On the meaning of decidability issues in dependent types for the problem of output correctness

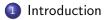
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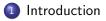


- 2 Dependent and Subtypes
- Operation 1 Proof-checking and Type-reconstruction



(Modal) Correctness & Interaction

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Proof-checking and Type-reconstruction



4 (Modal) Correctness & Interaction

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The background

- B.C. Smith, "Limits of correctness in computers", (1994): can computer systems satisfy correctly their designers aim?
 - the use of models in the construction of computer systems;
 - 2 levels of abstractions dealt with by models;
 - opartiality of representations by models;
 - the role of feedback in judging models;

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The Question

- Syntactic correctness: is it possible to formulate correct structural procedures to satisfy given specifications?
 - an appropriate language: dependent types (embedded operational semantics + treatment of information sources);
 - Iimits of correctness: decidability;
 - useful extensions: accessibility, feedback, multiple sources;

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2 Dependent and Subtypes

Proof-checking and Type-reconstruction



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Dependent Types: some known facts

- Curry-Howard Isomorphism: propositions-as-types and proofs-as-terms identities;
- Dependent Types: extension to the first-order setting, functional language (MLTT; LF);
- The program-meets-specification variant: dependency as routine-subroutines relation;
 - +: description of more complex programs;
 - +: more precise typing procedure, less bad-behaved terms;
 - -: increasing of computational information: complicated encoding.

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Dependent Types (2)

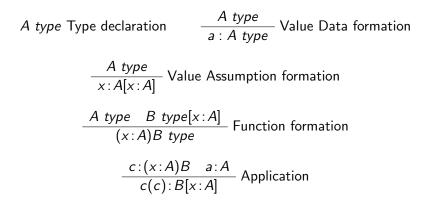
Definition (Language)

- *A*, *B*, · · · := *types*: specifications of possible values computable by a program;
- *a*, *b*, · · · := *terms*: instances of programs;
- $\Gamma, \Delta, \cdots := contexts$: subroutines;
- *a*: *A* := typed term declaration;
- [x : A] := variable declaration;
- **b** : **B**[x : A]: dependent terms are interpreted as programs calling subrotines;
- b: B[x/a: A]: substitution is the satisfaction of the call at runtime for the dependent routine.

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The Language



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The Language (2)

$$\frac{a:A \quad b:B[x:A]}{(x)b:B[x:A]}$$
 Abstraction
$$\frac{a:A \quad b:B}{(a,b):A \land B}$$
 Conjunction
$$\frac{a:A}{(a,b):A \land B} \quad \frac{b:B}{r(b):A \lor B}$$
 Disjunction
$$\frac{x:A \vdash b:B(x)}{(x)b:(\forall x:A)B(x)}$$
 Universal quantification
$$\frac{a:A \quad b:B(a)}{(a,b):(\exists x:A)B(x)}$$
 Existential quantification

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A simple Example

```
    A typed function to sort lists
```

```
sort:NatList => Sortedlist
let Sortedlist:=
match Natlist with
[] => []
l (x::Natlist) => let Sortedlist := <1,p>
l := Natlist insert p:Sorted l
```

 the type of functions mapping lists of natural numbers to sorted lists of natural numbers

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Subtyping: explicit vs. implicit information (cf. Turner (2007))

• Dependent Types as hidden computational information:

 $(\forall x : A, \exists y : B)S(x, y)$ – for each unvalued term x one gets a pair (x, y) depending on x, containing a proof plus related computational information

• Subtypes as explicit counterpart:

the pair (f, p), with program f and proof p that f is of type S(f), hence the existential type $(\exists x : [A] \Rightarrow [B])S(x)$;

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Language with Subtypes

$$\frac{A \ type \qquad B(x) \ type[x:A]}{\{x:A \mid B\} \ type}$$
Subset Formation
$$\frac{a:A \qquad b:B[x/a]}{a:\{x:A \mid B(x)\}}$$
Subset introduction
$$\frac{a:\{x:A \mid B(x)\} \qquad c(x):C(x)[x:A;y:B]}{c(a):C(a)}$$
Subset elimination
$$\frac{a:A[\Gamma] \qquad a:\{x:A \mid B(x)\}[\Gamma]}{b:B[\Gamma]}$$
Dependent Subsumption

