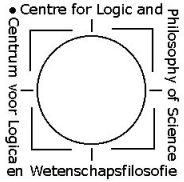
Alleged Assassins

Bjørn Jespersen & Giuseppe Primiero

Department of Computer Science, Technical University of Ostrava &
Department of Logic, Czech Academy of Sciences, Prague
FWO & Centre for Logic and Philosophy of Science, Ghent University



bjorn.jespersen@gmail.com
Giuseppe.Primiero@Ugent.be

TbiLLC 2011 – Kutaisi, Georgia

Outline

a is an alleged assassin

?

$$\frac{\textit{a} \text{ is an alleged assassin}}{?}$$

- what is the logical structure of the premise?
- what follows as conclusion?

Property Modification

- Let M be a modifier and F a property. Then (MF) is the result of the procedure of applying the function M to the argument F.

Property Modification

- Let M be a modifier and F a property. Then (MF) is the result of the procedure of applying the function M to the argument F.

- A full semantic theory of modification must include the following variants:
  - Subsective: $(M'F)a \therefore Fa$
  - Privative: $(M''F)a \therefore \neg Fa$
  - Intersective: $(M'''F)a \therefore M^*a \wedge Fa$
  - Modal: M'''' oscillates between subsection and privation

3 Negative Characterizations of M''''

- $$\frac{(MF)_c x}{M^*_c x \wedge F_c x}$$

3 Negative Characterizations of M''''

- $$\frac{(MF)_c x}{M^*_c x \wedge F_c x}$$
- $$\frac{F_c x \leftrightarrow G_c x \qquad F_c x \rightarrow (MF)_c x}{G_c x \rightarrow (MG)_c x}$$

3 Negative Characterizations of M''''

- $$\frac{(MF)_c x}{M^*_c x \wedge F_c x}$$
- $$\frac{F_c x \leftrightarrow G_c x \qquad F_c x \rightarrow (MF)_c x}{G_c x \rightarrow (MG)_c x}$$

- Fails to validate either of $Fa, \neg Fa$ as conclusion.

Task

A positive characterization of modal modification.

A solution to privative modification

- [Primiero and Jespersen, 2010] offers two analyses of *privative modification* using two variants of *procedural semantics*:

  Realism: Tichý's Transparent Intensional Logic

  Idealism: Martin-Löf's Constructive Type Theory

A solution to privative modification

- [Primiero and Jespersen, 2010] offers two analyses of *privative modification* using two variants of *procedural semantics*:

  Realism: Tichý's Transparent Intensional Logic

  Idealism: Martin-Löf's Constructive Type Theory

- Common basic idea is to analyze *modal modification* in terms of *possibility/contingency*:

  TIL: alethic

  CTT: epistemic

The Commmon Core

1. a notion of *construction*
2. a *functional* language
3. a *typed* universe
4. an *interpreted* syntax

What Distinguishes TIL from CTT

	TIL	CTT
Semantics	model-theoretic	proof-theoretic
Modifier	property to property	set to set

TIL [Duži et al., 2010]

Basic and Functional Types

- *Ground Types:* o, ι, τ, ω
- *Property:* $(o\iota)_{\tau\omega}$
- *Property modifier:* $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$
- *Proposition:* $o_{\tau\omega}$
- *Propositional modifier:* $(o_{\tau\omega}o_{\tau\omega})$

Sentential Meaning

"*a* is an alleged assassin"

$\lambda w \lambda t$ [[*Alleged Assassin*]$_{wt}$ *a*]

The speech act of allegation

$$\frac{\lambda w \lambda t\ [Alleges_{wt}\ b\ \lambda w' \lambda t' [F_{w't'}\ a]]}{\lambda w \lambda t\ [\exists x [\exists P [Alleges_{wt}\ x\ P]]]}\ \text{EG}$$

"*b* alleges that *a* is an *F*"
"somebody alleges something"

Introduction rule for *Alleged*

$$\lambda f[[\mathit{Alleged}\ f]_{wt}\ a] = \lambda f[\exists x[\mathit{Alleges}_{wt}\ x\ \lambda w \lambda t[f_{wt}\ a]]]$$

"being a property that *a* is alleged to have equals being a property that somebody alleges *a* to have"

Elimination Rule for *Alleged*

$$\frac{[[\textit{Alleged Assassin}]_{wt}\ a]}{\exists w'[\exists t'[\textit{Assassin}_{w't'}\ a]] \wedge \exists w''[\exists t''\neg[\textit{Assassin}_{w''t''}\ a]]}$$

Introduction rule for *Allegedly*

$$\lambda P[\textit{Allegedly } P] = \lambda P[\lambda w \lambda t[\exists x[\textit{Alleges}_{wt}\ x\ P]]]$$

"being an alleged proposition equals
being a proposition that somebody alleges"

