Dynamic Proof Theories For Normative Reasoning on the Basis of Consistency Considerations

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Central Problem

Answer the question:

> What are "actual" / "all-things-considered" / "proper" / etc. obligations

given:

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What are "actual" / "all-things-considered" / "proper" / etc. obligations

given:

$\mathsf{O}a, \mathsf{O}\neg a', \mathsf{O}b, \mathsf{O}(a|c), \ldots$

- a set of norms
- (optionally) a set of facts

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$a, d, \ldots$

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- a set of norms

$a, d, \ldots$

- (optionally) a set of facts
- (optionally) a set of constraints

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What are "actual" / "all-things-considered" / "proper" / etc. obligations

given:

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- a set of norms

$a, d, \ldots$

- (optionally) a set of facts

$\neg(a \wedge a'), \ldots$

- (optionally) a set of constraints

Central Problem

Answer the question:

What are "actual" / "all-things-considered" / "proper" / etc. obligations

given:

- a set of norms $Oa, O\neg a', Ob, O(a|c), \ldots$
- (optionally) a set of facts $a, d, \ldots$
- (optionally) a set of constraints $\neg(a \wedge a'), \ldots$

Problem: There may be conflicts/inconsistencies.

Input:

Obligations	Constraints
a	$\neg(a \wedge a')$
a'	
b	

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Consistent chunks: $a,\ b$ $a',\ b$

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Consistent chunks:

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Further reasoning:

$a \wedge b,$
$a \vee b,$
$a \vee a'$

$a' \wedge b,$
$a \vee b,$
$a \vee a'$

Input:

Obligations	Constraints
a	$\neg(a \wedge a')$
a'	
b	

Consistent chunks:

$a, \ b$ $\qquad$ $a', \ b$

Further reasoning:

$a \wedge b$, $a \vee b$, $a \vee a'$ $\qquad$ $a' \wedge b$, $a \vee b$, $a \vee a'$

What to do with the chunks?

Input:

Obligations	Constraints
a	$\neg(a \wedge a')$
a'	
b	

Consistent chunks:

$a,\ b$ | $a',\ b$

Further reasoning:

$a \wedge b,\ a \vee b,\ a \vee a'$ | $a' \wedge b,\ a \vee b,\ a \vee a'$

What to do with the chunks?

- intersection

Input:

Obligations	Constraints
a	$\neg(a \wedge a')$
a'	
b	

Consistent chunks:	$a,\ b$	$a',\ b$
Further reasoning:	$a \wedge b$, $a \vee b$, $a \vee a'$	$a' \wedge b$, $a \vee b$, $a \vee a'$

What to do with the chunks?

- intersection
- union

back to IO

Input:

Obligations	Facts
(a, b)	a
(a, c)	
$(b, \neg c)$	

back to IO

Input:

Obligations	Facts
(a, b)	a
(a, c)	
$(b, \neg c)$	

Consistent chunks:

(a, b), (a, c)

(a, b), $(b, \neg c)$

(a, c), $(b, \neg c)$

back to IO

Input:

Obligations	Facts
(a, b)	a
(a, c)	
$(b, \neg c)$	

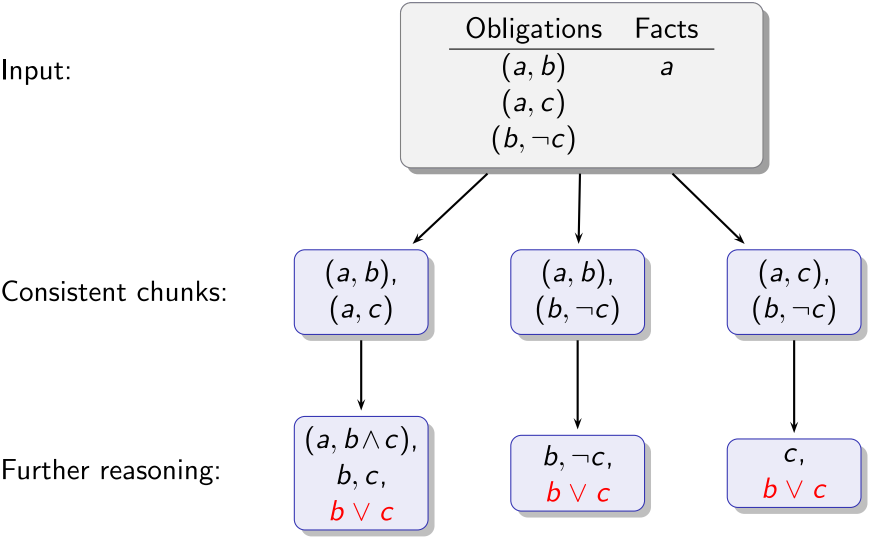
Consistent chunks:

(a, b), (a, c)	(a, b), $(b, \neg c)$	(a, c), $(b, \neg c)$

Further reasoning:

$(a, b \wedge c)$, b, c, $b \vee c$	b, $\neg c$, $b \vee c$	c, $b \vee c$

back to IO


Input:
Obligations | Facts
(a, b) | a
(a, c)
$(b, \neg c)$
Consistent chunks:
(a, b), (a, c)
(a, b), $(b, \neg c)$
(a, c), $(b, \neg c)$
Further reasoning:
$(a, b \wedge c)$, b, c, $b \vee c$
b, $\neg c$, $b \vee c$
c, $b \vee c$

- flat normative/knowledge/etc. base
    - Rescher-Manor, Horty

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    - Input/Output logic (Makinson, Van Der Torre)

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lack of proof theory

adaptive logics

Idea: apply

If $\bullet O$ then O.

"as much as possible".

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Monadic case:

$$\text{If } \bullet \mathrm{O}A \text{ then } \mathrm{O}A.$$

Idea: apply

explicit obligation

"actual" obligation

If $\bullet O$ then O.

"as much as possible".

Monadic case:

$$\text{If } \bullet \mathrm{O}A \text{ then } \mathrm{O}A.$$

Dyadic case:

$$\text{If } \bullet \mathrm{O}(A, B) \text{ then } \mathrm{O}(A, B).$$

Adaptive logic in the standard format

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   supraclassical core logic (reflexive, monotonic, transitive)

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   characterized by a logical form, in our case

$$\Omega = \{\bullet O \wedge \neg O \mid O \text{ is an O-formula}\}$$

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3. Strategy

   e.g., minimal abnormality and reliability

The Lower Limit Logic for the Monadic Case

- classical connectives $\wedge, \vee, \supset, \equiv, \neg$

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- "explicitness" operator $\bullet$

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  - where $\mathsf{O}A$ is well-formed, so is $\bullet\mathsf{O}A$

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  - where $\mathsf{O}A$ is well-formed, so is $\bullet\mathsf{O}A$
- "Ought-implies-can": $\vdash \Box A \supset \neg\mathsf{O}\neg A$
- classical propositional logic
- $\bullet$ is a "dummy"

"adaptive meaning"

