A constructive modal semantics for contextual verification.

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September 14, 2009 - LOG-IC09 – Potsdam

Overview

- Advantages and shortcomings of the constructive modelling of contexts for knowledge representation:
  - Ability to deal with verifications as first-class citizens;
  - The semantics highlights underlying properties of an appropriate syntax (e.g. type theory);
  - Constrained to variable evaluation;
  - One strong epistemic notion.

Modelling contexts constructively (1)

- Problem 1: Constructively, modelling contexts is possible only under variable-substitution;

> *"On a 3km long path, straight without obstacles, our veichle takes n(TIME) to reach the target"*

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[x=Straight, y=3Km, z=NoObstacles :: Path, n::Nat]
prop P = Veichle
Time(P(x, y, z)) ==> Value :: n
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- AIM1: Design a semantics to describe verificational procedure under contextual modification.

Modelling contexts constructively (2)

- Problem 2: Constructively, the system does not preserve weaker epistemic values (uncertainty, communicated contents and non-monotonicity)

> *"Performing computation at terminals* $1, 2$*, the output of network* $1 - 3$ *is C"*

- AIM2: Use an interpretation of context that does not collapse with common knowledge (i.e. Distributed Knowledge) .

Comparable Approaches

- Modal Contexts: S. Buvac, I.A. Mason, “Propositional Logic of Context”, in Proceedings of the Eleventh National Conference on Artificial Intelligence, AAAI Press, Richard Fikes and Wendy Lehnert (eds.), pp, 412–419, 1993.
- Staged Computation: Davies, R., Pfenning, F., “A modal Analysis of Staged Computation”, *Journal of the ACM*, vol.48, n.3, pp.555-604, 2001.
- Operational Semantics: Pientka, B., Dunfield, J., “Programming with Proofs and Explicit Contexts”, Proceedings of the 10th international ACM SIGPLAN Conference on Principles and Practice of Declarative Programming, Valencia, Spain, ACM, pp.163-173, 2008.

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The Constructive Semantics: $\mathcal{L}^{ver}$

We start with a (sub-system of) an intuitionistic language $\mathcal{L}^{ver}$:

- It enjoys a standard intuitionistic semantics with an ordering among knowledge states based on a monotonic verification function;
- It induces a monotone satisfaction relation in a finitistic setting;
- Monotonicity holds in the transitive frames;
- It does not include hypothetical reasoning but only material implication;

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The Constructive Semantics: $\mathcal{L}^{ver}$

$$\mathcal{L}^{ver} := A \mid \top \mid \bot \mid \neg A \mid A \& B \mid A \vee B \mid A \supset B.$$

Definition (Models of $\mathcal{L}^{ver}$)

A model for knowledge by verification is a tuple $M^{ver} = \{\mathbb{K}, \leq, R, v\}$, where $\mathbb{K}$ is a finite non-empty set ranging over $\{K_i, K_j, \dots\}$ and each K_i is a finite subset of $\mathcal{P}$; $\leq$ is a partial order relation on $\mathbb{K}$, induced by a reflexive and transitive binary accessibility relation R and v is a verification function $v : \mathcal{P} \mapsto 2^{\mathbb{K}}$.

The Constructive Semantics: the non-assumptive fragment (cont'd)

Definition (Satisfaction Relation)

$C1^{ver}$ $v_M, K_i \not\vDash \bot$;
$C2^{ver}$ for all $A \in \mathcal{P}$, $v_M, K_i \vDash A$ iff $A \in v(K_i)$;
$C3^{ver}$ $v_M, K_i \vDash A \vee B$ iff $v_M, K_i \vDash A$ or $v_M, K_i \vDash B$;
$C4^{ver}$ $v_M, K_i \vDash A \wedge B$ iff $v_M, K_i \vDash A$ and $v_M, K_i \vDash B$
$C5^{ver}$ $v_M, K_i \vDash A \supset B$ iff $v_M, K_i \vDash A$ implies $v_M, K_i \vDash B$
$C6^{ver}$ $v_M, K_i \vDash \neg A$ iff for any $K_{i-n} \subseteq K_i$ it holds that $v_M, K_{i-n} \vDash A \supset \bot$.

The Constructive Semantics: the non-assumptive fragment (cont'd)

Definition (Hereditariness of M^{ver})

For every $K_i, K_j \in \mathbb{K}$, a reflexive and transitive accessibility relation R holds such that $R(K_i, K_j)$ iff

1. $K_i \subseteq K_j$ and for every $A \in \mathcal{P}$ such that $K_i \vDash A$, then also $K_j \vDash A$;
2. and for every K_k such that $R'(K_j, K_k)$, then also $R''(K_i, K_k)$, hence $K_i \subseteq K_k \in \mathbb{K}$.

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The Constructive Semantics: $\mathcal{L}^{inf}$

Non-standard extension for non-monotonic contextual reasoning:

- knowledge states are by definition contextualized to sets of assumptions
  - *cf. assumptions-based truth maintenance system in de Kleer (1986)*;
  - *cf importation/discharge in McCarthy, Buvac (1994)*;
- an ordering $\leq^{\gamma}$ between contexts Γ, Γ' is given by formulating additional assumptions;
- it induces a symmetric accessibility relation on the knowledge states;
- it is non-monotonic, with the verification function based on assumptive content.

