An Andersonian Deontic Logic with Contextualized Sanctions

M. Beirlaen and C. Straßer

Centre for Logic and Philosophy of Science
Ghent University, Belgium
{Mathieu.Beirlaen, Christian.Strasser@UGent.be}

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Anderson's reduction

Idea: define deontic in terms of alethic operators

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SDL is the deontic fragment of **KDA** (Åqvist '02)

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Contextualizing sanctions: from s to S

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(IV) Work in progress

Part I

The logic **DSL**: definition

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$\Box$: **S5**-modality (equivalence class of accessible worlds)

DSL: grammar/ interpretation (2)

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$\mathsf{S}A$:

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$\mathsf{S}A$: "*A holds* and causes a sanction"

$A \Rightarrow \mathsf{S}A$: "*If A* were to hold, it *would* cause a sanction"

DSL: axiomatization (1)

DSL is obtained by adding to **S5** the following axiom schemata and rules:

$$\mathsf{S}A \supset A \qquad \text{(SR)}$$

$$\mathsf{S}A \supset (A \Rightarrow \mathsf{S}A) \qquad (\mathsf{S}{\Rightarrow})$$

$$(A \Rightarrow \mathsf{S}(A \wedge B)) \supset (A \Rightarrow \mathsf{S}A) \qquad \text{(SW)}$$

$$(\mathsf{S}A \wedge \mathsf{S}B) \supset \mathsf{S}(A \wedge B) \qquad (\mathsf{S}\wedge)$$

$$((A \vee B) \Rightarrow \mathsf{S}(A \vee B)) \equiv ((A \Rightarrow \mathsf{S}A) \wedge (B \Rightarrow \mathsf{S}B)) \qquad (\mathsf{S}\vee)$$

$$\text{If } \vdash_{\mathbf{EL}} A \supset B \text{ then } \vdash \mathsf{S}A \supset \mathsf{S}B \qquad (\mathsf{S}{\supset})$$

DSL: axiomatization (2)

$$SA \supset A \tag{SR}$$

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If A causes a sanction, then A.

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If *A* causes a sanction, then *A*.

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If *A* causes a sanction in some world, then *A* causes a sanction in every accessible world in which *A* holds.

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If A causes a sanction in some world, then A causes a sanction in every accessible world in which A holds.

$$SA \equiv (\mathsf{F}A \wedge A)$$

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$$SA \equiv (FA \wedge A)$$

$$((A \Rightarrow B) \wedge ((A \wedge B) \Rightarrow S(A \wedge B))) \supset (A \Rightarrow SA)$$

If:
A necessitates B,
and $A \wedge B$, if it holds, would cause a sanction,
then A is itself sufficient to cause the sanction.

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Example:
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Example:
Little Peter is told by his mom to either wash the dishes or bring out the garbage, otherwise he is not allowed to watch his favorite TV show this evening.

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$$(\mathsf{S}A \wedge \mathsf{S}B) \supset \mathsf{S}(A \wedge B) \qquad (\mathsf{S}\wedge)$$

If A causes a sanction and B causes a sanction, then $A \wedge B$ causes a sanction.

DSL: axiomatization (4)

$$((A \vee B) \Rightarrow \mathsf{S}(A \vee B)) \equiv ((A \Rightarrow \mathsf{S}A) \wedge (B \Rightarrow \mathsf{S}B)) \qquad (\mathsf{S}\vee)$$

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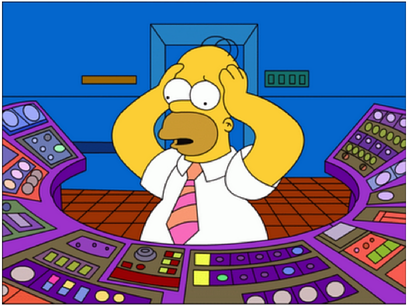

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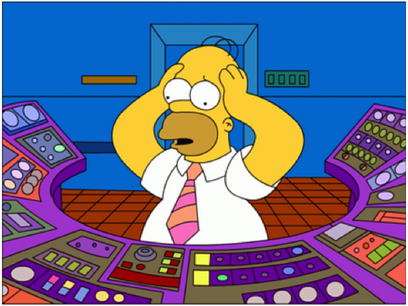

If he were to press button *a* or button *b*, Homer would cause a meltdown.