Adaptive Proofs

A line:

l $A \wedge B$ l',...,l''; R Δ

line-
number

Adaptive Proofs

A line:

formula

l $\quad A \wedge B \quad$ l',. . . ,l"; R $\quad \Delta$

line-number

Adaptive Proofs

A line:

formula

l $A \wedge B$ l',...,l''; R Δ

line-number

justification

Adaptive Proofs

A line:

formula		condition

l $A \wedge B$ l',. . . ,l''; R Δ

line-number

justification

Adaptive Proofs

A line:

formula condition

l $A \wedge B$ l',...,l''; R Δ

line-number justification

Conditional rule:

If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee \mathrm{Dab}\,(\Delta)$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Delta \end{array}$$

Adaptive Proofs

A line:

formula

condition

$$\text{l} \quad A \wedge B \qquad \text{l',...,l''; R} \quad \Delta$$

line-number

justification

Conditional rule:

If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee \mathrm{Dab}\,(\Delta)$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Delta \end{array}$$

collect abnormalities

Adaptive Proofs

A line:

formula   condition

$$l \quad A \wedge B \quad l',\ldots,l'';\ R \quad \Delta$$

line-number   justification

Conditional rule:

If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \vee \mathrm{Dab}(\Delta)$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Delta \end{array}$$

add new condition

collect abnormalities

1	$\bullet \mathsf{O} a$	PREM	$\emptyset$
2	$\bullet \mathsf{O} a'$	PREM	$\emptyset$
3	$\bullet \mathsf{O} c$	PREM	$\emptyset$
4	$\Box \neg (a \wedge a')$	PREM	$\emptyset$

premise introduction

1	$\bullet Oa$	PREM	$\emptyset$
2	$\bullet Oa'$	PREM	$\emptyset$
3	$\bullet Oc$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	Oa	1; RC	$\{\bullet Oa \wedge \neg Oa\}$

Note: $\bullet Oa \vdash_{\mathbf{LLL}} Oa \vee (\bullet Oa \wedge \neg Oa)$

1	$\bullet \mathsf{O} a$	PREM	$\emptyset$
2	$\bullet \mathsf{O} a'$	PREM	$\emptyset$
3	$\bullet \mathsf{O} c$	PREM	$\emptyset$
4	$\Box \neg (a \wedge a')$	PREM	$\emptyset$
5	$\mathsf{O} a$	1; RC	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$

unconditional rule

Recall: O is **KD**-modality

If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$ then

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

1	$\bullet Oa$	PREM	$\emptyset$
2	$\bullet Oa'$	PREM	$\emptyset$
3	$\bullet Oc$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	Oa	1; RC	$\{\bullet Oa \wedge \neg Oa\}$
6	$O(a \vee a')$	5; RU	$\{\bullet Oa \wedge \neg Oa\}$
7	Oa'	2; RC	$\{\bullet Oa' \wedge \neg Oa'\}$
8	$O(a \vee a')$	7; RU	$\{\bullet Oa' \wedge \neg Oa'\}$

analogous to lines 5 and 6

1	$\bullet \mathsf{O} a$	PREM	$\emptyset$
2	$\bullet \mathsf{O} a'$	PREM	$\emptyset$
3	$\bullet \mathsf{O} c$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	$\mathsf{O} a$	1; RC	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$
7	$\mathsf{O} a'$	2; RC	$\{\bullet \mathsf{O} a' \wedge \neg \mathsf{O} a'\}$
8	$\mathsf{O}(a \vee a')$	7; RU	$\{\bullet \mathsf{O} a' \wedge \neg \mathsf{O} a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

We shortcut:
$!A =_{\mathrm{df}} \bullet \mathsf{O} A \wedge \neg \mathsf{O} A$

this follows by O-aggregation and OIC

1	$\bullet\mathsf{O}a$	PREM	$\emptyset$
2	$\bullet\mathsf{O}a'$	PREM	$\emptyset$
3	$\bullet\mathsf{O}c$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	$\mathsf{O}a$	1; RC	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
7	$\mathsf{O}a'$	2; RC	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
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- One of our assumptions is false! We need a retraction mechanism.

1	$\bullet \mathsf{O}a$	PREM	$\emptyset$
2	$\bullet \mathsf{O}a'$	PREM	$\emptyset$
3	$\bullet \mathsf{O}c$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	$\mathsf{O}a$	1; RC	$\{\bullet \mathsf{O}a \wedge \neg \mathsf{O}a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet \mathsf{O}a \wedge \neg \mathsf{O}a\}$
7	$\mathsf{O}a'$	2; RC	$\{\bullet \mathsf{O}a' \wedge \neg \mathsf{O}a'\}$
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- One of our assumptions is false! We need a retraction mechanism.
- *marking* of lines which are retracted

1	$\bullet \mathsf{O} a$	PREM	$\emptyset$
2	$\bullet \mathsf{O} a'$	PREM	$\emptyset$
3	$\bullet \mathsf{O} c$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	$\mathsf{O} a$	1; RC	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet \mathsf{O} a \wedge \neg \mathsf{O} a\}$
7	$\mathsf{O} a'$	2; RC	$\{\bullet \mathsf{O} a' \wedge \neg \mathsf{O} a'\}$
8	$\mathsf{O}(a \vee a')$	7; RU	$\{\bullet \mathsf{O} a' \wedge \neg \mathsf{O} a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

- One of our assumptions is false! We need a retraction mechanism.
- *marking* of lines which are retracted
- determined by the minimal disjunctions of abnormalities which are derived on the empty condition

1	$\bullet O a$	PREM	$\emptyset$
2	$\bullet O a'$	PREM	$\emptyset$
3	$\bullet O c$	PREM	$\emptyset$
4	$\Box\neg(a \wedge a')$	PREM	$\emptyset$
5	$O a$	1; RC	$\{\bullet O a \wedge \neg O a\}$
6	$O(a \vee a')$	5; RU	$\{\bullet O a \wedge \neg O a\}$
7	$O a'$	2; RC	$\{\bullet O a' \wedge \neg O a'\}$
8	$O(a \vee a')$	7; RU	$\{\bullet O a' \wedge \neg O a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

- One of our assumptions is false! We need a retraction mechanism.
- *marking* of lines which are retracted
- determined by the minimal disjunctions of abnormalities which are derived on the empty condition
- exact definition depends on the strategy

Reliability and Minimal Abnormality

$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	$\mathsf{O}a$	1; RC	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
7	$\mathsf{O}a'$	2; RC	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
8	$\mathsf{O}(a \vee a')$	7; RU	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

- *Reliability* assumption contains a member of a minimal disjunction of abnormalities $\Rightarrow$ *retract*

Reliability and Minimal Abnormality

$\vdots$	$\vdots$	$\vdots$	$\vdots$
✓5	$\mathrm{O}a$	1; RC	$\{\bullet \mathrm{O}a \wedge \neg \mathrm{O}a\}$
✓6	$\mathrm{O}(a \vee a')$	5; RU	$\{\bullet \mathrm{O}a \wedge \neg \mathrm{O}a\}$
✓7	$\mathrm{O}a'$	2; RC	$\{\bullet \mathrm{O}a' \wedge \neg \mathrm{O}a'\}$
✓8	$\mathrm{O}(a \vee a')$	7; RU	$\{\bullet \mathrm{O}a' \wedge \neg \mathrm{O}a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