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Requirements on Program-meets-Specification

well-formedness:

 each involved value is well-formed (with subtyping, computation depends predicatively on type formation, impredicatively by universes or kinding);

2 termination:

 β-η-conversion rules are needed on components terms (termination property for routines);

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Dependent Types and Output Correctness

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Type-checking

• The efficiency of the program is based on the evaluation mechanism for the system. General formulation of the proof-checking problem:

Definition (Type-checking Problem)

Given a context Γ , term *a* and type *A*, is $\Gamma \vdash a: A$ a derivable expression?

Type-checking (2)

 In a dependent type format, accessibility of all x': A' in Γ is formally expressed as Type-reconstruction

Definition (Type-reconstruction Problem)

Given a term *a*, there exists a type *A* and a dependency context Γ such that $\vdash a : A[\Gamma]$ is a derivable expression?

Typability and Type-checking in Simple Types

• Typability and type-checking equivalent to unification, decidable properties:

ex. let a : A and b : B, any typing of x(yb)(y(fa)) forces $f : A \to B$.

• Inhabitation: to answer $\Gamma \vdash ?: A$, apply one of the following tactics:

- For $A = B \rightarrow C$, ask if $\Gamma, B \vdash ? : C$;
- For A = C pick $B_1 \rightarrow \cdots \rightarrow B_k \rightarrow C$ from Γ , where $k \ge 0$, then ask if $\Gamma \vdash ?: B_i$, for all *i*.

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Typability and Type-checking in Dependent Types

- Type-inhabitation and Type-reconstruction are undecidable properties: require *explicit* accessibility on contextual data;
- Type-checking not perfomed on the type of variables: soundness presupposes well-formed contexts examples: *Cayenne*, *DependentML*;
- Typability is decidable with β-reduction on all formulas plus a lemma on the reducibility of contexts or dependency-erasing functions (example: λP);

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Proof-checking and Type-reconstruction



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Modal Correctness

- Correctness on input is characterized by a full treatment of computational information;
 - reconstruction on abstracted information / termination on procedures
 - example: the model of completely presented types in Turner (1993)
- further solution: expressing correctness of an algorithm wrt its subroutines accessibility:
 - can all the subroutines be executed at runtime?
 - at which level of subprocesses does the program fail?

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Using modal contexts: labelling formulas

- a: A[□(x': A')] = the program a satisfies specification A by calling subroutine for specification A' evaluated at runtime in any context (and can be used safely by any other routine);
- a : A[◊(x : A')] = the program a satisfies specification A by calling subroutine for specification A' evaluated only in the present context (cannot be used safely by other routines).

Here "safely" means "without risk of incurring in loops".

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Remarks on using modal contexts

- the judgmental interpretation of □/◊J is not trivial (non propositional);
- meaning dependent on introduction/elimination rules for modalities;
- modalities from context are preserved to index construction of a staged program;
- growing formal literature (ex: Pfenning 2001); applications to code mobility (Moody 1993) and staged computation (Nanevski et al. 2008).

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Levels of failure (1)

Internal information failure: "which step in the program execution (routines, calls for sub-routines) fails?"

Definition (Internal Levels Of Failure)

IL1 correctness by subcalls recursion (accessibility);

IL2 correctness by termination procedures (evaluation at runtime).

Levels of failure (2)

External information failure: "which data is missing or fails on dependency, so that the termination process fails?"

Definition (External Levels Of Failure)

IL3 correctness by data dependency (well-formedness on dependency); IL4 correctness by data retrieval (failure-with-world).

Interaction

- prevention of program failure is syntactically based on completeness of data;
- control on modal format triggers the issue of human-machine connection as an higher level of reliability;
- further extension: priority relations on terminations.

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Conclusion

- "no [...] social process can take place among program verifiers" (De Millo et al. 1979)
- dependent programming offers ways to implement them.