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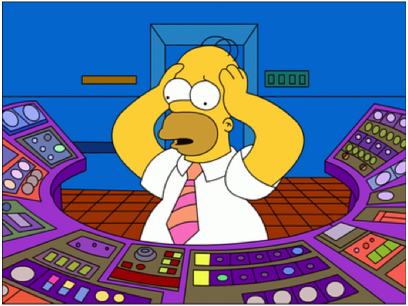

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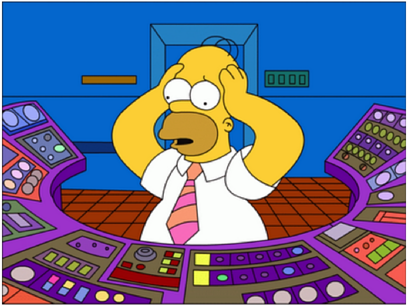

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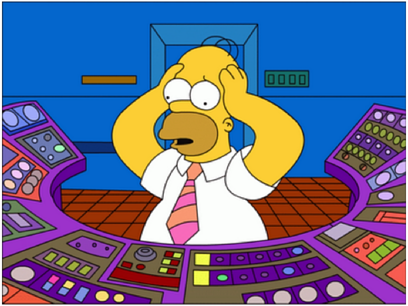

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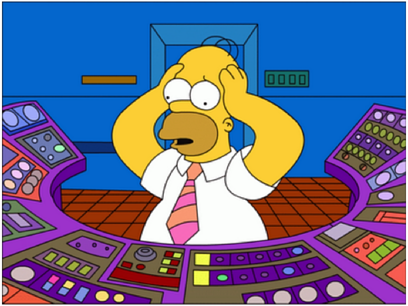

Pressing button *a* causes a meltdown. Pressing button *b* does not.

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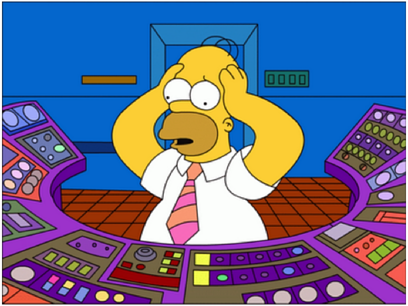

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$$\mathsf{S}a \not\supset \mathsf{S}(a \vee b)$$

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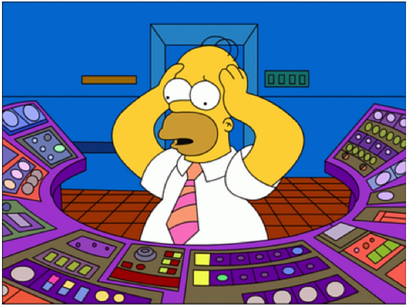

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Then, since $\mathsf{S}(A \vee B) \supset (\mathsf{S}A \vee \mathsf{S}B)$:

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Solution:

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Let $(\tau, \sigma) \in \{(\wedge, \vee), (\vee, \wedge)\}$. **EL** is defined by:

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Suppose:

$$\text{If } \vdash_{\mathbf{CL}} A \supset B \text{ then } \vdash \mathsf{S}A \supset \mathsf{S}B$$

Then, since $\mathsf{S}(A \vee B) \supset (\mathsf{S}A \vee \mathsf{S}B)$:

$$\mathsf{S}A \supset (\mathsf{S}(A \wedge B) \vee \mathsf{S}(A \wedge \neg B))$$

Solution:

$$\text{If } \vdash_{\mathbf{EL}} A \supset B \text{ then } \vdash \mathsf{S}A \supset \mathsf{S}B \qquad (\mathsf{S}\supset)$$

Let $(\tau, \sigma) \in \{(\wedge, \vee), (\vee, \wedge)\}$. **EL** is defined by:

$$\neg(A \,\tau\, B) \dashv\vdash (\neg A \,\sigma\, \neg B) \qquad (1)$$
$$\neg\neg A \dashv\vdash A \qquad (2)$$
$$((A \,\tau\, B) \,\tau\, C) \dashv\vdash (A \,\tau (B \,\tau\, C)) \qquad (3)$$
$$(A \,\tau\, B) \dashv\vdash (B \,\tau\, A) \qquad (4)$$
$$(A \,\tau (B \,\sigma\, C)) \dashv\vdash ((A \,\tau\, B) \,\sigma (A \,\tau\, C)) \qquad (5)$$
$$(A \,\tau\, A) \dashv\vdash A \qquad (6)$$
$$((A \,\tau\, \neg A) \,\sigma\, B) \vdash B \qquad (7)$$
$$((A \,\tau\, B) \,\sigma\, A) \vdash A \qquad (8)$$
$$((A \,\tau\, \neg A) \,\tau\, B) \vdash (A \,\tau\, \neg A) \qquad (9)$$
$$\text{If } A \vdash B \text{ then } C \vdash C^{A/B} \qquad (10)$$

Where $C^{A/B}$ is the product of substituting any amount of subformulas A in C by B.