- *Reliability* assumption contains a member of a minimal disjunction of abnormalities $\Rightarrow$ *retract*

Reliability and Minimal Abnormality

$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	$\mathsf{O}a$	1; RC	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
7	$\mathsf{O}a'$	2; RC	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
8	$\mathsf{O}(a \vee a')$	7; RU	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
9	$!a \vee !a'$	1,2,4; RU	$\emptyset$

- *Reliability* assumption contains a member of a minimal disjunction of abnormalities $\Rightarrow$ *retract*
  - application context: where conflict is likely to be a sign of erroneous issuing of norms by the authority
  - e.g., authority may have made a mistake in issuing $\mathsf{O}a'$ (that explains the conflict)
  - there may additionally be a high cost for realizing an erroneous norm (e.g., a big financial investment)

Reliability and Minimal Abnormality

$\vdots$	$\vdots$	$\vdots$	$\vdots$
✓5	$\mathsf{O}a$	1; RC	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
6	$\mathsf{O}(a \vee a')$	5; RU	$\{\bullet\mathsf{O}a \wedge \neg\mathsf{O}a\}$
✓7	$\mathsf{O}a'$	2; RC	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
8	$\mathsf{O}(a \vee$	7; RU	$\{\bullet\mathsf{O}a' \wedge \neg\mathsf{O}a'\}$
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$\{\{!a\}, \{!a'\}\}$

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- *Minimal Abnormality*
  - minimal choice sets

Reliability and Minimal Abnormality

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  - application context: where conflict is likely to be a sign of erroneous issuing of norms by the authority
  - e.g., authority may have made a mistake in issuing $\mathsf{O}a'$ (that explains the conflict)
  - there may additionally be a high cost for realizing an erroneous norm (e.g., a big financial investment)
- *Minimal Abnormality*
  - minimal choice sets
  - A is derived "safely" if for each minimal choice φ, A is derived on a condition Δ such that $\varphi \cap \Delta = \emptyset$

Counting Strategy

1	$\bullet \mathrm{O}a$	PREM	$\emptyset$
2	$\bullet \mathrm{O}b$	PREM	$\emptyset$
3	$\bullet \mathrm{O}c$	PREM	$\emptyset$
4	$\bullet \mathrm{O}d$	PREM	$\emptyset$

Counting Strategy

1	$\bullet \mathsf{O} a$	PREM	$\emptyset$
2	$\bullet \mathsf{O} b$	PREM	$\emptyset$
3	$\bullet \mathsf{O} c$	PREM	$\emptyset$
4	$\bullet \mathsf{O} d$	PREM	$\emptyset$
5	$\Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$	PREM	$\emptyset$

Counting Strategy

1	$\bullet\mathsf{O}a$	PREM	$\emptyset$
2	$\bullet\mathsf{O}b$	PREM	$\emptyset$
3	$\bullet\mathsf{O}c$	PREM	$\emptyset$
4	$\bullet\mathsf{O}d$	PREM	$\emptyset$
5	$\Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$	PREM	$\emptyset$
6	$!a \vee !b$	1,2,5; RU	$\emptyset$
7	$!a \vee !c$	1,3,5; RU	$\emptyset$
8	$!a \vee !d$	1,4,5; RU	$\emptyset$

Counting Strategy

1	$\bullet \mathsf{O}a$	PREM	$\emptyset$
2	$\bullet \mathsf{O}b$	PREM	$\emptyset$
3	$\bullet \mathsf{O}c$	PREM	$\emptyset$
4	$\bullet \mathsf{O}d$	PREM	$\emptyset$
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6	$!a \vee !b$	1,2,5; RU	$\emptyset$
7	$!a \vee !c$	1,3,5; RU	$\emptyset$
8	$!a \vee !d$	1,4,5; RU	$\emptyset$
✓9	$\mathsf{O}a$	1; RC	$\{!a\}$
10	$\mathsf{O}b$	2; RC	$\{!b\}$
11	$\mathsf{O}c$	3; RC	$\{!c\}$
12	$\mathsf{O}d$	4; RC	$\{!d\}$

Counting Strategy

1	$\bullet \mathsf{O}a$	PREM	$\emptyset$
2	$\bullet \mathsf{O}b$	PREM	$\emptyset$
3	$\bullet \mathsf{O}c$	PREM	$\emptyset$
4	$\bullet \mathsf{O}d$	PREM	$\emptyset$
5	$\Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$	PREM	$\emptyset$
6	$!a \vee !b$	1,2,5; RU	$\emptyset$
7	$!a \vee !c$	1,3,5; RU	$\emptyset$
8	$!a \vee !d$	1,4,5; RU	$\emptyset$
✓9	$\mathsf{O}a$	1; RC	$\{!a\}$
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- marking like for Minimal Abnormality: just now consider the quantitatively minimal choice sets
  - minimal choice sets (w.r.t. $\subset$): $\{!a\}$ and $\{!b, !c, !d\}$
  - minimal choice sets (w.r.t. cardinality): $\{!a\}$

Duality between Maximal Consistent Subsets and the Minimal Choice Sets

$$\bullet \mathsf{O}a, \bullet \mathsf{O}b, \bullet \mathsf{O}c, \bullet \mathsf{O}d, \Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$$

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Maximal consistent subsets:

1. $\{a\}$
2. $\{b, c, d\}$

Duality between Maximal Consistent Subsets and the Minimal Choice Sets

$$\bullet \bigcirc a, \bullet \bigcirc b, \bullet \bigcirc c, \bullet \bigcirc d, \Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$$

Maximal consistent subsets:

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Maximal choice sets:

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Maximal consistent subsets:

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Maximal choice sets:

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2. $\{!a\}$

Where $\mathcal{O}$ and $\mathcal{C}$ are sets of propositional formulas:

- Let $\Gamma^{\mathcal{O},\mathcal{C}} = \{\bullet\mathsf{O}A \mid A \in \mathcal{O}\} \cup \{\Box A \mid A \in \mathcal{C}\}$.
- We say that $\mathcal{O}' \subseteq \mathcal{O}$ is *consistent w.r.t.* $\mathcal{C}$ iff $\mathcal{O}' \cup \mathcal{C}$ is consistent.
- $\mathcal{O}'$ is $\prec$*-maximally consistent w.r.t.* $\mathcal{C}$ iff it is consistent w.r.t. $\mathcal{C}$ and there is no $\mathcal{O}'' \prec \mathcal{O}'$ that is consistent w.r.t. $\mathcal{C}$.

Duality between Maximal Consistent Subsets and the Minimal Choice Sets

$$\bullet \mathsf{O}a, \bullet \mathsf{O}b, \bullet \mathsf{O}c, \bullet \mathsf{O}d, \Box(a \supset (\neg b \wedge \neg c \wedge \neg d))$$

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Note the following duality:

- for each maximal consistent subset $\mathcal{O}'$ w.r.t. $\mathcal{C}$ there is a maximal choice set φ of $\Gamma^{\mathcal{O},\mathcal{C}}$ such that $\mathcal{O}' = \mathcal{O} \setminus \{A \mid !A \in \varphi\}$

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- and vice versa

Theorem

$\Gamma^{\mathcal{O},\mathcal{C}} \vdash_{\mathbf{AL^m}} \mathsf{O}A$ *iff* A *is implied by all* $\subset$*-maximally consistent subsets of* $\mathcal{O}$.

