s}}$ is defined by:

$$\Omega_{\mathbf{LA^r_s}} = \{(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \mid \text{No predicate that occurs in } B \text{ occurs in } A\}$$

For $\mathbf{MLA^s_s}$, the lower limit logic is $\mathbf{D}$, the strategy is the *simple strategy* and the set of abnormalities $\Omega_{\mathbf{MLA^s_s}}$ is, relying on the previously introduced abbreviation, defined by:

$$\Omega_{\mathbf{MLA^s_s}} = \{\Box(\forall\alpha(A(\alpha) \supset B(\alpha)) \wedge (B(\beta) \wedge \neg A(\beta))) \vee \Box\forall\alpha B(\alpha) \vee \bigvee_{i=1}^{n} \Box\forall\alpha(A_i^{-1}(\alpha) \supset B(\alpha)) \mid \text{No predicate that occurs in } B \text{ occurs in } A\}$$

[26] This section is limited to what I need to present these specific logics. For a more general formal presentation of adaptive logics in standard format, see Batens (2007).

Proof Theory The proof theory of these logics is characterized by the three generic inference rules introduced in section 2 and the following definitions.

Within adaptive logics, proofs are considered to be chains of subsequent stages. A *stage of a proof* is a sequence of lines obtained by application of the three generic rules. As such, every proof starts off with the first stage which is an empty sequence. Each time a line is added to the proof by applying one of the inference rules, the proof comes to its next stage, which is the sequence of lines written so far extended with the new line.

Definition 1 (Minimal Dab-formula at stage s). *A Dab-formula $Dab(\Delta)$*[27] *is a* minimal Dab-formula at stage s *if and only if $Dab(\Delta)$ is derived on the empty condition at stage s, and there is no $\Delta' \subset \Delta$ for which $Dab(\Delta')$ is derived on the empty condition at stage s.*

Definition 2 (Set of unreliable formulas $U_s(\Gamma)$ at stage s). *The* set of unreliable formulas $U_s(\Gamma)$ *at stage s is the union of all Δ for which $Dab(\Delta)$ is a minimal Dab-formula at stage s.*

Definition 3 (Marking for the reliability strategy). *Line i with condition Θ is* marked for the reliability strategy *at stage s of a proof if and only if $\Theta \cap U_s(\Gamma) \neq \emptyset$.*

Definition 4 (Marking for the simple strategy). *Line i with condition Θ is* marked for the simple strategy *at stage s of a proof, if stage s contains a line of which an $A \in \Theta$ is the formula and $\emptyset$ the condition.*

Definition 5 (Derivation of a formula at stage s). *A formula A is* derived from Γ at stage *s of a proof if and only if A is the formula of a line that is unmarked at stage s.*

Definition 6 (Final derivation of a formula at stage s). *A formula A is* finally derived from Γ *at stage s of a proof if and only if A is derived at line i, line i is not marked at stage s and every extension of the proof in which i is marked may be further extended in such a way that line i is unmarked.*[28]

Definition 7 (Final Derivability for $\mathbf{LA^r_s}$). $\Gamma \vdash_{\mathbf{LA^r_s}} A$ *($A \in Cn_{\mathbf{LA^r_s}}(\Gamma)$) if and only if A is finally derived in an $\mathbf{LA^r_s}$-proof from Γ.*

Definition 8 (Final Derivability for $\mathbf{MLA^s_s}$). *For $\Gamma \subset \mathcal{W}_\Gamma$: $\Gamma \vdash_{\mathbf{MLA^s_s}} A$ ($A \in Cn_{\mathbf{MLA^s_s}}(\Gamma)$) if and only if A is finally derived in a $\mathbf{MLA^s_s}$-proof from Γ.*

Semantics The semantics of an adaptive logic is obtained by a selection on the models of the lower limit logic. For a more elaborate discussion of the following definitions, I refer to the original articles and the aforementioned theoretical overviews of adaptive logics.

[27] Recall, $Dab(\Theta)$ is the (classical) disjunction of the abnormalities in a finite subset Θ of the set of abnormalities Ω.

[28] Using the simple strategy, it is not possible that a marked line becomes unmarked at a later stage of a proof. Therefore, the final criterium reduces for this strategy to the requirement that the line remains unmarked in every extension of the proof.

Definition 9. *A* $\mathbf{CL}$*-model M of the premise set* Γ *is* reliable *if and only if* $\{A \in \Omega \mid M \models A\} \subseteq \Delta_1 \cup \Delta_2 \cup \ldots$ *with* $\{Dab(\Delta_1), Dab(\Delta_2), \ldots\}$ *the set of minimal* Dab*-consequences of* Γ.

Definition 10. *A* $\mathbf{D}$*-model M of the premise set* Γ *is* simply all right *if and only if* $\{A \in \Omega \mid M \models A\} = \{A \in \Omega \mid \Gamma \vdash_{\mathbf{D}} A\}$.

Definition 11 (Semantic Consequence of $\mathbf{LA^r_s}$). $\Gamma \models_{\mathbf{LA^r_s}} A$ *if and only if* A *is verified by all reliable models of* Γ.

Definition 12 (Semantic Consequence of $\mathbf{MLA^s_s}$). *For* $\Gamma \subset \mathcal{W}_\Gamma$*:* $\Gamma \models_{\mathbf{MLA^s_s}} A$ *if and only if* A *is verified by all simply all right models of* Γ.

The fact that these two logics are in standard format warrants that the following theorems hold:

Theorem 1 (Soundness and Completeness of $\mathbf{LA^r_s}$). $\Gamma \vdash_{\mathbf{LA^r_s}} A$ *if and only if* $\Gamma \models_{\mathbf{LA^r_s}} A$.

Theorem 2 (Soundness and Completeness of $\mathbf{MLA^s_s}$). $\Gamma \vdash_{\mathbf{MLA^s_s}} A$ *if and only if* $\Gamma \models_{\mathbf{MLA^s_s}} A$.

10 Conclusion

In this overview paper, I covered quite some ground. I started by discussing the possibility of modeling abduction by means of formal logics, after which I expanded this discussion by identifying four main abduction patterns. Next, I discussed the benefits of using the adaptive logics framework to model abductive reasoning. Zooming in to the details of singular fact abduction, I consequently argued that the problem of multiple explanatory hypotheses can be solved according to two different strategies, which are called theoretical and practical abduction. Finally, I concluded this paper by presenting two full adaptive logics in standard format: $\mathbf{LA^r_s}$ for practical singular fact abduction and $\mathbf{MLA^s_s}$ for theoretical singular fact abduction.

However, if we look at the prospect of modeling abductive reasoning by means of formal (adaptive) logics, we have so far only scratched the tip of the iceberg. At present, apart from a single exception, only logics have been devised for singular fact abduction, which is in fact the most easy of the various patterns of abduction. Yet the complications that already arise on this level should warn us that the road ahead will be steep and arduous.

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