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The Constructive Semantics: $\mathcal{L}^{inf}$

Definition (Informational Context)

For any $K_i \in \mathbb{K}$, an informational context Γ for K_i is a set of functions $\gamma := \mathcal{V} \mapsto 2^{\mathbb{K}}$ denoted by $(\Gamma)K_i$. A function $\gamma : x \mapsto A$ is valid for $(\Gamma)K_i$ if for any $K_{i-n} \subseteq K_i$ it holds $v_{M^{ver}}, K_i \not\models \neg A$.

$$\mathcal{L}^{inf} := A \mid \top \mid \bot \mid A \& B \mid A \vee B \mid A \supset B \mid \Box A \mid \Diamond A$$

The Constructive Semantics: the assumption-based fragment (cont'd)

Definition (Models of $\mathcal{L}^{inf}$)

A model for contextual knowledge is a tuple $M^{inf} = \{\mathbb{K}, \leq^{\gamma}, R, v\}$, where $\mathbb{K}$ is a finite non-empty set ranging over $\{K_i, K_j, \dots\}$ and each K_i is a subset of $\mathcal{P}$; $\leq^{\gamma}$ is a partial order relation on contexts Γ, Γ', where respectively $(\Gamma)K_i$ and $(\Gamma')K_j$ and the relation is induced by a function $\gamma := x \mapsto A$ such that $\Gamma \leq^{\gamma} \Gamma'$ iff for some $A \in \mathcal{P}$, it holds that $(\Gamma)K_i \nvDash A$ then $(\Gamma')K_j \vDash A$; R is a symmetric accessibility relation on $\mathbb{K}$ induced by $\leq^{\gamma}$ and v is the contextual verification function $v : \mathcal{P} \mapsto 2^{\mathbb{K}}$.

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The Constructive Semantics: the assumption-based fragment (cont'd)

Definition (Satisfaction Relation)

$C1^{inf}$ $v_M, K_i \vDash^{\Gamma} A$ iff $v_{M^{ver}}, K_i \nvDash \neg A$ and there is $\gamma \in \Gamma := x \mapsto A \in v(K_i)$;

$C2^{inf}$ $v_M, K_i \vDash^{\Gamma} \bot$ iff $v_M, K_{i-n} \vDash^{\Gamma} A$ and $\gamma' \in \Gamma' := x \mapsto \neg A \in v(K_i)$

$C3^{inf}$ $v_M, K_i \vDash^{\Gamma} A \vee B$ iff $v_M, K_i \vDash^{\Gamma} A$ or $v_M, K_i \vDash^{\Gamma} B$;

$C4^{inf}$ $v_M, K_i \vDash^{\Gamma} A \wedge B$ iff $v_M, K_i \vDash^{\Gamma} A$ and $v_M, K_i \vDash^{\Gamma} B$;

$C5^{inf}$ $v_M, K_i \vDash^{\Gamma} A \supset B$ iff $\gamma \in \Gamma := x \mapsto A \in v(K_i)$ implies $K_i \vDash^{\Gamma} B$;

$C6^{inf}$ $v_M, K_i \vDash^{\Gamma} \Box A$ iff for any function γ and any knowledge state $K_j \supset K_i$ it holds $K_j \vDash^{\Gamma \leq \gamma} A$;

$C7^{inf}$ $v_M, K_i \vDash^{\Gamma} \Diamond A$ iff there is a function γ and a knowledge state $K_j \supset K_i$ for which it holds $K_i \vDash^{\Gamma \leq \gamma} A$.

The Constructive Semantics: the assumption-based fragment (cont'd)

Lemma (Contextual Monotonicity)

If $K_i \vDash^{\Gamma} \top$ and for some γ it holds $\Gamma \leq^{\gamma} \Gamma' \vDash \top$ then if $K_i \models^{\Gamma} A$ then $K_j \models^{\Gamma'} A$.

Global and Local Validity

- modelling verifications valid under any/some contextual extension;
- the former will reduce to verification in the non-assumptive fragment;
- the latter will describe a weaker epistemic condition;
- we implement modal frames to this aim.

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Global and Local Contexts

Definition (Global Context)

For any context Γ, the global context $\Box\Gamma$ is given by $\bigcup\{\Box A_1, \ldots, \Box A_n\}$ such that $\gamma := x \mapsto A_i \in \Gamma$.

Definition (Local Context)

For any context Γ, the local context $\Diamond\Gamma$ is given by $\bigcup\{\circ A_1, \ldots, \circ A_n \mid \circ = \{\Box, \Diamond\}\}$ and $\gamma := x \mapsto A_i \in \Gamma$ and $\exists A_i$ such that $\Diamond A_i$.

Global and Local Contexts (II)

Derivability in such frames becomes dependent from possibile extensions of contexts:

Definition (Semantic consequence from a global context)

$K_i \models^{\Box\Gamma} A$ iff for every γ, it holds $K_i \models^{\Gamma\leq\gamma} A$. We denote by $\models^{\Box\Gamma} A$ a semantic consequence of every K_i with global context $\Box\Gamma$.

Definition (Semantic consequence from a local context)

$K_i \models^{\Diamond\Gamma} A$ iff for some γ it holds $K_i \models^{\Gamma\leq\gamma} A$. We denote by $\models^{\Diamond\Gamma} A$ a semantic consequence of every K_i with local context $\Diamond\Gamma$.

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Remarks and Open Issues

- $\mathcal{M}^{(\mathcal{L}^{ver} \cup \mathcal{L}^{inf})}$ is provably equivalent to the class of models corresponding to contextual KT with $\Box$ and $\Diamond$;
- separating different epistemic states is crucial to the evolution of constructive systems;
- it can help in designing systems for multi-staged information (security and reliability);
- a multi-modal format and a signature system are the next required elements.