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Definition
$A \in \mathcal{O}$ is *free in* $\mathcal{O}$ *w.r.t.* $\mathcal{C}$ iff $A \in \mathcal{O}'$ for all $\subset$-maximally consistent subsets $\mathcal{O}'$ of $\mathcal{O}$ w.r.t. $\mathcal{C}$.

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Definition

$A \in \mathcal{O}$ is *free in* $\mathcal{O}$ *w.r.t.* $\mathcal{C}$ iff $A \in \mathcal{O}'$ for all $\subset$-maximally consistent subsets $\mathcal{O}'$ of $\mathcal{O}$ w.r.t. $\mathcal{C}$.

E.g., b is free in $\mathcal{O} = \{a, \neg a, b\}$ w.r.t. $\mathcal{C} = \emptyset$.

Theorem
$\Gamma^{\mathcal{O},\mathcal{C}} \vdash_{\mathbf{AL^m}} \mathsf{O}A$ *iff* A *is implied by all* $\subset$*-maximally consistent subsets of* $\mathcal{O}$*.*

Theorem
$\Gamma^{\mathcal{O},\mathcal{C}} \vdash_{\mathbf{AL^c}} \mathsf{O}A$ *iff* A *is implied by all* $\prec_{\text{card}}$*-maximally consistent subsets of* $\mathcal{O}$ *w.r.t.* $\mathcal{C}$*.*

Definition
$A \in \mathcal{O}$ is *free in* $\mathcal{O}$ *w.r.t.* $\mathcal{C}$ iff $A \in \mathcal{O}'$ for all $\subset$-maximally consistent subsets $\mathcal{O}'$ of $\mathcal{O}$ w.r.t. $\mathcal{C}$.

Theorem
$\Gamma^{\mathcal{O},\mathcal{C}} \vdash_{\mathbf{AL^r}} \mathsf{O}A$ *iff* A *is implied by the set of free members of* $\mathcal{O}$ *w.r.t.* $\mathcal{C}$*.*

Taking into account implicit obligations

Taking into account implicit obligations

E.g., $\bullet O(a \wedge b), \bullet O \neg a \nvdash_{\mathbf{AL}} Ob$.

- new operator: $\circ$ characterized by
  - If $A \vdash_{\mathbf{CL}} B$ then $\vdash \bullet OA \supset \circ OB$.

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1	$\bullet O(a \wedge b)$	PREM	$\emptyset$
2	$\bullet O\neg a$	PREM	$\emptyset$
3	$O(a \wedge b)$	1; RC	$\{!a\}$

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1	$\bullet\mathsf{O}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet\mathsf{O}\neg a$	PREM	$\emptyset$
3	$\mathsf{O}(a \wedge b)$	1; RC	$\{!a\}$
4	$\mathsf{O}b$	3; RU	$\{!a\}$

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1	$\bullet\mathsf{O}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet\mathsf{O}\neg a$	PREM	$\emptyset$
3	$\mathsf{O}(a \wedge b)$	1; RC	$\{!a\}$
4	$\mathsf{O}b$	3; RU	$\{!a\}$
5	$\mathsf{O}\neg a$	2; RC	$\{!(\neg a)\}$

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1	$\bullet O(a \wedge b)$	PREM	$\emptyset$
2	$\bullet O\neg a$	PREM	$\emptyset$
3	$O(a \wedge b)$	1; RC	$\{!a\}$
4	Ob	3; RU	$\{!a\}$
5	$O\neg a$	2; RC	$\{!(\neg a)\}$
6	$\circ Ob$	1; RU	$\emptyset$

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1	$\bullet\mathsf{O}(a \wedge b)$	PREM	$\emptyset$
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3	$\mathsf{O}(a \wedge b)$	1; RC	$\{!a\}$
4	$\mathsf{O}b$	3; RU	$\{!a\}$
5	$\mathsf{O}\neg a$	2; RC	$\{!(\neg a)\}$
6	$\circ\mathsf{O}b$	1; RU	$\emptyset$
7	$\mathsf{O}b$	6; RC	$\{\dagger b\}$

$\dagger b = \circ\mathsf{O}b \wedge \neg\mathsf{O}b$?

Taking into account implicit obligations

E.g., $\bullet\mathrm{O}(a \wedge b), \bullet\mathrm{O}\neg a \nvdash_{\mathbf{AL}} \mathrm{O}b$.

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1	$\bullet\mathrm{O}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet\mathrm{O}\neg a$	PREM	$\emptyset$
3	$\mathrm{O}(a \wedge b)$	1; RC	$\{!a\}$
4	$\mathrm{O}b$	3; RU	$\{!a\}$
5	$\mathrm{O}\neg a$	2; RC	$\{!(\neg a)\}$
6	$\circ\mathrm{O}b$	1; RU	$\emptyset$
7	$\mathrm{O}b$	6; RC	$\{\dagger b\}$
8	$\mathrm{O}(a \vee \neg b)$	1; RC	$\{\dagger(a \vee \neg b)\}$
9	$\mathrm{O}(\neg a \vee \neg b)$	2; RC	$\{\dagger(\neg a \vee \neg b)\}$
10	$\mathrm{O}\neg b$	8,9; RU	$\{\dagger(a \vee \neg b), \dagger(\neg a \vee \neg b$

Taking into account implicit obligations

1	$\bullet\mathsf{O}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet\mathsf{O}\neg a$	PREM	$\emptyset$
3	$\mathsf{O}(a \wedge b)$	1; RC	$\{!a\}$
4	$\mathsf{O}b$	3; RU	$\{!a\}$
5	$\mathsf{O}\neg a$	2; RC	$\{!(\neg a)\}$
6	$\circ\mathsf{O}b$	1; RU	$\emptyset$
7	$\mathsf{O}b$	6; RC	$\{\dagger b\}$
$\checkmark$8	$\mathsf{O}(a \vee \neg b)$	1; RC	$\{\dagger(a \vee \neg b)\}$
$\checkmark$9	$\mathsf{O}(\neg a \vee \neg b)$	2; RC	$\{\dagger(\neg a \vee \neg b)\}$
$\checkmark$10	$\mathsf{O}\neg b$	8,9; RU	$\{\dagger(a \vee \neg b), \dagger(\neg a \vee \neg b$
11	$\dagger b \vee \dagger(a \vee \neg b) \vee \dagger(\neg a \vee \neg b)$	1,2; RU	$\emptyset$
12	$!(a \wedge b) \vee !\neg a$	1,2; RU	$\emptyset$
13	$\dagger a \vee \dagger\neg a$	1,2; RU	$\emptyset$
14	$\dagger(a \vee \neg b) \vee \dagger(\neg a \vee \neg b)$	13; RU	$\emptyset$

