SUPERVENIENCE: ITS LOGIC AND ITS INFERENTIAL ROLE IN CLASSICAL GENETICS*

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Abstract

Supervenience is mostly conceived of as a purely philosophical concept. Nevertheless, I will argue, it played an important and very fruitful inferential role in classical genetics. Gregor Mendel *assumed* that phenotypic traits supervene on underlying factors, and this assumption allowed him to successfully predict and explain the phenotypical regularities he had experimentally discovered. Therefore it is interesting to explicate how we reason about supervenience relations.

I will tackle the following two questions. Firstly, can a reliable method (a logic) be found for inferring supervenience claims from data? Secondly, can a reliable method (a logic) be found to empirically test supervenience claims? I will answer these questions within the framework of the adaptive logics programme.

1. *What is supervenience?*

Supervenience is a concept that is well-known in present-day analytic philosophy, in intellectual domains as different as ethics, aesthetics, metaphysics, philosophy of science, etc. It is generally thought of as being a relation between (sets of) properties. A set of properties A supervenes one a set of properties B if and only if any two objects that are B-indiscernible (i.e., exactly alike according to the properties in B) are also A-indiscernible (see Kim, 1993a; Savallos and Yalçin, 1995). For example, to believe that the moral is supervenient on the natural is to believe that if two objects (persons, acts, states of affairs, . . .) are alike in all natural respects, they are also alike in all moral respects (Kim, 1984, 57).

Usually, A and B are closed under Boolean property forming operations (mostly complementation, conjunction and disjunction).[1] Some of their members are called *maximal properties* (Kim, 1984, 58–59). These are the strongest consistent properties constructible in them. Maximal properties are jointly exhaustive and mutually exclusive (so every object must have one and only one of them). For example, suppose that B is the Boolean closure of $\{P, Q, R\}$. Then the eight B-maximal properties are $(P \wedge Q \wedge R), (P \wedge Q \wedge \sim R), \ldots, (\sim P \wedge \sim Q \wedge \sim R)$. Two objects are B-indiscernible if and only if they have the same B-maximal property. So the definition of supervenience can be stated more precisely as

Definition 1: A supervenes *on B if and only if for any x and y if x and y have the same B-maximal property then x and y have the same A-maximal property.*

There are many different concepts of supervenience. The definition just given concerns *protosupervenience* (Bacon, 1995) or *de facto supervenience* (McLaughlin, 1995). Other concepts, like weak supervenience or strong supervenience can be obtained from it by adding modal strength (see McLaughlin (1995) for definitions of these concepts). In the rest of this paper, when I use 'supervenience' or 'supervenes' this should be read as 'de facto supervenience' or 'protosupervenes'.

Protosupervenience is not a popular concept. Many philosophers think it's too weak to express interesting metaphysical claims. Because they lack modal strength, relations of protosupervenience do not carry over to other possible worlds. I admit that this is a serious flaw in metaphysical discussions. Nevertheless, *de facto* supervenience deserves our attention as it played an important inferential role in classical genetics.

Supervenience has two crucial features: determination and multiple realizability. Definition 1 states that once an object's B-properties are fixed, it's A-properties are fixed too. In other words, the B-properties *determine* the A-properties. But definition 1 does not rule out the possibility that two objects are A-indiscernible and yet are B-discernible. In other words, it explicitly allows for the *multiple realizability* of A-properties by B-properties.

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[1] So, for example, if $G \in A$ and $H \in A$, then also $\sim G \in A$, $G \wedge H \in A$, $G \vee \sim H \in A$, $\ldots$ Note, however, that the metaphysical status of disjunctive and of negative properties has frequently been debated (see Kim, 1993a).

2. *Mendel's experimental treatment of heredity and hybridization*

For centuries, philosophers, scientists and breeders paid much attention to the problems of heredity and hybridization. At the end of the 18th century and especially in the 19th century, research into these phenomena became academically institutionalized, partly due to the huge economic advantages it promised to offer. (Orel, 1996, 7–35) In the 1850^s and 1860^s, Gregor Mendel (1822–1884) studied heredity and hybridization both experimentally and theoretically. He presented his findings to the Brünn Natural History Society in 1865. In 1866 his paper, "Versuche über Pflanzen-Hybriden" (Mendel, 1933), was published.[2] Mendel's results remained largely unnoticed until they were 'rediscovered' by Carl Correns and by Hugo de Vries in 1900.[3] From then onwards they were rapidly circulated throughout Europe and the States, largely due to William Bateson's proselytism (Orel, 1996, 282–293).

Mendel's experiments were very fruitful compared to those of his predecessors. This was partly due to the following methodological decision: instead of studying the overall similarities and differences between subsequent generations, he focussed on pairs of discrete traits (mainly in *Pisum* or pea plants). He selected seven pairs of opposing traits (for instance long stems and short stems, or green and yellow pods, or round and wrinkled seeds) and in each experiment he considered only a definite number of them. (Stern and Sherwood, 1966, 5–8)

This methodological decision was very fruitful (Meijer, 1983, 128–129). It allowed him to discover a set of fairly simple phenotypic regularities. First, he found out that if opposing traits are united by fertilization, only one of them is manifest in the resulting hybrids (e.g., after crossing true-breeding long-stemmed plants with short-stemmed ones, all hybrids, now called the F_1-generation, have long stems).[4] This was the case for all seven pairs of traits. The manifest traits he called *dominating*, the opposing ones *recessive* (Stern and Sherwood, 1966, 9–10). Secondly, he found out that after selfing

[2] In 1966, Curt Stern and Eva R. Sherwood published a very good translation: "Experiments on Plant Hybrids" (in Stern and Sherwood, 1966, 1–48).

[3] The prevailing story about this dramatic 'rediscovery' is not very veracious. On the one hand, it is most likely that Hugo de Vries knew about Mendel's work before 1900. On the other hand, both de Vries and Correns seem to have accepted only part of Mendel's explanation. (see Orel, 1996, 284, 289)

[4] A true-breeding plant is a plant that only produces offspring with the same traits after selfing (in contrast to hybrid plants, cf. infra). Selfing or self-fertilizing means that seeds are fertilized with pollen from the same plant (either naturally or artificially). *Pisum* plants are self-fertilizing in nature.

these hybrids, the recessive traits reappear in the hybrid progeny (now called the F_2-generation) along with the dominating traits. The average ratio of dominant traits to recessive traits in his experiments approximated 3:1 (Stern and Sherwood, 1966, 10–13). Thirdly, selfing the F_2-generation yielded the following results. Plants with the recessive trait begot recessive progeny only. Of the dominant plants, only one third yielded exclusively dominant progeny. Like the recessive progeny, this was true-breeding. The remaining dominant plants from the F_2-generation behaved like the F_1-hybrids, i.e. they begot both dominant and recessive progeny with an average ratio of 3:1 (Stern and Sherwood, 1966, 14–15). Fourthly and finally, experiments involving more than one pair of opposing traits (now called multihybrid experiments) indicated that

> the behavior of each pair of differing traits in a hybrid association is independent of all other differences in the two parental plants. (Stern and Sherwood, 1966, 22)

Mendel's methodological decision not only allowed him to discover these phenotypic regularities. It also helped him to find a simple explanation. It consisted of two main parts. On the one hand he assumed that phenotypic traits are caused by some underlying factors, which he sometimes called *innere Beschaffenheit*, other times *Factoren* or *Anlagen* (see Mendel, 1933, 23–24). On the other hand, he made assumptions about the behavior of these factors, i.e. about how they are transmitted from generation to generation. In the beginning of the 20th century, these assumptions would be called 'Mendel's Laws'.

In the next section, I will pursue the relation between traits and their underlying factors. Mendel's laws I will shortly handle in section 9.

3. *The relation between factors and traits*

Mendel assumed that observable traits were caused or determined by unobservable factors. This approach was certainly not new. Since antiquity, philosophers and scientists have searched for mechanisms underlying hereditary phenomena. But Mendel's focuss on discrete traits gave rise to a relatively simpler picture. Now what exactly was the relation between traits and their underlying factors?