DSL: axiomatization (6)

Some examples:

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$$A \wedge \neg(B \vee C) \;{}_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; A \wedge (\neg B \wedge \neg C) \;{}_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; (A \wedge \neg C) \wedge \neg B$$

DSL: axiomatization (6)

Some examples:

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$$A \vee \neg(B \wedge C) \;_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; A \vee (\neg B \vee \neg C)$$

DSL: axiomatization (6)

Some examples:

$$A \wedge \neg(B \vee C) \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} A \wedge (\neg B \wedge \neg C) \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} (A \wedge \neg C) \wedge \neg B$$

$$A \vee \neg(B \wedge C) \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} A \vee (\neg B \vee \neg C)$$

$$A \wedge A \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} A \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} A \vee A \;_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}} A \vee (A \wedge A)$$

DSL: axiomatization (6)

Some examples:

$$A \wedge \neg(B \vee C) \;{}_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}}\; A \wedge (\neg B \wedge \neg C) \;{}_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}}\; (A \wedge \neg C) \wedge \neg B$$

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$$\neg(A \wedge B) \;_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; \neg A \vee \neg B$$

$$(A \vee \neg A) \wedge B \vdash_{\mathbf{EL}} B$$

DSL: axiomatization (6)

Some examples:

$$A \wedge \neg(B \vee C) \;{}_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; A \wedge (\neg B \wedge \neg C) \;{}_{\mathbf{EL}}\dashv\vdash_{\mathbf{EL}}\; (A \wedge \neg C) \wedge \neg B$$

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$$(A \vee \neg A) \wedge B \vdash_{\mathbf{EL}} B$$

$$B \nvdash_{\mathbf{EL}} (A \vee \neg A) \wedge B$$

DSL: axiomatization (6)

Some examples:

$$A \wedge \neg(B \vee C) \;{}_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}}\; A \wedge (\neg B \wedge \neg C) \;{}_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}}\; (A \wedge \neg C) \wedge \neg B$$

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$$\neg(A \wedge B) \;{}_{\mathbf{EL}}\!\dashv\vdash_{\mathbf{EL}}\; \neg A \vee \neg B$$

$$(A \vee \neg A) \wedge B \vdash_{\mathbf{EL}} B$$

$$B \nvdash_{\mathbf{EL}} (A \vee \neg A) \wedge B$$

EL is the fragment of **CL** that does not allow for the introduction of new propositional variables

Part II

The logic **DSL**: further properties

Deontic properties of **DSL**

The following properties *fail* in **DSL**:

$$OA, O\neg A \vdash B$$
$$OA, O\neg A \vdash OB$$
$$\text{If } \vdash_{\mathbf{CL}} A \supset B, \text{ then } \vdash_{\mathbf{DSL}} OA \supset OB$$
$$\text{If } \vdash_{\mathbf{CL}} A \equiv B, \text{ then } \vdash_{\mathbf{DSL}} OA \equiv OB$$
$$OA \vdash O(A \vee B)$$

Deontic properties of **DSL**

The following properties *fail* in **DSL**:

$$OA, O\neg A \vdash B$$
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$$\text{If } \vdash_{\mathbf{CL}} A \supset B, \text{ then } \vdash_{\mathbf{DSL}} OA \supset OB$$
$$\text{If } \vdash_{\mathbf{CL}} A \equiv B, \text{ then } \vdash_{\mathbf{DSL}} OA \equiv OB$$
$$OA \vdash O(A \vee B)$$

The following properties *hold* in **DSL**:

$$O(A \wedge B) \vdash OA \wedge OB$$
$$OA, OB \vdash O(A \vee B)$$
$$OA, OB \vdash O(A \wedge B)$$
$$O(A \vee B), O\neg A \vdash OB$$
$$O(A \vee B), \neg\Diamond A \vdash OB$$
$$\text{If } \vdash_{\mathbf{CL}} A, \text{ then } \vdash OA$$
$$\text{If } \vdash_{\mathbf{EL}} A \equiv B, \text{ then } \vdash OA \equiv OB$$