- $\dagger(\bigvee_I A_i) = (\circ\mathsf{O}\bigvee_I A_i \wedge \neg\mathsf{O}\bigvee_I A_i) \vee \bigvee_{\emptyset \neq J \subset I}(\circ\mathsf{O}\bigvee_J A_j \wedge \neg\mathsf{O}\bigvee_J A_j)$
- e.g.,
  $\dagger(a \vee \neg b) = (\circ\mathsf{O}(a \vee \neg b) \wedge \neg\mathsf{O}(a \vee \neg b)) \vee (\circ\mathsf{O}a \wedge \neg\mathsf{O}a) \vee (\circ\mathsf{O}\neg b \wedge \neg\mathsf{O}\neg b)$

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$$\text{If } A \vdash_{\mathbf{CL}} B, \text{ then } \bullet \mathsf{P}A \vdash \circ \mathsf{P}A.$$

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1	$\bullet \mathsf{P}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet \mathsf{O} \neg a$	PREM	$\emptyset$
3	$\circ \mathsf{P} a$	1; RU	$\emptyset$
4	$\circ \mathsf{P} b$	1; RU	$\emptyset$
5	$\circ \mathsf{O} \neg a$	2; RU	$\emptyset$

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1	$\bullet \mathsf{P}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet \mathsf{O}\neg a$	PREM	$\emptyset$
3	$\circ \mathsf{P}a$	1; RU	$\emptyset$
4	$\circ \mathsf{P}b$	1; RU	$\emptyset$
5	$\circ \mathsf{O}\neg a$	2; RU	$\emptyset$
6	$(\circ \mathsf{O}\neg a \wedge \neg \mathsf{O}\neg a) \vee (\circ \mathsf{P}a \wedge \neg \mathsf{P}a)$	3,5; RU	$\emptyset$

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2	$\bullet \mathsf{O}\neg a$	PREM	$\emptyset$
3	$\circ \mathsf{P}a$	1; RU	$\emptyset$
4	$\circ \mathsf{P}b$	1; RU	$\emptyset$
5	$\circ \mathsf{O}\neg a$	2; RU	$\emptyset$
6	$(\circ \mathsf{O}\neg a \wedge \neg \mathsf{O}\neg a) \vee (\circ \mathsf{P}a \wedge \neg \mathsf{P}a)$	3,5; RU	$\emptyset$
7	$!\neg a \vee (\bullet \mathsf{P}(a \wedge b) \wedge \neg \mathsf{P}(a \wedge b))$	1,2; RU	$\emptyset$

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1	$\bullet \mathsf{P}(a \wedge b)$	PREM	$\emptyset$
2	$\bullet \mathsf{O}\neg a$	PREM	$\emptyset$
3	$\circ \mathsf{P}a$	1; RU	$\emptyset$
4	$\circ \mathsf{P}b$	1; RU	$\emptyset$
5	$\circ \mathsf{O}\neg a$	2; RU	$\emptyset$
6	$(\circ \mathsf{O}\neg a \wedge \neg \mathsf{O}\neg a) \vee (\circ \mathsf{P}a \wedge \neg \mathsf{P}a)$	3,5; RU	$\emptyset$
7	$!\neg a \vee (\bullet \mathsf{P}(a \wedge b) \wedge \neg \mathsf{P}(a \wedge b))$	1,2; RU	$\emptyset$
8	$\mathsf{P}b$	4; RC	$\{\circ \mathsf{P}b \wedge \neg \mathsf{P}b\}$

Excursus: Discussive Context and Rescher-Manor Consequence Relations

- Discussant 1 states: a

Excursus: Discussive Context and Rescher-Manor Consequence Relations

- Discussant 1 states: a
- Discussant 2 states: $\neg a \vee b$

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  - Discussant 1: $\bullet \mathsf{O} a$ (she explicitly supports a)

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- Question: should we derive b?
- Problem: Discussant 2 may not agree with/support a but nevertheless not state $\neg a$ (e.g., due to lack of knowledge)
- in our framework this can be expressed (under different readings of O and P)
  - Discussant 1: $\bullet \mathsf{O} a$ (she explicitly supports a)
  - Discussant 2: $\bullet \mathsf{O}(\neg a \vee b)$ (he explicitly supports $\neg a \vee b$)

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- Discussant 2 states: $\neg a \vee b$
- Question: should we derive b?
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- in our framework this can be expressed (under different readings of O and P)
  - Discussant 1: $\bullet \mathsf{O} a$ (she explicitly supports a)
  - Discussant 2: $\bullet \mathsf{O}(\neg a \vee b)$ (he explicitly supports $\neg a \vee b$)
  - Discussant 2: $\bullet \mathsf{P} \neg a$ (he explicitly states lack of support for $\neg a$)

note: this is different from $\bullet \mathsf{O} \neg a$!

Excursus: Discussive Context and Rescher-Manor Consequence Relations

- Discussant 1 states: a
- Discussant 2 states: $\neg a \vee b$
- Question: should we derive b?
- Problem: Discussant 2 may not agree with/support a but nevertheless not state $\neg a$ (e.g., due to lack of knowledge)
- in our framework this can be expressed (under different readings of O and P)
  - Discussant 1: $\bullet \mathsf{O} a$ (she explicitly supports a)
  - Discussant 2: $\bullet \mathsf{O}(\neg a \vee b)$ (he explicitly supports $\neg a \vee b$)
  - Discussant 2: $\bullet \mathsf{P} \neg a$ (he explicitly states lack of support for $\neg a$)
  - we get: $\mathsf{O}(\neg a \vee b)$ (e.g., "$\neg a \vee b$ is supportable for all discussants")

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- Problem: Discussant 2 may not agree with/support a but nevertheless not state $\neg a$ (e.g., due to lack of knowledge)
- in our framework this can be expressed (under different readings of O and P)
  - Discussant 1: $\bullet \mathrm{O} a$ (she explicitly supports a)
  - Discussant 2: $\bullet \mathrm{O}(\neg a \vee b)$ (he explicitly supports $\neg a \vee b$)
  - Discussant 2: $\bullet \mathrm{P} \neg a$ (he explicitly states lack of support for $\neg a$)
  - we get: $\mathrm{O}(\neg a \vee b)$ (e.g., "$\neg a \vee b$ is supportable for all discussants")
  - but not $\mathrm{O} b$.

