Generalized Conversational Relevance.

Relevance Conditions for Asserting Disjunctions.

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Outline

1. Introduction
   - Gricean Pragmatics
   - Generalized Conversational Relevance
   - Relevance Conditions for Asserting Disjunctions
   - Distinctive Properties of these Relevance Conditions
   - Aim of this talk
2. The Adaptive Logics Approach
   - Introduction
   - The Lower Limit Logic
   - Representing Relevantly Assertable Sentences
   - The Adaptive Logic $\mathbf{RIT^s}$
   - Appendix
3. Conclusion

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Introduction

Gricean Pragmatics

The Cooperative Principle

Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1989, p. 26)

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- These specify the main characteristics of communicative acts governed by the Cooperative Principle.

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  - = by reconciling seemingly uncooperative assertions with the Cooperative Principle.

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    $\Rightarrow$ the more cooperative an assertion, the easier a hearer will be able to grasp its meaning, notice it,...

I will focus on speakers!

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Be relevant!

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↔ Particularized Conversational Relevance

- = The relevance conditions that refer to the extra–linguistic context, e.g. the shared background knowledge of speaker and hearer.

= analogous to the distinction between particularized and generalized *conversational implicatures* (hearer's perspective)

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  - Otherwise, the speaker isn't as informative as she could be.

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For an atomic disjunction $A \vee B$ to be relevantly assertable, two conditions have to be satisfied:

- Neither A nor B may be known by the speaker.
  - Otherwise, the speaker isn't as informative as she could be.
- The speaker has to know whether A and B are co–consistent (i.e. whether $A \wedge B$ is consistent).
  - If A and B are not co–consistent, $A \vee B$ is a tautology.

    $\Rightarrow$ informational content = empty

A Relevantly Assertable Atomic Disjunction

A speaker s may assert an atomic disjunction $A \vee B$ in case (1) she knows that $A \vee B$ is the case, (2) she doesn't know that A is the case, (3) she doesn't know that B is the case, and (4) she knows that $A \wedge B$ is consistent.

Introduction

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A Relevantly Assertable Formula

A speaker s may assert a formula A in case the conditions (1)–(4) of atomic disjunctions are satisfied for all disjunctive subformulas of A.

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- People may gain a better insight in what they already know (i.e. people are not logically omniscient).

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- New information may become available.
  - = Non–monotonicity!
- People may gain a better insight in what they already know (i.e. people are not logically omniscient).
  - = A proof theoretic feature (not a metatheoretic one)!

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- [Appendix: I will discuss the related approach of Verhoeven (2007).]

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Adaptive Logics?

Adaptive Logics are formal logics that were developed to explicate dynamic (reasoning) processes (both monotonic and non–monotonic ones).

e.g. Induction, abduction, default reasoning,...

The Adaptive Logics Approach

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$\hookrightarrow$ The *lower limit logic* of the logic **RIT**^{s}.

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The Adaptive Logics Approach

The Lower Limit Logic

The logic **KC** is a standard bimodal logic!

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The Modal Language Schema of **KC**

Language	*Letters*	*Log. Symbols*	*Def. Symbols*	*Set of Formulas*
$\mathcal{L}$	$\mathcal{S}$	$\neg, \wedge, \vee$	$\supset, \equiv$	$\mathcal{W}$
$\mathcal{L}^{\mathcal{M}}$	$\mathcal{S}, \bot$	$\neg, \wedge, \vee, K, C$	$\supset, \equiv$	$\mathcal{W}^{\mathcal{M}}$

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Two Modal (Necessity) Operators

- *KA* will be used to express that the formula *A* is *known* by the speaker.
- *CA* will be used to express that the formula *A* is *consistent*.

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Remark: The corresponding "possibility" operators are left out!

Proof Theory of **KC**

= the axiom system of **CL**, extended by the following (modal) axiom schemas

The Adaptive Logics Approach

Proof Theory of **KC**

= the axiom system of **CL**, extended by the following (modal) axiom schemas

MAK1	$K(A \supset B) \supset KA \supset KB$	MAC1	$C(A \supset B) \supset CA \supset CB$
NECK	From $\vdash A$ follows $\vdash KA$	NECC	From $\vdash A$ follows $\vdash CA$
MAK2	$KA \supset A$		
MAK3	$KA \supset KKA$		
MAK4	$A \supset K\neg K\neg A$		

$A\bot$	$\bot \supset A$

The Adaptive Logics Approach

The Lower Limit Logic

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  - R^K is a reflexive, symmetric and transitive accessibility relation,
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  - $v : \mathcal{S} \times W \mapsto \{0, 1\}$ is an assignment function.