For ease of reference, let me introduce four predicates: P = 'has the dominating trait', P' = 'has the recessive trait', P = 'has the potential for the

dominating trait', and P$'$ = 'has the potential for the recessive trait'.[5] With these four predicates it is fairly easy to express the relation between traits and their underlying factors (see Stern and Sherwood, 1966, 23–32).

(3.1) $(\forall x)((\mathrm{P}'x \& \sim\mathrm{P}x) \supset P'x)$

In other words, if a plant has the potential for the recessive trait but not for the dominating one, it will have the recessive trait. Dominating traits can be realized in two different ways. The next two formulas concern what Mendel called 'constant dominating traits' and 'hybrid traits' respectively (Stern and Sherwood, 1966, 16). Hybrid and constant dominating traits are observationally indistinguishable.[6]

(3.2) $(\forall x)((\mathrm{P}x \& \sim\mathrm{P}'x) \supset Px)$

(3.3) $(\forall x)((\mathrm{P}x \& \mathrm{P}'x) \supset Px)$

This list of formulas exhausts all possible cases, since every plant has to have at least one of these potentials:

(3.4) $\sim(\exists x)(\sim\mathrm{P}x \& \sim\mathrm{P}'x)$

From this short exposition it is clear what is the relation between traits and factors. A plant's internal make-up *determines* its observable traits. But the latter (at least the dominating ones) are *multiply realizable*. In other words, traits *supervene* upon factors.

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[5] Note that Mendel did not typographically distinguish between traits and factors. Note also that Mendel typically used small letters to refer to recessive traits. I deviate from this convention in order to dovetail with the language of first order predicate logic (cf. infra).

[6] Note that I do not refer to diploidy (the fact that the somatic cells of pea plants contain two copies of every chromosome and hence also two alleles of every gene, one of which is inherited from the father, the other from the mother) here. Although his theory is often presented in terms of *pairs of factors or alleles* in modern genetics textbooks (cf. Klug et al., 2006, 41), Mendel was not wedded to this idea. Most plausibly, he believed that pea plants received one factor from each parent, but he thought they were uncountable fluids, rather than countable particles. If the two factors were identical, he regarded them as one (cf. mixing water with water yields . . . water). Only if the factors were different they existed in pairs (cf. after mixing water and oil you still end up with two different fluids). (Meijer, 1983, 139–142, 147)

4. *Specific claims of supervenience*

My use of the concept of supervenience in the last section was rather unusual. Claims like 'the mental supervenes on the physical' or 'the moral supervenes on the natural' or 'properties of wholes supervene on the properties of their parts' have been frequently debated during the last three centuries. They are very general and abstract, and they seem inappropriate for empirical testing. Mendel's claims that 'trait such-and-so supervenes on factors such-and-so', by contrast, were highly specific and could be experimentally tested (at least indirectly — see section 9). So it might be objected that I wrongly applied the concept of supervenience. For two reasons, however, such an objection would be undeserved.

On the one hand, supervenience claims do not have to be general. Traditional definitions, such as definition 1, refer to sets of properties A and B without requiring that they cover a complete ontological level (such as 'the mental' or 'the physical'). This demand may be added (in which case supervenience claims are indeed quite unsuitable for empirical testing), but this is not necessary.

On the other hand, specific supervenience claims may also be valuable. According to Jaegwon Kim,

> the only direct way of explaining why a general supervenience relation holds ... is to appeal to the presence of specific supervenience relations — that is, appropriate correlations between specific supervenient properties and their supervenient bases. ...Moreover, such correlations seem to be the best, and most natural, *evidential ground* for supervenience claims — often the only kind of solid evidence we could have for *empirical* supervenience claims. (Kim, 1993b, 159, his italics)[7]

So far, I have tried to show that supervenience claims implicitly played a role in scientific practice, more specifically in the work of Gregor Mendel. I have also shown that his supervenience claims differed from those traditionally encountered in the philosophical literature. They were specific, rather than general. In the rest of this paper, I want to address the following two questions:

(1) Is it possible to develop a reliable method for inferring specific supervenience claims from a set of data?
(2) Is it possible to develop a reliable method for testing specific supervenience claims by confronting them with the data?

[7] See also Kim (1988, 122–123).

To that end, I will present two logics of supervenience that deal with specific supervenience claims. These will be written as $\mathsf{PS}^{\{\delta\}}_{\{\pi_1,\ldots,\pi_m\}}$, in which $\pi_1,\ldots,\pi_m$ and δ are primitive predicates of adicity 1. They must be read as '$\{\delta\}$ supervenes on $\{\pi_1,\ldots,\pi_m\}$'. Contrary to what is customary in the literature about supervenience, I do not close these sets under Boolean operations (in section 5, the reader will notice that this poses no technical problems).

5. *Maximal Properties, Carnapian Divisions and the Logical Form of Supervenience Claims*

Before I present the logic of supervenience I will shortly dwell on the logical form of supervenience claims. In view of definition 1, I will start with maximal properties or predicates, making use of the Carnapian concept of 'division' (Carnap, 1951).

Definition 2: *A is a* basic formula *(a* BF-*formula) iff the following condition is fulfilled:*
    (a) *A consists of a primitive predicate of adicity 1 and one variable,[8] or*
    (b) *A consists of a negation sign, a primitive predicate of adicity 1 and one variable.*

So Px, Ry, and $\sim Qz$ are BF-formulas if P, Q and R are primitive.

Definition 3: *B is a* conjunction of basic formulas *(a* CBF-*formula) iff the following four conditions are fulfilled:*
    (a) *B is the conjunction of one or more* BF-*formulas,[9]*
    (b) *all of its conjuncts are different,*
    (c) *none of its conjuncts is the negation of another conjunct, and*
    (d) *all of its conjuncts contain the same variable.*

So $(Rx \wedge \sim Qx)$ is a CBF-formula, whereas $(Px \wedge Px)$, $(Px \wedge \sim Px)$ and $(Px \wedge \sim Qz)$ are not.

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[8] A predicate is primitive in a certain language iff it cannot be defined with the help of other predicates of that language.

[9] If there is only one BF-formula, the resulting CBF-formula is not really a conjunction. Nevertheless, I allow it as a limit case.

CBF-formulas can be abbreviated by defining CBF-*predicates*. For example, one can introduce a predicate M and stipulate that '$M\alpha$' means '$(P\alpha \wedge (R\alpha \wedge \sim Q\alpha))$'. CBF-*divisions* are special sets of such CBF-predicates.

Definition 4: *Suppose that M_1, M_2, ..., M_k are k CBF-predicates ($k \geq 2$). These predicates form a CBF-division iff the following conditions hold (CL stands for standard classical logic):*

(a) $\vdash_{\mathsf{CL}} (\forall\alpha)(M_1\alpha \vee M_2\alpha \vee \ldots \vee M_k\alpha)$.
(b) *For all M_i and M_j ($i \neq j$):* $\vdash_{\mathsf{CL}} (\forall\alpha)\sim(M_i\alpha \wedge M_j\alpha)$.

The members of a CBF-division are jointly exhaustive and mutually exclusive. Moreover, all of them are consistent (see the third condition of definition 3). This means that they denote maximal properties.

Now we have the full formal apparatus to fix the logical form of specific supervenience claims. Let PS_B^A stand for 'A supervenes on B' (in which A is a set containing one primitive predicate of adicity 1 and B is a set of (one or more) such predicates). Let S be the set of all such supervenience claims (relative to some language).[10] Where $\Pi^B = \{M_1, \ldots, M_n\}$ and $\Pi^A = \{M_1', M_2'\}$ are some B- and A-CBF-division respectively, this comes down to: S contains all formulas of the form:

$$\bigwedge \{(\forall\alpha)(M_i\alpha \supset M_1'\alpha) \vee (\forall\alpha)(M_i\alpha \supset M_2'\alpha) | M_i \in \Pi^B$$
$$\text{and } M_1', M_2' \in \Pi^A\}$$

This formula clearly shows that supervenience claims in fact are clusters of inductive generalizations. They are inductive generalizations, since it is claimed, for each B-maximal property M_i, that all M_i-objects have some other property in common. They are clusters of such generalizations since our faith in each of them is closely tied to our faith in the others. Suppose we would find out that two objects have the same B-maximal property M_i, yet are A-discernible. This would lead to the conclusion that A does not supervene on B. But then our trust in each of the other inductive generalizations (viz. those regarding the other B-maximal properties) would erode too.