Alternative axiomatization of **DSL**

$$\mathcal{W}_{\Box}^{\mathsf{O}} := \mathcal{W} \mid \mathsf{O}\mathcal{W} \mid \Box\mathcal{W}_{\Box}^{\mathsf{O}} \mid \neg\mathcal{W}_{\Box}^{\mathsf{O}} \mid \mathcal{W}_{\Box}^{\mathsf{O}} \vee \mathcal{W}_{\Box}^{\mathsf{O}}$$

Alternative axiomatization of **DSL**

$$\mathcal{W}^{\mathsf{O}}_{\Box} := \mathcal{W} \mid \mathsf{O}\mathcal{W} \mid \Box\mathcal{W}^{\mathsf{O}}_{\Box} \mid \neg\mathcal{W}^{\mathsf{O}}_{\Box} \mid \mathcal{W}^{\mathsf{O}}_{\Box} \vee \mathcal{W}^{\mathsf{O}}_{\Box}$$

DSLO is axiomatized by strengthening **S5** by:

Alternative axiomatization of **DSL**

$$\mathcal{W}^{\mathsf{O}}_{\Box} := \mathcal{W} \mid \mathsf{O}\mathcal{W} \mid \Box\mathcal{W}^{\mathsf{O}}_{\Box} \mid \neg\mathcal{W}^{\mathsf{O}}_{\Box} \mid \mathcal{W}^{\mathsf{O}}_{\Box} \vee \mathcal{W}^{\mathsf{O}}_{\Box}$$

DSLO is axiomatized by strengthening **S5** by:

$$(\mathsf{O}A \wedge \mathsf{O}B) \supset \mathsf{O}(A \wedge B) \tag{AND}$$

$$\mathsf{O}(A \wedge B) \supset \mathsf{O}A \tag{ADE}$$

$$(\mathsf{O}A \wedge \mathsf{O}B) \supset \mathsf{O}(A \vee B) \tag{OR}$$

$$((B \Rightarrow A) \wedge \mathsf{O}(A \vee B)) \supset \mathsf{O}A \tag{DINH}$$

$$\mathsf{O}A \supset \Box\mathsf{O}A \tag{ON}$$

$$(\neg A \Rightarrow \mathsf{O}A) \supset \mathsf{O}A \tag{OW}$$

$$\text{If } A \vdash_{\mathbf{EL}} B \text{ then } \vdash \mathsf{O}A \supset \mathsf{O}B \tag{EINH}$$

Alternative axiomatization of **DSL**

$$\mathcal{W}_{\Box}^{\mathsf{O}} := \mathcal{W} \mid \mathsf{O}\mathcal{W} \mid \Box\mathcal{W}_{\Box}^{\mathsf{O}} \mid \neg\mathcal{W}_{\Box}^{\mathsf{O}} \mid \mathcal{W}_{\Box}^{\mathsf{O}} \vee \mathcal{W}_{\Box}^{\mathsf{O}}$$

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$$\text{If } A \vdash_{\mathbf{EL}} B \text{ then } \vdash \mathsf{O}A \supset \mathsf{O}B \tag{EINH}$$

$SA =_{\mathrm{df}} \mathsf{O}\neg A \wedge A$

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$$(\mathsf{O}A \wedge \mathsf{O}B) \supset \mathsf{O}(A \wedge B) \quad \text{(AND)}$$

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$$(\mathsf{O}A \wedge \mathsf{O}B) \supset \mathsf{O}(A \vee B) \quad \text{(OR)}$$

$$((B \Rightarrow A) \wedge \mathsf{O}(A \vee B)) \supset \mathsf{O}A \quad \text{(DINH)}$$

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$$\text{If } A \vdash_{\mathbf{EL}} B \text{ then } \vdash \mathsf{O}A \supset \mathsf{O}B \quad \text{(EINH)}$$

$\mathsf{S}A =_{\text{df}} \mathsf{O}\neg A \wedge A$
$\mathsf{F}A =_{\text{df}} \mathsf{O}\neg A$

Alternative axiomatization of **DSL**

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$$(\neg A \Rightarrow \mathsf{O}A) \supset \mathsf{O}A \tag{OW}$$

$$\text{If } A \vdash_{\mathbf{EL}} B \text{ then } \vdash \mathsf{O}A \supset \mathsf{O}B \tag{EINH}$$

$\mathsf{S}A =_{\mathrm{df}} \mathsf{O}\neg A \wedge A$
$\mathsf{F}A =_{\mathrm{df}} \mathsf{O}\neg A$

Theorem

$\Gamma \vdash_{\mathbf{DSL}} A$ *iff* $\Gamma \vdash_{\mathbf{DSLO}} A$.