Elimination rule for *Allegedly*

$$\frac{[\mathit{Allegedly}\ P]_{wt}}{\exists w'[\exists t[P_{w't'}]] \wedge \exists w''[\exists t''[\neg P_{w''t''}]]}$$

Two initial comments

1. Given the judgemental structure of formulas in CTT, we can model only the propositional modifier:
   - from '*a* is an alleged assassin' to 'Allegedly, *a* is an assassin'

2. The standard constructive syntax does not allow to deal with the contingency required by modal modifiers:
   - an extended language is required

Language [Primiero, 2012],[Primiero, 2011]

Definition (Alphabet)

The syntax is defined by the following alphabet:

$\mathcal{K} : \{type, type_{inf}\}$ (verifiers, possibly terminating processes)

$Types := A \mid \bot \mid A \wedge B \mid A \vee B \mid A \rightarrow B \mid A \supset B.$

$Terms := x_i \mid a_i \mid (a_i, b_j) \mid (x_i(b_j)) \mid a_i(b_j).$

$Contexts := \Gamma_i \mid \Delta_i \mid \Box_i\Gamma \mid \Diamond_i\Gamma$

$Judgements := \Delta_i; \Gamma_i \vdash A\ type \mid \Box_i(A\ true) \mid \Diamond_i(A\ true) \mid \circ_{i,j}\Gamma \vdash \circ_{i,j}(A\ true).$

Language [Primiero, 2012],[Primiero, 2011]

Definition (Alphabet)

The syntax is defined by the following alphabet:

$\mathcal{K} : \{type, type_{inf}\}$ (verifiers, possibly terminating processes)

$Types := A \mid \bot \mid A \wedge B \mid A \vee B \mid A \rightarrow B \mid A \supset B.$

$Terms := x_i \mid a_i \mid (a_i, b_j) \mid (x_i(b_j)) \mid a_i(b_j).$

$Contexts := \Gamma_i \mid \Delta_i \mid \Box_i\Gamma \mid \Diamond_i\Gamma$

$Judgements := \Delta_i; \Gamma_i \vdash A\ type \mid \Box_i(A\ true) \mid \Diamond_i(A\ true) \mid \circ_{i,j}\Gamma \vdash \circ_{i,j}(A\ true).$

Modal Modification Rule: Introduction

Allegedly *[a is an assassin]*

$$\frac{\textit{Assassin type}[\Gamma] \qquad \textit{Property}_i\ \textit{type}_{inf} \in \Gamma \qquad \textit{Alleged}(x)[x : \textit{Assassin}]}{\Box\Gamma, \Diamond(\textit{Property}_i) \vdash a : \textit{Assassin}[x_i / p_i : \textit{Property}_i]}$$

Modal Modification Rule: Elimination I

It is proven that [a is an assassin]

$$\frac{\Box\Gamma, \Diamond(\mathit{Property}_i) \vdash a : \mathit{Assassin}[x_i/p_i : \mathit{Property}_i] \qquad p_i : \mathit{Property}_i}{\Box(\Gamma, p_i : \mathit{Property}_i) \vdash a : \mathit{Assassin}}$$

$$\frac{A\ \mathit{type}_{\mathit{inf}} \qquad x : A \vdash B\ \mathit{type}_{\mathit{inf}} \qquad a : A}{(x(b))(a) = b[a/x] : B\ \mathit{type}[a/x]}\ \beta\text{-conversion}$$

Modal Modification Rule: Elimination II

The allegation that [a is an assassin] is false.

$$\frac{\Box\Gamma, x_i : \mathit{Property}_i \vdash a : \mathit{Assassin}[x_i / p_i : \mathit{Property}_i] \qquad p_i : \mathit{Property}_i \to \bot}{a : \mathit{Assassin} \to \bot}$$

Summary of this presentation

1. Oscillation between subsection and privation
2. Alethic vs. Epistemic Possibility
3. For TIL, if $(M''''F)a$ is true, then at some pair *wt* (empirical parameters) *Fa* is true and at another such pair *Fa* is false
4. For CTT, if $(M''''F)a$ *true* is an admissible judgment to make, then conditions for *Fa true* are known to be satisfiable, but not all are asserted as verified

References I

Duži, M., Jespersen, B., and Materna, P. (2010).

Procedural Semantics for Hyperintensional Logic, volume 17 of *Logic, Epistemology and the Unity of Sciences*.

Springer Verlag.

Primiero, G. (2011).

A multi-modal type system and its procedural semantics for safe distributed programming.

In *Intuitionistic Modal Logic and Applications Workshop (IMLA11)*, Nancy.

Manuscript.

Primiero, G. (2012).

A contextual type theory with judgemental modalities for reasoning from open assumptions.

Logique & Analyse, forthcoming.

Primiero, G. and Jespersen, B. (2010).

Two kinds of procedural semantics for privative modification.

Lecture Notes in Artificial Intelligence, 6284:252–71.