Introducing Indexes

- instead of $\bullet$ $[\circ]$ we use $\bullet_i$ $[\circ_i]$ where $i \in I$ for some index set I

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- Various interpretations possible

Introducing Indexes

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i indicates a specific authority

- abnormalities: $\Omega = \bigcup_I \Omega_i$ where $\Omega_i = \{\bullet_i OA \wedge \neg A\}$

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similar for the strengthenings/variants

i indicates a specific authority

- abnormalities: $\Omega = \bigcup_I \Omega_i$ where $\Omega_i = \{\bullet_i \mathsf{O} A \wedge \neg A\}$
- e.g.,

$$\left.\begin{array}{c} \bullet_1 \mathsf{O}a, \\ \bullet_2 \mathsf{O}a, \\ \bullet_3 \mathsf{O}a, \\ \bullet_4 \mathsf{O}\neg a \end{array}\right. \vdash_{\mathbf{AL^c}} \mathsf{O}a$$

i indicating time

- use lexicographic/reverse-lexicographic ALs

1	$\bullet_1 \mathsf{O}(a \wedge c)$	PREM	$\emptyset$
2	$\bullet_2 \mathsf{O}\neg a$	PREM	$\emptyset$
3	$\bullet_3 \mathsf{O}b$	PREM	$\emptyset$

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where $!^i A =_{\mathrm{df}} \bullet_i \mathsf{O}A \wedge \neg \mathsf{O}A$

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2	$\bullet_2 O\neg a$	PREM	$\emptyset$
3	$\bullet_3 Ob$	PREM	$\emptyset$
4	$!^1(a \wedge c) \vee !^2\neg a$	1,2; RU	$\emptyset$
5	$\circ_1 Oa$	1; RU	$\emptyset$
6	$\circ_1 Oc$	1; RU	$\emptyset$
7	$\circ_2 O\neg a$	2; RU	$\emptyset$

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3	$\bullet_3 \mathsf{O}b$	PREM	$\emptyset$
4	$!^1(a \wedge c) \vee !^2\neg a$	1,2; RU	$\emptyset$
5	$\circ_1 \mathsf{O}a$	1; RU	$\emptyset$
6	$\circ_1 \mathsf{O}c$	1; RU	$\emptyset$
7	$\circ_2 \mathsf{O}\neg a$	2; RU	$\emptyset$
8	$\dagger^1 a \vee \dagger^2 \neg a$	5,7; RU	$\emptyset$

where $\dagger^i A =_{\mathrm{df}} \circ_i \mathsf{O}A \wedge \neg \mathsf{O}A$

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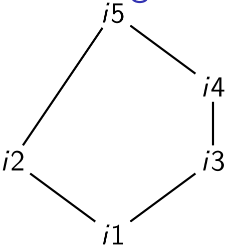
7	$\circ_2 \mathsf{O}\neg a$	2; RU	$\emptyset$
8	$\dagger^1 a \vee \dagger^2 \neg a$	5,7; RU	$\emptyset$
9	$\mathsf{O}\neg a$	2; RC	$\{!^2\neg a\}$
✓10	$\mathsf{O}(a \wedge c)$	1; RC	$\{!^1(a \wedge c)\}$
✓11	$\mathsf{O}a$	5; RC	$\{\dagger^1 a\}$
12	$\mathsf{O}c$	6; RC	$\{\dagger^1 c\}$
13	$\mathsf{O}b$	3; RC	$\{!^3 b\}$

Concerning the reverse-lexicographic order on Ω^2, $\{!^1(a \wedge c), \dagger^1 a\}$ is the minimal choice set at this stage.

i indicating the degree of authority



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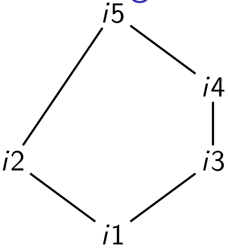

- Suppose we have: $\bullet_{i1} \mathrm{O} a$, $\bullet_{i4} \mathrm{O} \neg a$, $\bullet_{i2} \mathrm{O} b$, $\bullet_{i3} \mathrm{O} b'$, $\bullet_{i5} \mathrm{O} c, \Box \neg (b \wedge b')$.

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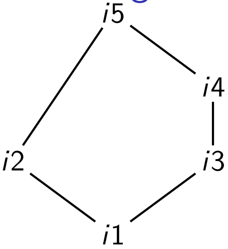

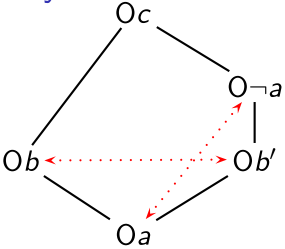

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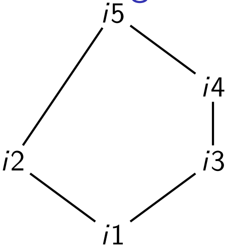

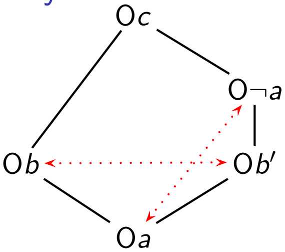

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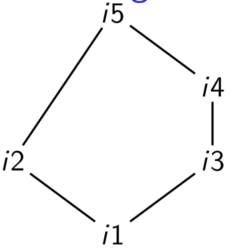

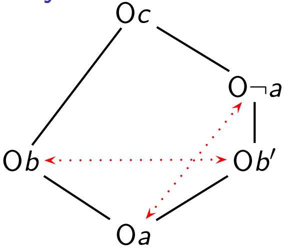

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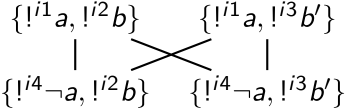

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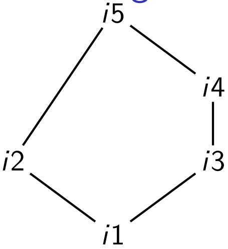

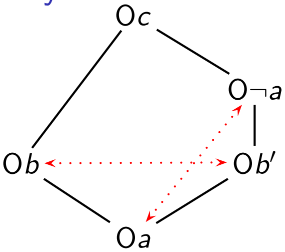

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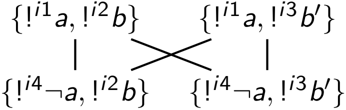

- we get: $\mathsf{O}a, \mathsf{O}(b \vee b'), \mathsf{O}c$

More complex combinations

- ${}_{i}\bullet_{j}OA$
- i indicates time
- j indicates the degree of authority

The dyadic case: Illustration of the main idea

Factual Input

A_1

A_2

A_3

$\vdots$

Conditional Input

$\bullet O(A_1, D_1)$

$\bullet O(A_2, D_2)$

$\bullet O(A_3, D_3)$

$\vdots$

Constraints

$\Box C_1$

$\Box C_2$

$\Box C_3$

$\vdots$

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$\bullet \bigcirc(A_3, D_3)$

$\vdots$

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A_3

$\vdots$

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$\bullet \mathrm{O}(A_3, D_3)$

$\vdots$

Constraints

$\Box C_1$

$\Box C_2$

$\Box C_3$

$\vdots$

actualize
"as much as
possible"

Actualized Conditionals

$\mathrm{O}(A_2, D_2)$

$\vdots$

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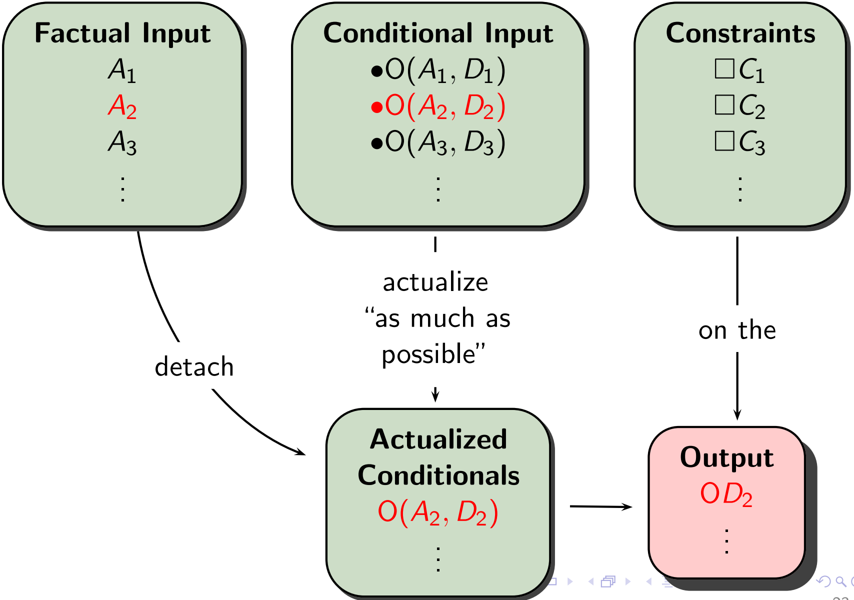