Part III

DSL and the ‘paradoxes’

Ross' paradox

P: "posting the letter"
B: "burning the letter"

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Ross' paradox concerns the validity of the inference from (i) to (ii):

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(i) OP

Ross' paradox

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Ross' paradox concerns the validity of the inference from (i) to (ii):

(i) $\mathsf{O}P$

(ii) $\mathsf{O}(P \vee B)$

Ross' paradox

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B: "burning the letter"

Ross' paradox concerns the validity of the inference from (i) to (ii):

(i) $\neg P \Rightarrow \mathsf{S}\neg P$

(ii) $\neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$

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$$\mathsf{O}P \nvdash_{\mathbf{DSL}} \mathsf{O}(P \vee B)$$

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Ross' paradox concerns the validity of the inference from (i) to (ii):

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$$\mathsf{O}P \nvdash_{\mathbf{DSL}} \mathsf{O}(P \vee B)$$
$$\neg P \Rightarrow \mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$$

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$$\neg P \Rightarrow \mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$$
$$\mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \mathsf{S}\neg(P \vee B)$$

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(ii) $\neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$

$$\begin{aligned} \mathsf{O}P &\nvdash_{\mathbf{DSL}} \mathsf{O}(P \vee B) \\ \neg P \Rightarrow \mathsf{S}\neg P &\nvdash_{\mathbf{DSL}} \neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B) \\ \mathsf{S}\neg P &\nvdash_{\mathbf{DSL}} \mathsf{S}\neg(P \vee B) \\ \mathsf{S}\neg P &\nvdash_{\mathbf{DSL}} \mathsf{S}(\neg P \wedge \neg B) \end{aligned}$$

Ross' paradox

P: "posting the letter"
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Ross' paradox concerns the validity of the inference from (i) to (ii):

(i) $\neg P \Rightarrow \mathsf{S}\neg P$

(ii) $\neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$

$$\mathsf{O}P \nvdash_{\mathbf{DSL}} \mathsf{O}(P \vee B)$$

$$\neg P \Rightarrow \mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \neg(P \vee B) \Rightarrow \mathsf{S}\neg(P \vee B)$$

$$\mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \mathsf{S}\neg(P \vee B)$$

$$\mathsf{S}\neg P \nvdash_{\mathbf{DSL}} \mathsf{S}(\neg P \wedge \neg B)$$

$$\neg P \Rightarrow \mathsf{S}\neg P \vdash_{\mathbf{DSL}} \neg(P \vee B) \Rightarrow \mathsf{S}\neg P$$

The good Samaritan

H: "x helps y who has been robbed"
R: "y has been robbed"

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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$

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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$

(ii) $R \Rightarrow s$

The good Samaritan

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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$
(ii) $R \Rightarrow \mathsf{s}$

$$H \Rightarrow R, R \Rightarrow \mathsf{s} \vdash_{\mathbf{KDA}} H \Rightarrow \mathsf{s}$$

The good Samaritan

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R: "y has been robbed"

The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

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(ii) $R \Rightarrow \mathsf{s}$

(iii) $H \Rightarrow \mathsf{s}$

$$H \Rightarrow R, R \Rightarrow \mathsf{s} \vdash_{\mathbf{KDA}} H \Rightarrow \mathsf{s}$$

The good Samaritan

H: "x helps y who has been robbed"
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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$

(ii) $R \Rightarrow SR$

(iii) $H \Rightarrow SH$

$$H \Rightarrow R, R \Rightarrow \mathsf{s} \vdash_{\mathbf{KDA}} H \Rightarrow \mathsf{s}$$

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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$

(ii) $R \Rightarrow SR$

(iii) $H \Rightarrow SH$

$$H \Rightarrow R, R \Rightarrow \mathsf{s} \vdash_{\mathbf{KDA}} H \Rightarrow \mathsf{s}$$

$$H \Rightarrow R, R \Rightarrow SR \vdash_{\mathbf{DSL}} H \Rightarrow SR$$

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H: "x helps y who has been robbed"
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The good Samaritan paradox concerns the inference from (i) and (ii) to (iii) (in **KDA**):