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[10] Note that I just restricted the members of S to supervenience claims whose supervening set is a singleton. In spite of this restriction, I can still express more complex claims of supervenience, because the following holds in general: $\mathsf{PS}_{\{\pi_1,\ldots,\pi_m\}}^{\{\delta_1,\ldots,\delta_n\}}$ iff $\mathsf{PS}_{\{\pi_1,\ldots,\pi_m\}}^{\{\delta_1\}}$ $\wedge$ $\mathsf{PS}_{\{\pi_1,\ldots,\pi_m\}}^{\{\delta_2\}}$ $\wedge \ldots \wedge$ $\mathsf{PS}_{\{\pi_1,\ldots,\pi_m\}}^{\{\delta_n\}}$ (I leave it to the reader to check this fact).

6. *The Adaptive Logics Programme*

The logics of supervenience that I will present below are adaptive. Adaptive logics have some very important advantages. Firstly, they have a dynamic proof theory that more closely resembles actual reasoning processes than does that of e.g. classical logic. They display an internal dynamics: in an adaptive proof, conclusions may be drawn on certain conditions and they may again be dropped if these conditions are violated. Secondly, the semantics and the meta-theory of the adaptive logics guarantee that given an adaptive logic and a premise set, the set of final consequences is fixed (Batens, 2004). For these reasons, adaptive logics provide a suitable conceptual framework for addressing the questions from page 6.

In this section, I will give a rough sketch of the *adaptive logics programme* and of the standard format for adaptive logics. For a more extensive introduction, see Batens (2001, 2004). Since supervenience claims can be conceived of as clusters of inductive generalizations, I will frequently refer to IL^r and IL^m, the adaptive logics of induction that were presented in Batens and Haesaert (2001).

An adaptive logic interprets a premise set 'as normally as possible'. What counts as normal or abnormal depends on the specific adaptive logic one is dealing with. The logics of induction interpret a premise set as uniformly as possible. How they do this will become clear in the rest of this section.

All (flat) adaptive logics AL can be characterized by a triple: a lower limit logic, a set of abnormalities and a strategy.[11] The *lower limit logic* (LLL) is a monotonic logic. It is the stable part of AL. From a proof theoretic point of view, the lower limit logic delineates the rules of inference that hold unconditionally. From a semantic point of view, the adaptive models of the premise set Γ are a subset of its lower limit models. Therefore, $Cn_{LLL}(\Gamma) \subseteq Cn_{AL}(\Gamma)$. IL^r and IL^m both have CL as their lower limit logic.

The *set of abnormalities* (Ω) consists of the formulas, characterized by some logical form, that are presupposed to be false, unless and until proven otherwise. In the case of IL^r and IL^m, the abnormalities are formulas of the form $\exists A \wedge \exists \sim A$ in which A is purely functional (i.e. no individual constants, propositional letters or quantifiers occur in it) and in which $\exists A$ abbreviates the existential closure of A.

The lower limit logic and the set of abnormalities Ω together specify an upper limit logic (ULL). The ULL is obtained by adding to the characterization of the lower limit logic an axiom, a semantic clause, or an inference rule that rules out abnormalities. So the semantics of the upper limit logic

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[11] Flat adaptive logics, unlike prioritized ones, treat all premises on a par. I will not discuss prioritized adaptive logics here; see Batens (2004).

consists of the LLL-models that verify no abnormality. The upper limit logic of IL^r and IL^m is UCL: the result of extending CL with the axiom $\exists A \supset \forall A$. In the UCL-models, the interpretation of every predicate of adicity n is either the empty set $\emptyset$ or the set of all n-tuples of members of the domain — they are called the *uniform models*.

An adaptive logic presupposes all abnormalities (all $A \in \Omega$) to be false, unless and until proven otherwise. An abnormality is true if it is LLL-derivable from the premises. However, it is possible that a set of premises LLL-entails a disjunction of abnormalities, but none of its disjuncts.

Call a disjunction of abnormalities a *Dab-formula* and write it as $Dab(\Delta)$, in which Δ is a finite subset of Ω. The Dab-formulas that are LLL-derivable from Γ are called the Dab-consequences of Γ. $Dab(\Delta)$ is a *minimal Dab-consequence* of Γ iff $\Gamma \vdash_{LLL} Dab(\Delta)$ and there is no $\Delta' \subset \Delta$ such that $\Gamma \vdash_{LLL} Dab(\Delta')$.

If $Dab(\Delta)$ is a minimal Dab-consequence of Γ, then it is LLL-derivable that *some* member of Δ behaves abnormally, but not *which one* does. How an adaptive logic treats these minimal Dab-consequences depends on its *adaptive strategy*. The two adaptive strategies that are mostly used are *Reliability* and *Minimal Abnormality*. They each lead to a different semantics and proof theory. (In the following paragraphs, AL^r and AL^m denote adaptive logics having Reliability and Minimal Abnormality as their strategy respectively).

The AL-models of a premise set Γ are a subset of its LLL-models, but both strategies have a different selection criterion. Let $Ab(\mathcal{M}) = \{A \mid A \in \Omega$ and $\mathcal{M} \models A\}$ and let $U(\Gamma) = \{A \mid A \in \Delta$ for some minimal *Dab*-consequence Δ of $\Gamma\}$.

Definition 5: *A* LLL-*model* $\mathcal{M}$ *of* Γ *is* reliable *iff* $Ab(\mathcal{M}) \subseteq U(\Gamma)$.

Definition 6: $\Gamma \models_{AL^r} A$ *iff* A *is verified by all reliable models of* Γ.

Definition 7: *A* LLL-*model* $\mathcal{M}$ *of* Γ *is* minimally abnormal *iff there is no* LLL-*model* $\mathcal{M}'$ *of* Γ *such that* $Ab(\mathcal{M}') \subset Ab(\mathcal{M})$.

Definition 8: $\Gamma \models_{AL^m} A$ *iff* A *is verified by all minimally abnormal models of* Γ.

The proof theory of adaptive logics is dynamic. Formulas that are derived at some stage s of a proof may be considered as not derived at some later stage s' (where adding a new line to the proof brings it to the next stage). The motor for this dynamics is given by the Derivability Adjustment Theorem, which describes the relation between the lower limit logic, the upper limit logic and the set of abnormalities.

Theorem 1: $\Gamma \vdash_{\mathsf{ULL}} A$ *iff there is some finite* $\Delta \subset \Omega$ *such that* $\Gamma \vdash_{\mathsf{LLL}} A \vee Dab(\Delta)$ *(Derivability Adjustment Theorem)*

Proof. See Batens (2004). □

If $\Gamma \vdash_{\mathsf{LLL}} A \vee Dab(\Delta)$ then either A is LLL-derivable from Γ or some abnormality (some member of Δ) is true. Since adaptive logics presuppose abnormalities to be false, unless and until proven otherwise, one may derive A from Γ provided that the members of Δ behave normally. How this last sentence is to be interpreted again depends on the adaptive strategy.

The lines of a dynamic proof consist of five elements: (i) a line number, (ii) a derived formula A, (iii) the line numbers of the formulas from which A is derived, (iv) the rule by which A is derived, and (v) the set of formulas that should behave normally (depending on the strategy chosen) in order for A to be so derivable. In an adaptive proof, the following rules may be used (I list them in generic form):

- PREM : if $A \in \Gamma$, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) " – ", (iv) "PREM", and (v) $\emptyset$.
- RU : If $B_1, \ldots, B_m \vdash_{\mathsf{LLL}} A$ and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the line numbers of the B_i, (iv) "RU", and (v) $\Delta_1 \cup \ldots \cup \Delta_m$.
- RC : If $B_1, \ldots, B_m \vdash_{\mathsf{LLL}} A \vee Dab(\Theta)$ and $B_1, \ldots, B_m$ occur in the proof on the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the line number of the B_i, (iv) "RC", and (v) $\Theta \cup \Delta_1 \cup \ldots \cup \Delta_m$.