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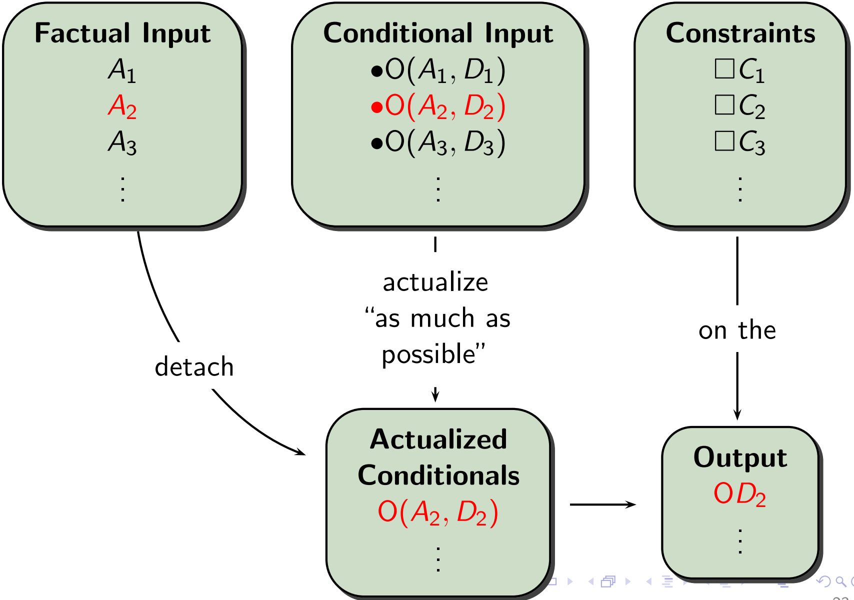

Lower Limit Logic

- mostly as for the monadic case

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- some axioms characterizing $\mathsf{O}(A, B)$ such as:

$$\vdash \mathsf{O}(A, B) \wedge \mathsf{O}(A', B) \supset \mathsf{O}(A \vee A', B)$$

Simple example for reliability

$\Gamma =$
$\{i_1 \wedge i_2, i_3, \bullet\mathsf{O}(i_1, a \wedge b), \bullet\mathsf{O}(i_2, \neg a \wedge b), \bullet\mathsf{O}(i_3, c), \bullet\mathsf{O}(i_1, \neg d), \Box d\}$.

1	$i_1 \wedge i_2$	PREM	$\emptyset$
2	i_1	1; RU	$\emptyset$
3	i_3	PREM	$\emptyset$
4	$\bullet\mathsf{O}(i_1, a \wedge b)$	PREM	$\emptyset$
5	$\bullet\mathsf{O}(i_2, \neg a \wedge b)$	PREM	$\emptyset$

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4	$\bullet O(i_1, a \wedge b)$	PREM	$\emptyset$
5	$\bullet O(i_2, \neg a \wedge b)$	PREM	$\emptyset$
6	$!(i_1, a \wedge b) \vee !(i_2, \neg a \wedge b)$	1,4,5; RU	$\emptyset$

$!(A, B) =_{\mathrm{df}} \bullet O(A, B) \wedge \neg O(A, B)$

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6	$!(i_1, a \wedge b) \vee !(i_2, \neg a \wedge b)$	1,4,5; RU	$\emptyset$
^{6}7	$\mathsf{O}(i_1, a \wedge b)$	4;RC	$\{!(i_1, a \wedge b)\}$
^{6}8	$\mathsf{O}(a \wedge b)$	2,4; RC	$\{!(i_1, a \wedge b)\}$

Simple example for reliability

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6	$!(i_1, a \wedge b) \vee !(i_2, \neg a \wedge b)$	1,4,5; RU	$\emptyset$
^{6}7	$O(i_1, a \wedge b)$	4;RC	$\{!(i_1, a \wedge b)\}$
^{6}8	$O(a \wedge b)$	2,4; RC	$\{!(i_1, a \wedge b)\}$
9	$\bullet O(i_3, c)$	PREM	$\emptyset$
10	Oc	3,9; RC	$\{!(i_3, c)\}$

Simple example for reliability

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$\{i_1 \wedge i_2, i_3, \bullet O(i_1, a \wedge b), \bullet O(i_2, \neg a \wedge b), \bullet O(i_3, c), \bullet O(i_1, \neg d), \Box d\}.$

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4	$\bullet O(i_1, a \wedge b)$	PREM	$\emptyset$
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6	$!(i_1, a \wedge b) \vee !(i_2, \neg a \wedge b)$	1,4,5; RU	$\emptyset$
^{6}7	$O(i_1, a \wedge b)$	4;RC	$\{!(i_1, a \wedge b)\}$
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10	Oc	3,9; RC	$\{!(i_3, c)\}$
11	$\bullet O(i_1, \neg d)$	PREM	$\emptyset$
12	$O\neg d$	2,11; RU	$\{!(i_1, \neg d)\}$

Simple example for reliability

$\Gamma =$
$\{i_1 \wedge i_2, i_3, \bullet\mathsf{O}(i_1, a \wedge b), \bullet\mathsf{O}(i_2, \neg a \wedge b), \bullet\mathsf{O}(i_3, c), \bullet\mathsf{O}(i_1, \neg d), \Box d\}$.