(i) $H \Rightarrow R$
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$$H \Rightarrow R, R \Rightarrow \mathsf{s} \vdash_{\mathbf{KDA}} H \Rightarrow \mathsf{s}$$
$$H \Rightarrow R, R \Rightarrow SR \vdash_{\mathbf{DSL}} H \Rightarrow SR$$
$$H \Rightarrow R, R \Rightarrow SR \nvdash_{\mathbf{DSL}} H \Rightarrow SH$$

Part IV

Work in progress

Permissions

$$\mathsf{P}A =_{\mathrm{df}} \neg\mathsf{O}\neg A$$

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$\mathsf{P}A \equiv \Diamond(A \wedge \neg\mathsf{s})$

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Suppose $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B)$

Permissions

$\mathsf{P}A =_{\mathrm{df}} \neg\mathsf{O}\neg A$

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Suppose $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B)$

$\neg\mathsf{O}(A \vee B) \equiv \neg\mathsf{O}\neg(\neg A \wedge \neg B) \equiv \mathsf{P}(\neg A \wedge \neg B)$

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$\mathsf{P}A =_{\mathrm{df}} \neg\mathsf{O}\neg A$

$\mathsf{P}A \equiv \Diamond(A \wedge \neg \mathsf{s})$

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Suppose $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B)$

$$\neg\mathsf{O}(A \vee B) \equiv \neg\mathsf{O}\neg(\neg A \wedge \neg B) \equiv \mathsf{P}(\neg A \wedge \neg B)$$

Hence $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B) \vdash \mathsf{P}(\neg A \wedge \neg B)$

Permissions

$\mathsf{P}A =_{\mathrm{df}} \neg\mathsf{O}\neg A$

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Hence $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B) \vdash \mathsf{P}(\neg A \wedge \neg B)$

Alternative: $\mathsf{P}\left(\bigwedge_I A_i\right) = \Diamond\left(\bigwedge_I A_i \wedge \neg \bigvee_{\emptyset \neq J \subseteq I} \mathsf{S}\left(\bigwedge_J A_j\right)\right)$

Permissions

$\mathsf{P}A =_{\mathrm{df}} \neg\mathsf{O}\neg A$

$\mathsf{P}A \equiv \Diamond(A \wedge \neg \mathsf{s})$

$\mathsf{P}A \equiv \Diamond(A \wedge \neg \mathsf{S}A)$

Suppose $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B)$

$\quad \neg\mathsf{O}(A \vee B) \equiv \neg\mathsf{O}\neg(\neg A \wedge \neg B) \equiv \mathsf{P}(\neg A \wedge \neg B)$

Hence $\mathsf{O}A \wedge \neg\mathsf{O}(A \vee B) \vdash \mathsf{P}(\neg A \wedge \neg B)$

Alternative: $\mathsf{P}\left(\bigwedge_I A_i\right) = \Diamond\left(\bigwedge_I A_i \wedge \neg \bigvee_{\emptyset \neq J \subseteq I} \mathsf{S}\left(\bigwedge_J A_j\right)\right)$

Strong/positive permission, free choice permission?

Future work

Devise a semantics for **DSL**

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- Devise a semantics for **DSL**
- Contextualize Kanger's constant q, abbreviating that 'all normative demands are met'

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- Devise a semantics for **DSL**
- Contextualize Kanger's constant q, abbreviating that 'all normative demands are met'
  - 'Dual' to Anderson's reduction, yet different properties in our setting

Thank you!

References

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Explicit Disjunctive Obligations

(a)	You should post the letter.		
(b)	Thus, implicitly: You should post the letter or burn it.	(a')	You should post the letter or email it.
(c)	You cannot post the letter.	(b')	You cannot post the letter. (e.g. the post is closed already)
(d)	You should burn the letter.	(c')	You should email it.

- (d) is counter-intuitive

Note that we get in **DSL**:

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Explicit Disjunctive Obligations

(a)	You should post the letter.		
(b)	Thus, implicitly: You should post the letter or burn it.	(a')	You should post the letter or email it.
(c)	You cannot post the letter.	(b')	You cannot post the letter. (e.g. the post is closed already)
(d)	You should burn the letter.	(c')	You should email it.

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- (c') is intuitive
- explicit disjunctions have a different logic than derived ones

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Deontic and alethic modalities in **DSL**

$$\Box A \qquad \vdash_{\mathbf{DSL}} OA$$

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