Lines in a dynamic proof may be marked. These marks may change from one stage of the proof to the next because marked lines may be unmarked again. The second element of a marked line is not considered as derived from the premises. Marking is governed by a *marking definition* that is characteristic for the adaptive strategy. Together with the rules and the definition of (final) derivability this marking definition governs the proof theory.

At any stage of the proof, zero or more Dab-formulas are LLL-derived from the premises. Some of them are minimal at that stage. Let $U_s(\Gamma)$ be the union of all Δ for which $Dab(\Delta)$ is a minimal Dab-consequence at stage s. So $U_s(\Gamma)$ is the set of unreliable formulas at stage s. Let $\Phi_s^\circ(\Gamma)$ be the set of all sets that contain one disjunct out of each minimal Dab-formula at stage s. Let $\Phi_s^\star(\Gamma)$ contain, for any $\varphi \in \Phi_s^\circ(\Gamma)$, the set $Cn_{\mathsf{LLL}}(\varphi) \cap \Omega$. Finally, let $\Phi_s(\Gamma)$ contain those members of $\Phi_s^\star(\Gamma)$ that are not proper supersets of other members of $\Phi_s^\star(\Gamma)$ (see Batens, 2001).

Definition 9: *Marking for* AL^r: *Line* i *is marked at stage* s *iff, where* Δ *is its fifth element,* $\Delta \cap U_s(\Gamma) \neq \emptyset$.

Definition 10: *Marking for* AL^m*: Line i is marked at stage s iff, where A is its second element and Δ is its fifth element, (i) there is no $\varphi \in \Phi_s(\Gamma)$ such that $\varphi \cap \Delta = \emptyset$, or (ii) for some $\varphi \in \Phi_s(\Gamma)$, there is no line k that has A as its second element and has as its fifth element some Θ such that $\varphi \cap \Theta = \emptyset$.*

Given these marking definitions and the generic rules, derivability may seem rather problematic for adaptive logics. Nevertheless both derivability at a stage and final derivability are well defined.

Definition 11: *A formula A is* derived *at stage s of a proof from Γ iff A is the second element of a non-marked line at stage s.*

Definition 12: *A is* finally derived *from Γ on line i of a proof at stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) any extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.*

All formulas that can be finally derived at some stage s are finally derivable.

Definition 13: *$\Gamma \vdash_{\mathsf{AL}} A$ (A is* finally derivable *from Γ) iff A is finally derived on a line of an AL-proof from Γ.*

In section 8.2, I will present an example of a dynamic proof in which the difference between derivability at a stage and final derivability, as well as the difference between the reliability and the minimal abnormality strategy will be illustrated.

7. *The logic of supervenience*

Supervenience claims can be regarded as clusters of inductive generalizations (see page 8). This suggests that the logic of supervenience can be obtained by simply modifying the logics of induction IL^r and IL^m. The result of this modification I will call LPS: the logic of protosupervenience.

IL^r and IL^m are characterized as follows: (i) they have CL as their lower limit logic, (ii) they interpret a premise set 'as uniformly as possible' (i.e., abnormalities are formulas of the form $\exists A \wedge \exists {\sim} A$, in which A is a purely functional formula), (iii) their upper limit logic UCL is CL together with the axiom $\exists A \supset \forall A$, and (v) their adaptive strategy is either reliability (IL^r) or minimal abnormality (IL^m).

7.1. LPS

LPS presupposes that negations of supervenience claims are false, unless and until proven otherwise. Such negations state that there are two or more individuals that are B-indiscernible yet A-discernible. So the set of abnormalities is

$$\Omega = \{\sim\mathsf{PS}_B^A \mid \mathsf{PS}_B^A \in \mathrm{S}\}.$$

In other words, where Π^B and Π^A are defined as on page 8 and α denotes a variable, the members of Ω are equivalent to formulas of the form

$$\bigvee\{(\exists\alpha)(M_i\alpha \wedge M_1'\alpha) \wedge (\exists\alpha)(M_i\alpha \wedge M_2'\alpha) | M_i \in \Pi^B$$

$$\text{and } M_1', M_2' \in \Pi^A\}$$

The upper limit logic should presuppose that no abnormality is true. So it can be obtained by adding to CL the axiom-scheme (abbreviated as PS_B^A)

$$\bigwedge\{(\forall\alpha)(M_i\alpha \supset M_1'\alpha) \vee (\forall\alpha)(M_i\alpha \supset M_2'\alpha) | M_i \in \Pi^B$$

$$\text{and } M_1', M_2' \in \Pi^A\}.$$

This upper limit logic, call it DCL, has an interesting semantics, which strongly resembles the UCL-semantics. The DCL-models are not uniform, but *dual*. They divide their domain $\mathcal{D}$ in two parts ($\mathcal{D}'$ and $\mathcal{D} - \mathcal{D}'$) such that the interpretation of every primitive predicate of adicity 1 is either $\mathcal{D}'$ or $\mathcal{D} - \mathcal{D}'$.[12]

As was made clear in section 6, it has to be shown that DCL is connected in the right way to the lower limit logic CL and the set of abnormalities Ω by the Derivability Adjustment Theorem. I will do this in section 7.4, after I have presented the DCL-semantics in more detail.

7.2. *The semantics of* DCL

Let $\mathcal{L}$ be the usual predicative language-schema. Let $\mathcal{S}$ be the set of sentential letters, $\mathcal{C}$ and $\mathcal{V}$ the set of letters for individual constants and variables respectively, $\mathcal{F}$ the set of open and closed formulas, and $\mathcal{P}^r$ the set of letters

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[12] Note that UCL-models are a subset of the DCL-models, viz. the models for which $\mathcal{D}' = \mathcal{D}$.

for predicates of rank r. To simplify the semantic handling of the quantifiers, I extend $\mathcal{L}$ to the pseudo-language schema $\mathcal{L}^+$ by introducing a set of pseudo-constants, $\mathcal{O}$, that has at least the cardinality of the largest model one wants to consider. Let $\mathcal{W}^+$ be the set of wffs (closed formulas) of $\mathcal{L}^+$.

A DCL-model is a triple $\mathcal{M} = \langle D, D', v \rangle$ in which D is a set, $D' \subseteq D$, and v is an assignment function defined by:

D1.1 $\quad v : \mathcal{C} \cup \mathcal{O} \mapsto D$ (where $D = \{v(\alpha) | \alpha \in \mathcal{C} \cup \mathcal{O}\}$)

D1.2 $\quad v : \mathcal{S} \mapsto \{0, 1\}$

D1.3 $\quad v : \mathcal{P}^1 \mapsto \{D', D - D'\}$

D1.4 $\quad v : \mathcal{P}^r \mapsto \wp(D^r)$ (the power set of the r-th Cartesian product of D, for $r > 1$)

The valuation function $v_{\mathcal{M}} : \mathcal{W}^+ \mapsto \{0, 1\}$ determined by the model $\mathcal{M}$ is defined as for CL:

D2.1 $\quad v_{\mathcal{M}} : \mathcal{F} \mapsto \{0, 1\}$

D2.2 $\quad$ where $A \in \mathcal{S}, v_{\mathcal{M}}(A) = v(A)$

D2.3 $\quad v_{\mathcal{M}}(\pi\alpha_1 \ldots \alpha_r) = 1$ iff $\langle v(\alpha_1), \ldots, v(\alpha_r) \rangle \in v(\pi)$

D2.4 $\quad v_{\mathcal{M}}(\alpha = \beta) = 1$ iff $v(\alpha) = v(\beta)$

D2.5 $\quad v_{\mathcal{M}}(\sim A) = 1$ iff $v_{\mathcal{M}}(A) = 0$

D2.6 $\quad v_{\mathcal{M}}(A \vee B) = 1$ iff $v_{\mathcal{M}}(A) = 1$ or $v_{\mathcal{M}}(B) = 1$

D2.7 $\quad v_{\mathcal{M}}((\exists\alpha)A(\alpha)) = 1$ iff $v_{\mathcal{M}}(A(\beta)) = 1$ for some $\beta \in \mathcal{C} \cup \mathcal{O}$

Clauses for $\wedge, \supset, \equiv$ and $\forall$ are as usual.