1	$i_1 \wedge i_2$	PREM	$\emptyset$
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9	$\bullet\mathsf{O}(i_3, c)$	PREM	$\emptyset$
10	$\mathsf{O}c$	3,9; RC	$\{!(i_3, c)\}$
$^{15}11$	$\bullet\mathsf{O}(i_1, \neg d)$	PREM	$\emptyset$
12	$\mathsf{O}\neg d$	2,11; RU	$\{!(i_1, \neg d)\}$
13	$\Box d$	PREM	$\emptyset$
14	$\neg\mathsf{O}\neg d$	13; RU	$\emptyset$
15	$!(i_1, \neg d)$	1,11,14; RU	$\emptyset$

Excursus: Input/Output logics

Naive approach to produce output

$$\boxed{\mathrm{out}(\mathcal{G}, \mathcal{A}) = Cn\{B \mid \text{for some } A \in Cn_{\mathbf{CL}}(\mathcal{A}), (A, B) \in \mathcal{G}\}}$$

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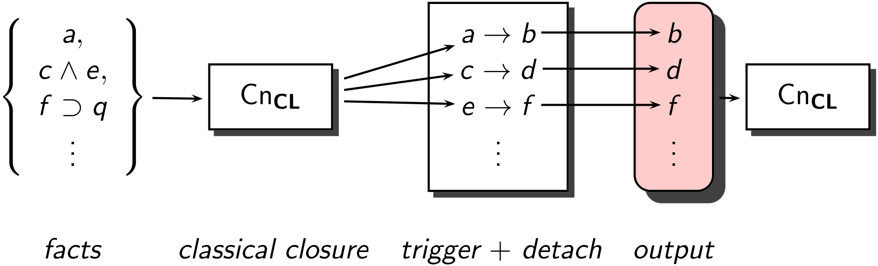

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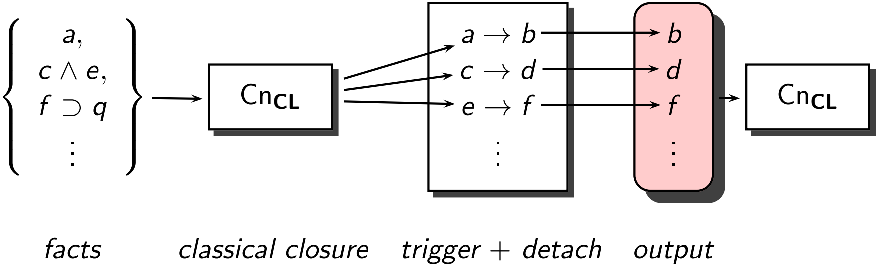


Problem: conflicting output (i.e., conflicting obligations or conflicting with constraints) Solution: work with consistent chunks

- we have representational theorems for all the 8 standard I/O-systems with constraints
- hence, we provide a proof theory for I/O-logics

Summary

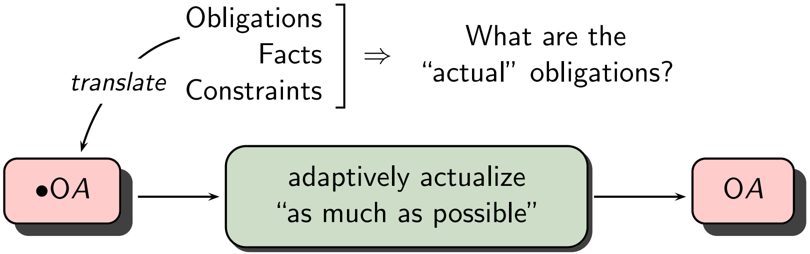
$$\left.\begin{array}{r}\text{Obligations}\\ \text{Facts}\\ \text{Constraints}\end{array}\right] \Rightarrow \begin{array}{c}\text{What are the}\\ \text{“actual” obligations?}\end{array}$$

Summary

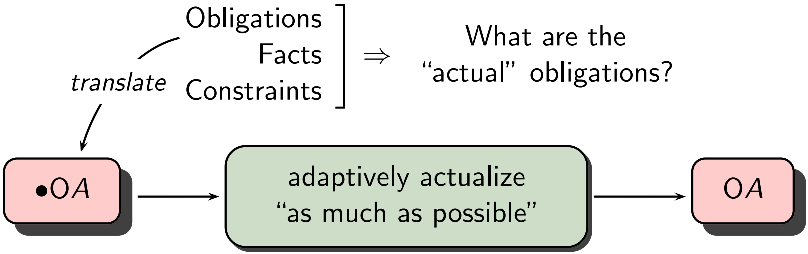
$$\left.\begin{array}{r}\text{Obligations}\\ \text{Facts}\\ \text{Constraints}\end{array}\right] \Rightarrow \begin{array}{c}\text{What are the}\\ \text{“actual” obligations?}\end{array}$$

$\bullet OA$

Summary

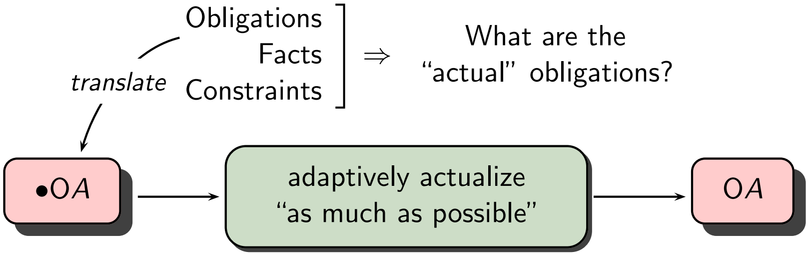

Obligations
Facts
Constraints
translate
⇒
What are the
"actual" obligations?
•OA
adaptively actualize
"as much as possible"
OA

Summary



- adaptive strategies disambiguate this

Summary



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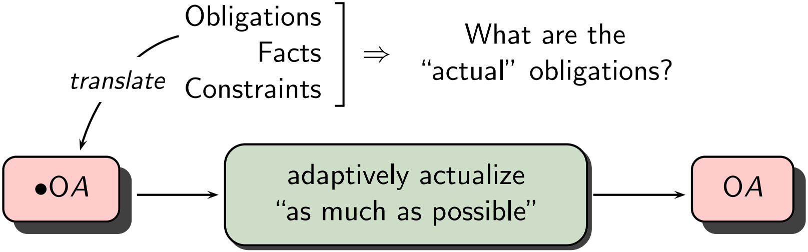

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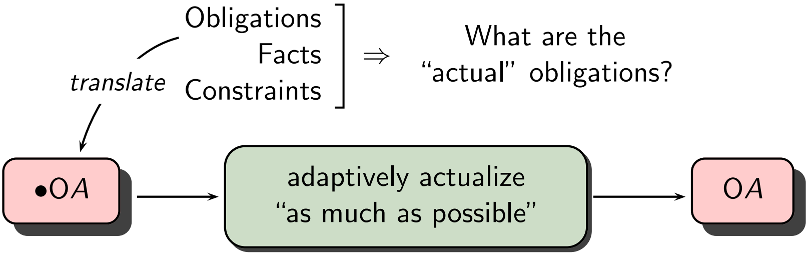

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- various strengthenings and enhancements (time, degree of authority, etc.)

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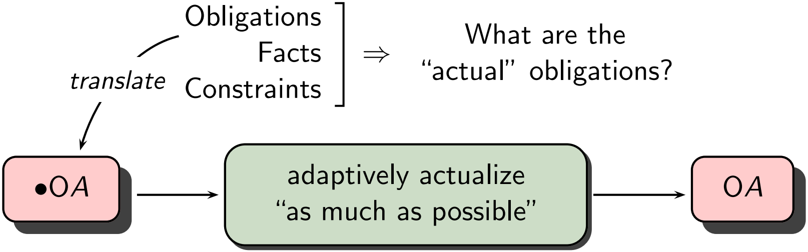

- adaptive strategies disambiguate this
- more or less cautious variants
- monadic or dyadic variant
- various strengthenings and enhancements (time, degree of authority, etc.)
- representational theorems for approaches ala Rescher/Manor, Horty, and I/O-logic with constraints