7.3. *Soundness and Completeness for* DCL

Theorem 2: *If* $\Gamma \vdash_{\mathsf{DCL}} A$, *then* $\Gamma \models_{\mathsf{DCL}} A$ (Soundness for DCL)

Proof. Consider an axiomatic DCL-proof of A from Γ. The lines of this proof consist of premises, CL-axioms, applications of CL-inference rules (such as modus ponens and universal generalization) and axioms of the form PS_B^A. Consider a DCL-model $\mathcal{M} = \langle D, D', v \rangle$ such that $v_{\mathcal{M}}$ verifies all members of Γ. Since all DCL-models are CL-models, $v_{\mathcal{M}}$ verifies all CL-axioms and if $v_{\mathcal{M}}(B_1) = \ldots = v_{\mathcal{M}}(B_n) = 1$ and C follows from $B_1, \ldots, B_n$ by some CL-inference rule, $v_{\mathcal{M}}(C) = 1$. So $v_{\mathcal{M}}(A) = 1$ if $v_{\mathcal{M}}$ verifies all DCL-axioms of the form PS_B^A.

To prove that $v_{\mathcal{M}}$ verifies all DCL-axioms of the form PS_B^A, suppose that $v_{\mathcal{M}}(\mathsf{PS}_B^A) = 0$ for some A and B (A being a singleton and $B = \{P_1, \ldots, P_n\}$). This means that for some $\beta, \gamma \in \mathcal{C} \cup \mathcal{O}$, $M \in \Pi^B$ and $M', \sim M' \in \Pi^A$, $v_{\mathcal{M}}(M\beta) = v_{\mathcal{M}}(M\gamma) = 1$ (the individuals denoted have

the same B-maximal property) but $v_{\mathcal{M}}(M'\beta) = v_{\mathcal{M}}(\sim M'\gamma) = 1$ (they do not have the same A-maximal property). Since A is a singleton, M' is a primitive predicate of rank 1. In accordance with D1.3, let $v(M') = D'$. So $v(\beta) \in D'$, but $v(\gamma) \notin D'$ (*).[13]

$M\beta$ abbreviates a CBF-formula of the form $\pm P_1\beta \wedge \pm P_2\beta \wedge \ldots \wedge \pm P_n\beta$ in which each occurrence of '$\pm$' may, but need not, stand for '$\sim$' (analogously for γ). Since $v_{\mathcal{M}}(M\beta) = v_{\mathcal{M}}(M\gamma) = 1$, the following holds for each P_i: either $v(\beta), v(\gamma) \in v(P_i)$ or $v(\beta), v(\gamma) \notin v(P_i)$ (if P_i is preceded by '$\sim$'). By D1.3, $v(P_i) = D'$ or $v(P_i) = D - D'$. So either $\beta, \gamma \in D'$ or $\beta, \gamma \notin D'$, which contradicts (*). $\qquad\square$

Theorem 3: *If* $\Gamma \models_{\mathsf{DCL}} A$, *then* $\Gamma \vdash_{\mathsf{DCL}} A$ (Completeness for DCL)[14]

Proof. Suppose that $\Gamma \nvdash_{\mathsf{DCL}} A$, and that A and all members of Γ are wffs of $\mathcal{L}$. Consider a sequence $B_1, B_2, \ldots$ that contains all wffs of $\mathcal{L}^+$ and in which each wff of the form $(\exists\alpha)A$ is followed immediately by an instance $A(o_i/\alpha)$ for some $o_i \in \mathcal{O}$ that does not occur in any previous member of the list. Then define

$$
\begin{aligned}
\Delta_0 &= Cn_{\mathsf{DCL}}(\Gamma) \\
\Delta_{i+1} &= Cn_{\mathsf{DCL}}(\Delta_i \cup \{B_{i+1}\}) \text{ if } A \notin Cn_{\mathsf{DCL}}(\Delta_i \cup \{B_{i+1}\}), \text{ and} \\
\Delta_{i+1} &= \Delta_i \text{ otherwise} \\
\Delta &= \Delta_0 \cup \Delta_1 \cup \ldots
\end{aligned}
$$

Each of the following is provable: (i) $\Gamma \subseteq \Delta$ (by the definition of Δ), (ii) $A \notin \Delta$ (idem), (iii) Δ is deductively closed (idem), (iv) Δ is maximally non-trivial and (v) Δ is ω-complete.

For the proof of (iv), remark first that $A \supset C \in \Delta$ for all C. Indeed, if $A \supset C \notin \Delta$, then there is a Δ_i such that $\Delta_i \cup \{A \supset C\} \vdash_{\mathsf{DCL}} A$. But then $\Delta_i \vdash_{\mathsf{DCL}} (A \supset C) \supset A$ (by the deduction theorem[15]) and hence $\Delta_i \vdash_{\mathsf{DCL}} A$ (by the CL-axiom $((A \supset B) \supset A) \supset A$), which is impossible. If $E \notin \Delta$, then there is a Δ_i such that $\Delta_i \cup E \vdash_{\mathsf{DCL}} A$. Since $A \supset C \in \Delta$ for all C, $\Delta \cup E$ is trivial.

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[13] I could just as well have stipulated that $v(M') = D - D'$, without thereby affecting the results of this proof.

[14] This completeness proof strongly resembles the one given in Batens (1999) for the paraconsistent logic CLuN.

[15] I leave it to the reader to check that the deduction theorem holds for DCL.

For the proof of (v), suppose that $(\exists\alpha)C \in \Delta$, but there is no $\beta \in \mathcal{C} \cup \mathcal{O}$ such that $C(\beta/\alpha) \in \Delta$. Let $(\exists\alpha)C = B_i$. So $B_{i+1} = C(o_j/\alpha)$ for some o_j that does not occur in $\langle B_1, B_2, \ldots, B_i\rangle$. Since o_j is no part of $\mathcal{L}$, o_j does not occur in A nor in some member of Γ. From the supposition it follows that $(\exists\alpha)C \in \Delta_i$ and $C(o_j/\alpha) \notin \Delta_{i+1}$. So by the definition of Δ, $\Delta_i \cup \{C(o_j/\alpha)\} \vdash_{\mathsf{DCL}} A$, and hence $\Delta_i \vdash_{\mathsf{DCL}} C(o_j/\alpha) \supset A$ (by the deduction theorem). Since $\Delta_i \vdash_{\mathsf{DCL}} (\exists\alpha)C$ and o_j does not occur in any member of Δ_i, $\Delta_i \vdash_{\mathsf{DCL}} A$. But then $A \in \Delta_i$, which is impossible.

Δ may contain wffs of the form $\alpha = \beta$ ($\alpha, \beta \in \mathcal{C}\cup\mathcal{O}$). For each $\alpha \in \mathcal{C}\cup\mathcal{O}$ I define the equivalence class $[\alpha] = \{\beta \mid \alpha = \beta \in \Delta\}$. For the $\alpha \in \mathcal{V}$, let $[\alpha] = [\beta]$ for some arbitrary $\beta \in \mathcal{C}$. Now I define a CL-model $\mathcal{M}$ as follows:

(1) $D = \{[\alpha] \mid \alpha \in \mathcal{C} \cup \mathcal{O}\}$
(2) where $\alpha \in \mathcal{C} \cup \mathcal{O} \cup \mathcal{V} : v(\alpha) = [\alpha]$
(3) where $A \in \mathcal{S} : v(A) = 1$ iff $A \in \Delta$, and
(4) where $\pi \in \mathcal{P}^r : v(\pi) = \{\langle[\alpha_1], \ldots, [\alpha_r]\rangle \mid \pi\alpha_1, \ldots, \alpha_r \in \Delta\}$

I have to show that (*) $v_{\mathcal{M}}(A) = 1$ iff $A \in \Delta$. By the clauses (3) and (4) this trivially holds for all $A \in \mathcal{S}$ and for all $\pi\alpha_1, \ldots, \alpha_r$. It also holds for wffs of the form $\alpha = \beta$ since $v_{\mathcal{M}}(\alpha = \beta) = 1$ iff $v(\alpha) = v(\beta)$ iff $[\alpha] = [\beta]$ iff $\alpha = \beta \in \Delta$.

Suppose that (*) holds for some A. Then it also holds for $\sim A$, $A \wedge B$, $A \vee B$, $A \supset B$, $A \equiv B$, $(\exists\alpha)A$ and $(\forall\alpha)A$. The proof of these cases proceeds as for CL. For example, if $\sim A \in \Delta$, then $A \notin \Delta$ (because Δ is not trivial), so $v_{\mathcal{M}}(A) = 0$ and hence $v_{\mathcal{M}}(\sim A) = 1$. If $v_{\mathcal{M}}(\sim A) = 1$, then $v_{\mathcal{M}}(A) = 0$. So $A \notin \Delta$. Since Δ is maximally non-trivial, $\Delta \cup \{A\}$ is trivial. So $\sim A \in \Delta$.

So there is a CL-model $\mathcal{M}$ that verifies all members of Δ, and hence of Γ, but falsifies A. Now it only has to be shown that it is in fact a DCL-model (i.e. that D1.3 holds for it). Take some arbitrary $\pi, \pi' \in \mathcal{P}^1$. Let $v(\pi) = D'$ and $v(\pi') = D''$.

For all $\alpha \in \mathcal{C} \cup \mathcal{O}$, either $\pi\alpha \in \Delta$ or $\sim\pi\alpha \in \Delta$ (suppose that $\pi\alpha, \sim\pi\alpha \notin \Delta$, then $v_{\mathcal{M}}(\pi\alpha) = v_{\mathcal{M}}(\sim\pi\alpha) = 0$, which is impossible). Analogously, either $\pi'\alpha \in \Delta$ or $\sim\pi'\alpha \in \Delta$.

Case 1: $\pi\alpha \in \Delta$ and $\pi'\alpha \in \Delta$. So by clause (4), $[\alpha] \in D'$ and $[\alpha] \in D''$. Now for all $\beta \in \mathcal{C} \cup \mathcal{O}$, if $[\beta] \in D'$, then $\pi\beta \in \Delta$. But then also $\pi'\beta \in \Delta$ and hence $[\beta] \in D''$, because $\pi\alpha, \pi'\alpha, \pi\beta \in \Delta$ and $\pi\alpha, \pi'\alpha, \pi\beta \vdash_{\mathsf{DCL}} \pi'\beta$. So $D' \subseteq D''$. Analogously, for all $\beta \in \mathcal{C} \cup \mathcal{O}$, if $[\beta] \in D''$, then $\pi'\beta \in \Delta$. Hence $\pi\beta \in \Delta$ and $[\beta] \in D'$. So $D'' \subseteq D'$. It follows that $D'' = D'$.

Case 2: $\pi\alpha \in \Delta$ and $\sim\pi'\alpha \in \Delta$ (so $\pi'\alpha \notin \Delta$, because Δ is not trivial). By clause (4), $[\alpha] \in D'$ and $[\alpha] \in D - D''$. Now for all $\beta \in \mathcal{C} \cup \mathcal{O}$, if

$[\beta] \in D'$, then $\pi\beta \in \Delta$. But then also $\sim\pi'\beta \in \Delta$ and $[\beta] \in D - D''$. So $D' \subseteq D - D''$. Analogously, for all $\beta \in \mathcal{C} \cup \mathcal{O}$, if $[\beta] \in D - D''$, then $\pi'\beta \notin \Delta$ and $\sim\pi'\beta \in \Delta$. But then $\pi\beta \in \Delta$ and $[\beta] \in D'$. So $D - D'' \subseteq D'$. It follows that $D'' = D - D'$.

Case 3: $\sim\pi\alpha \in \Delta$ and $\pi'\alpha \in \Delta$. I leave it to the reader to prove that $D'' = D - D'$.

Case 4: $\sim\pi\alpha \in \Delta$ and $\sim\pi'\alpha \in \Delta$. I leave it to the reader to prove that $D'' = D'$.

So for every $\pi \in \mathcal{P}^1 : v(\pi) \in \{D', D - D'\}$, for some $D' \subseteq D$. This means that clause D1.3 holds and that $\mathcal{M}$ is a DCL-model $\langle D, D', v \rangle$ that verifies all members of Γ but falsifies A. So $\Gamma \nvDash_{\mathsf{DCL}} A$. $\square$

7.4. LPS *revisited*

Relying on the proofs of soundness and completeness for DCL, I can now prove that the Derivability Adjustment Theorem holds.

Theorem 4: $\Gamma \vdash_{\mathsf{DCL}} A$ *iff there is some finite* $\Delta \subset \Omega$ *such that* $\Gamma \vdash_{\mathsf{CL}} A \vee Dab(\Delta)$ (Derivability Adjustment Theorem)

Proof. For the left-right direction, suppose that $\Gamma \vdash_{\mathsf{DCL}} A$. It follows that all DCL-models of Γ verify A. All other CL-models of Γ verify some member of Ω. Hence there is a $\Delta' \subseteq \Omega$ such that all CL-models of Γ verify a member of $\Delta' \cup \{A\}$. By the right compactness of CL[16], there is a finite $\Delta \subseteq \Delta'$ such that all CL-models of Γ verify a member of $\Delta \cup \{A\}$. So all CL-models of Γ verify $A \vee Dab(\Delta)$.

For the right-left direction, suppose that $\Gamma \vdash_{\mathsf{CL}} A \vee Dab(\Delta)$, and hence that $\Gamma \models_{\mathsf{CL}} A \vee Dab(\Delta)$. It follows that all CL-models of Γ, and hence all DCL-models of Γ, verify $A \vee Dab(\Delta)$. But all DCL-models of Γ falsify $Dab(\Delta)$. Consequently, $\Gamma \models_{\mathsf{DCL}} A$ and hence $\Gamma \vdash_{\mathsf{DCL}} A$. $\square$

The proof theoretic rules of LPS are as specified in section 6 (PREM, RU and RC), as are its marking definitions and its semantics (of course I'm implicitly distinguishing between LPS^r and LPS^m).

7.5. *The Problem with* LPS

Although the above characterization of LPS seems intuitively clear and fits the standard format for adaptive logics, it has a tremendous disadvantage —

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[16] Right compactness: every model of Γ verifies a member of Δ iff every model of Γ verifies a member of some finite $\Delta' \subseteq \Delta$.

a disadvantage that neither IL^r nor IL^m has. No one wants to derive from the empty premise set, $\emptyset$, that "All P are Q", for any P and Q. Fortunately, $\emptyset \nvdash_{\mathsf{IL}^r} (\forall x)(Px \supset Qx)$ and $\emptyset \nvdash_{\mathsf{IL}^m} (\forall x)(Px \supset Qx)$ (see Batens and Haesaert, 2001, 267). Analogously, one doesn't want to derive from $\emptyset$ that PS^A_B for any sets A and B.[17] However, to give one example, $\emptyset \vdash_{\mathsf{LPS}} ((\forall x)(Px \supset Qx) \vee (\forall x)(Px \supset \sim Qx)) \wedge ((\forall x)(\sim Px \supset Qx) \vee (\forall x)(\sim Px \supset \sim Qx))$. The reason is that $\sim\mathsf{PS}^{\{Q\}}_{\{P\}}$, which is equivalent to $((\exists x)(Px \wedge Qx) \wedge (\exists x)(Px \wedge \sim Qx)) \vee ((\exists x)(\sim Px \wedge Qx) \wedge (\exists x)(\sim Px \wedge \sim Qx))$, is not a member of any minimal Dab-consequence from $\emptyset$. In fact, no abnormality is CL-derivable from $\emptyset$, since none of them is verified by all CL-models of $\emptyset$ (the model with only one object in its domain falsifies all abnormalities).

The problem with LPS is even worse than that. Not only is any claim of supervenience derivable from $\emptyset$. It is also the case that if the premises say nothing about some property T (say nothing about its instances), then for every set of properties B it is derivable that $\{T\}$ supervenes on B. This implies that applying LPS to a premise set will always lead to unwarranted, and hence unwanted supervenience claims. Therefore, the question whether it is possible to develop a reliable method for inferring (specific) supervenience claims from a set of data has to be answered negatively.

8. *Solving the Problem:* RLPS^r *and* RLPS^m

The problem with LPS suggests that instead of blindly applying the conditional rule RC to a set of data, we should rather use our logic to test a well-chosen set of supervenience claims. To that end, we have to slightly modify or *restrict* LPS. Restricted LPS, or RLPS, takes as premises an ordered pair of sets $\Sigma = \langle \Gamma, \Gamma^* \rangle$. Γ can contain any kind of premises except supervenience claims; the latter are relegated to Γ^*. So $\Gamma^* \subset \mathsf{S}$ contains the supervenience claims we want to reason about. These modifications have an influence both on the proof theory and on the semantics of RLPS.

8.1. *The proof theory of* RLPS^r *and* RLPS^m

The derivability adjustment theorem for RLPS is the same as for LPS. However, it is no longer the case that we accept as adaptive consequences of Γ all A such that for some $\Delta \subset \Omega$, (i) $\Gamma \vdash_{\mathsf{CL}} A \vee Dab(\Delta)$ and (ii) all members of

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[17] Of course, if $A \subseteq B$, it is trivially true that PS^A_B. So we should not be reluctant to derive this supervenience claim from the empty premise set. Notwithstanding this trivial case, however, we should only accept specific supervenience claims if they are empirically warranted (cf. Jaegwon Kim's quote on page 6).

Δ behave normally. Instead, Δ may only contain negations of members of Γ^* or be the empty set. Given this modification, these are the rules of RLPS (again, they are presented in generic form):

- PREM : if $A \in \Gamma$, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) " – ", (iv) "PREM", and (v) $\emptyset$.
- RU : If $B_1, \ldots, B_m \vdash_{\mathsf{CL}} A$ and $B_1, \ldots, B_m$ occur in the proof with the conditions $\Delta_1, \ldots, \Delta_m$ respectively, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) the line numbers of the B_i, (iv) "RU", and (v) $\Delta_1 \cup \ldots \cup \Delta_m$.
- SUP : If $\vdash_{\mathsf{CL}} A \vee Dab(\Theta)$ and $A \in \Gamma^*$, then one may add a line consisting of (i) the appropriate line number, (ii) A, (iii) " – ", (iv) "SUP", and (v) Θ.

The marking definitions of RLPS^r and RLPS^m are as usual.

8.2. *Example*

Consider the premise set $\Sigma = \langle \Gamma, \Gamma^* \rangle$, with $\Gamma = \{Pa \wedge {\sim}Qa \wedge {\sim}Sa, Qb \wedge {\sim}Rb, Pc \wedge {\sim}Rc, Pd \wedge Rd, {\sim}Qe \wedge Re\}$ and $\Gamma^* = \{\mathsf{PS}^{\{Q\}}_{\{P\}}, \mathsf{PS}^{\{Q\}}_{\{R\}}, \mathsf{PS}^{\{S\}}_{\{P\}}\}$. In this section I will illustrate the dynamics of adaptive proofs, make clear that some formulas are finally derivable while others are not, and show that RLPS^r and RLPS^m may lead to different conclusions from the same Σ.

1	$Pa \wedge {\sim}Qa \wedge {\sim}Sa$	–	PREM	$\emptyset$
2	$Qb \wedge {\sim}Rb$	–	PREM	$\emptyset$
3	$Pc \wedge {\sim}Rc$	–	PREM	$\emptyset$
4	$Pd \wedge Rd$	–	PREM	$\emptyset$
5^{16}	$\mathsf{PS}^{\{Q\}}_{\{P\}}$	–	SUP	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$
6^{16}	$(\forall x)(Px \supset Qx) \vee (\forall x)(Px \supset {\sim}Qx)$	5	RU	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$
7^{16}	$(\forall x)(Px \supset {\sim}Qx)$	1,6	RU	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$
8^{16}	${\sim}Qc$	3,7	RU	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$
$9^{(16)}$	${\sim}Qd$	4,7	RU	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$
10^{16}	${\sim}Pb$	2,7	RU	$\{{\sim}\mathsf{PS}^{\{Q\}}_{\{P\}}\}$

At this point, I have conditionally derived that ${\sim}Qc$, ${\sim}Qd$ and ${\sim}Pb$. However, it might later turn out that these predictions are untenable (indeed, it will turn out at stage 16 that according to the reliability strategy all of them are, as is indicated by the superscript that is added to their line numbers; according to minimal abnormality, ${\sim}Qd$ is not untenable).

Now I will introduce another member of Γ^* and see whether I can derive some more predictions.

11^{16}	$PS^{\{Q\}}_{\{R\}}$	–	SUP	$\{\sim PS^{\{Q\}}_{\{R\}}\}$
12^{16}	Qc	2, 3, 11	RU	$\{\sim PS^{\{Q\}}_{\{R\}}\}$
13	$\sim Qe \wedge Re$	–	PREM	$\emptyset$
$14^{(16)}$	$\sim Qd$	4,11,13	RU	$\{\sim PS^{\{Q\}}_{\{R\}}\}$

Some of the predictions that that are derived at this stage are mutually inconsistent (Qc, $\sim Qc$). Fortunately, this poses no problem. Both consequences will have to be dropped in view of the *Dab*-consequence derived at line 16.

15	$Qc \vee \sim Qc$	–	RU	$\emptyset$
16	$\sim PS^{\{Q\}}_{\{P\}} \vee \sim PS^{\{Q\}}_{\{R\}}$	1,2,3,15	RU	$\emptyset$

According to RLPS^r, all lines that have either $\sim PS^{\{Q\}}_{\{P\}}$ or $\sim PS^{\{Q\}}_{\{R\}}$ or both as a member of their fifth element, have to be marked (including lines 9 and 14). So with the reliability strategy, we get rid of the incompatible predictions Qc and $\sim Qc$.

The markings definition of RLPS^m is slightly different. It demands that the same lines are marked, except for 9 and 14. The reason is that it interprets line 16 as minimally abnormal, i.e. it presupposes that either $\sim PS^{\{Q\}}_{\{P\}}$ is true, or $\sim PS^{\{Q\}}_{\{R\}}$, but not both. If $\sim PS^{\{Q\}}_{\{R\}}$ is false, $\sim Qd$ is derivable on line 14; if $\sim PS^{\{Q\}}_{\{P\}}$ is false, $\sim Qd$ is derivable on line 9. So $\sim Qd$ is RLPS^m-derivable at stage 16. Moreover, since the formula on line 16 is a minimal *Dab*-consequence of Γ, $\sim Qd$ is *finally* derivable according to RLPS^m. Since no minimal Dab-consequence has $\sim PS^{\{S\}}_{\{P\}}$ as a disjunct, $\sim Sc$ and $\sim Sd$ are also finally derivable (both with RLPS^r and RLPS^m) and $PS^{\{S\}}_{\{P\}}$ is empirically corroborated:

17	$PS^{\{S\}}_{\{P\}}$	–	SUP	$\{\sim PS^{\{S\}}_{\{P\}}\}$
18	$\sim Sc$	1,3,17	RU	$\{\sim PS^{\{S\}}_{\{P\}}\}$
19	$\sim Sd$	1,4,17	RU	$\{\sim PS^{\{S\}}_{\{P\}}\}$

RLPS^r and RLPS^m lead to different consequence sets. In general, Cn_{RLPS^r} $(\Sigma) \subseteq Cn_{\text{RLPS}^m}(\Sigma)$. Whether the one should be preferred to the other depends on the context in which you apply them. Both have a nice dynamic proof theory as well as a nice semantics (see below). The advantage of RLPS^r is that its proof theory is easier.

8.3. *The semantics of* RLPS^r *and* RLPS^m

The semantics of RLPS^r and RLPS^m are obtained by relaxing the constraints imposed on the LLL-models. The selection of the models still is based on the abnormalities they verify, but only those abnormalities that are negations of members of Γ^* are taken into account.

The abnormal part of a model $\mathcal{M}$ is defined as $Ab(\mathcal{M}) = \{A \mid A \in \Omega$ and $\mathcal{M} \models A\}$ and $U(\Gamma) = \{A \mid A \in \Delta$ for some minimal *Dab*-consequence Δ of $\Gamma\}$, as before. So $U(\Gamma) \subseteq \Omega$. However, only for some $A \in U(\Gamma)$ there is a corresponding $B \in \Gamma^*$ which it contradicts. So let $\Omega^* = \{\sim A \mid A \in \Gamma^*\}$. Now it is easy to select the models in view of the Ω^*-members they verify.

Definition 14: $\mathcal{M} \in \mathsf{M}_\Gamma$ *is* reliable *iff* $Ab(\mathcal{M}) \cap \Omega^* \subseteq U(\Gamma) \cap \Omega^*$

Definition 15: $\Gamma \models_{\text{RLPS}^r} A$ *iff A is verified by all reliable models of* Γ.

Definition 16: $\mathcal{M} \in \mathsf{M}_\Gamma$ *is* minimally abnormal *iff there is no* $\mathcal{M}' \in \mathsf{M}_\Gamma$ *such that* $Ab(\mathcal{M}') \cap \Omega^* \subset Ab(\mathcal{M}) \cap \Omega^*$.

Definition 17: $\Gamma \models_{\text{RLPS}^m} A$ *iff A is verified by all minimally abnormal models of* Γ.

9. *The Inferential Role of Supervenience in Classical Genetics*

My presentation of LPS and RLPS has shown that the unconstrained inference of supervenience claims is problematic, but that nonetheless such claims can be reliably tested by confronting them with empirical data. But what does this tell us about the inferential role of supervenience in classical genetics?

In sections 2 and 3 I have argued that Mendel *assumed* that traits supervene on underlying factors. But this poses a problem. Factors were unobservable, and their existence could not be proven directly. So how could Mendel confront his supervenience claims with his data?

Mendel not only assumed that traits supervene on factors, he also put forward assumptions about the behavior of the latter. Factors are passed on from

generation to generation via the gametes. The internal make-up of gametes depends on the make-up of the parent plant (Stern and Sherwood, 1966, 23–31). If the latter is a hybrid (P&P′), half of its gametes will be P, the other half will be P′. If the parent plant is not hybrid (so either if it is P&∼P′ or ∼P&P′), then all of its reproductive cells have the same make-up (viz. P and P′ respectively). This set of assumptions would later be called Mendel's First Law, or the Law of Segregation. So far, I have only focussed on one set of traits and one set of factors. In the dihybrid case, Mendel assumed that the behavior of one set of factors is independent of the behavior of any other set. Later this assumption would be known as Mendel's Second Law, or the Law of Independent Assortment.

These three sets of assumptions, the supervenience assumption and the laws of segregation and of independent assortment could together be empirically tested. The supervenience assumption allowed Mendel to determine the internal make-up of his experimental plants. The laws of segregation and of independent assortment allowed him to predict the distribution of factors among the progeny. Finally, the distribution of traits among the progeny could be derived (again on the basis of the supervenience assumption). Mendel's experimental results accorded very nicely with these predictions.[18]

All this shows that Mendel's supervenience claims could indirectly be tested, and that they played an inferential role in his work. The supervenience assumption was a fruitful one. No scientist or breeder had ever been able to formulate, let alone to verify, such precise experimental predictions before Mendel. It paved the way to the successful experimental treatment and mechanistic explanation of hereditary phenomena in the first decades of the 20th century. Its merits are most clearly illustrated in the scientific work of Thomas Hunt Morgan (1866–1945). Morgan was one of the most influential geneticists of the first half of the 20th century. From 1911 onwards, he defended and elaborated the chromosome theory, which linked the Mendelian factors or genes together with chromosomes (Morgan et al., 1915; Morgan, 1926). The chromosome theory provided a deeper understanding of Mendel's results (viz. of the mechanism underlying segregation and independent assortment) and of its apparent exceptions (by referring to linkage and crossing over).

Before 1911, however, Morgan was a confirmed adversary of Mendelism and of the chromosome theory. In a little paper, "What are 'Factors' in

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[18] In fact, they accorded too nicely, as was argued by the famous statistician sir Ronald Fisher (Stern and Sherwood, 1966, 139–172).

Mendelian Explanations?" (Morgan, 1909), he strongly opposed the 'modern factor-hypothesis', i.e. the assumed supervenience of traits on factors. Although he acknowledged the value of Mendel's assumption to a certain extent,[19] he formulated two important criticisms. The first concerned the 'jugglery' that all too often involved Mendelian explanations.

> If one factor will not explain the facts, then two are invoked; if two prove insufficient, three will sometimes work out. (Morgan, 1909, 365)

The second one is more relevant for the present discussion. Mendel's conceptual framework, he stated, reformulates the old idea of preformation (Morgan, 1909, 366). According to *preformation theories*, the egg or the sperm contains a preformed embryo (Orel, 1996, 9, 12). In the Mendelian picture, they contain unit characters, or the factors responsible for these characters.[20] According to *epigenetic theories*, by contrast, they contain a structureless substance, out of which the embryo develops due to some *vis essentialis* (Orel, 1996, 9). Morgan argued in favor of this epigenetic view:

> The egg need not contain the *characters* of the adult, nor need the sperm. Each contains a particular material which in *the course of the development produces* in some unknown way the character of the adult. (Morgan, 1909, 367, original italics)

The difference between preformation and epigenetic theories can be explicated by means of the concept of supervenience. In the former, there is some preformed or structured material that *determines* the organism's traits. In the latter, no underlying material is able to determine these traits by itself. Although Morgan originally believed that the epigenetic view offered the best way to scientific advance, he made his most valuable contributions to the field of genetics after he accepted the supervenience of traits on factors.

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[19] He wrote:

> I am not unappreciative of the distinct advantages that this method has in handling the facts. I realize how valuable it has been to us to be able to marshal our results under a few simple assumptions (...). So long as we do not lose sight of the purely arbitrary and formal nature of our formulae, little harm will be done (...). (Morgan, 1909, 365)

[20] According to Morgan, the 'factors' were sometimes referred to as the actual characters themselves (Morgan, 1909, 366).

Let me conclude with a final question. To what extent was Mendel's super-venience assumption a tenable one? Accumulating evidence from molecular biology has shown it isn't tenable at all. In order to influence the phenotype of an organism, the genetic information stored in its DNA must first be transcribed, i.e. copied to mRNA sequences. Transcription is often regulated (induced, repressed, enhanced) depending on the metabolic needs of the cell and on the cell's environment and the resulting RNA sequences are often subject to post-transcriptional modification (which might influence their informational content, cf. alternative splicing). The information in these mRNA sequences is then translated, i.e. it is used to synthesize polypeptide chains. One or more of these chains may together form a protein. The function of this protein is not only determined by the constituting polypeptide chains, but also by its structural organization in three-dimensional space. Finally, most phenotypic traits are caused by a multitude of interacting proteins. (Klug et al., 2006, chapters 13, 14, and 17) All this leads to the conclusion that the phenotype doesn't supervene on the genotype (i.e. the linear ordering of complementary bases in the DNA). Mendel's assumption was fruitful, but clearly false.

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