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- The assignment function v of M is extended to a valuation function v_M in the usual way.
  - $v_M(KA, w) = 1$ iff, for all $w' \in W$, if $R^K ww'$ then $v_M(A, w') = 1$.
  - $v_M(CA, w) = 1$ iff, for all $w' \in W$, if $R^C ww'$ then $v_M(A, w') = 1$.

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There is no relation between the accessibility relations R^K and R^C!

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The Adaptive Logics Approach

Representing Relevantly Assertable Sentences

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  - $g(\neg A) = \neg g^*(A)$
  - $g(A \wedge B) = g(A) \wedge g(B)$
  - $g(A \vee B) = (g(A) \vee g(B)) \wedge \neg K(A) \wedge \neg K(B) \wedge KC(A \wedge B)$

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  - $g^*(A \wedge B) = (g^*(A) \wedge g^*(B)) \vee K(\neg A) \vee K(\neg B) \vee \neg KC\neg(A \vee B)$
  - $g^*(A \vee B) = g^*(A) \vee g^*(B)$

Representing a Knowledge Base

$\Gamma^K = \{KA \mid A \in \mathcal{W}\}$.

The Adaptive Logics Approach

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The formula $A \in \mathcal{W}$ is relevantly assertable by a speaker s with knowledge base Γ^K iff $\Gamma^K \vdash_{\mathbf{RIT^s}} K(g(A))$.

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The Adaptive Logics Approach

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In the following, premise sets will be restricted to knowledge bases!

$\Rightarrow \Gamma^{\not K}$

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General Characterization

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1. Lower Limit Logic (**LLL**)
2. Set of Abnormalities Ω
3. Adaptive Strategy

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1. Lower Limit Logic (**LLL**): the logic **KC**
2. Set of Abnormalities Ω
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The Adaptive Logics Approach

The Adaptive Logic **RIT^s**

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$$\Omega^K = \{KA \mid A \in \mathcal{W}\}$$

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3. Adaptive Strategy

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$

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Defeasible Inference Steps?

$$\frac{\Gamma \vdash_{\mathbf{LLL}} B \vee A \qquad (A \in \Omega)}{\Gamma \vdash_{\mathbf{LLL}} B \qquad (\textit{unless } A \text{ cannot be interpreted as false})}$$

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$\hookrightarrow =$ in case $\Gamma \vdash_{\mathbf{LLL}} Dab(\{A\} \cup \Delta)$

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i.e. $\Gamma \vDash_{\mathbf{RIT^s}} A$ iff A is verified by all elements of *some* selected sets of preferred **KC**–models of Γ.

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The Selected Sets of **KC**–Models of a Premise Set Γ

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  - $Ab(M) = \{A \in \Omega \mid A \text{ is verified by } M\}$.
- A **KC**–model M of Γ is a *minimally abnormal* model of Γ iff there is no **KC**–model M' of Γ such that $Ab(M') \subset Ab(M)$.
- All minimally abnormal **KC**–models of Γ that verify the same abnormalities are grouped together in distinct sets.
  = The selected sets of **KC**–models of Γ!

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$: Proof Theory (1)

General Features

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- Deduction Rules
- Marking Criterium
  - Dynamic proofs

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Deduction Rules

PREM — If $A \in \Gamma$:

$$\begin{array}{ll} \ldots & \ldots \\ \hline A & \emptyset \end{array}$$

RU — If $A_1, \ldots, A_n \vdash_{\mathbf{KC}} B$:

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}$$

RC — If $A_1, \ldots, A_n \vdash_{\mathbf{KC}} B \vee Dab(\Theta)$

$$\begin{array}{ll} A_1 & \Delta_1 \\ \vdots & \vdots \\ A_n & \Delta_n \\ \hline B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}$$

Definition

$Dab(\Delta) = \bigvee(\Delta)$ for $\Delta \subset \Omega$.

Marking Criterium: Normal Selections Strategy

- *Dab*–consequences

  $Dab(\Delta)$ is a *Dab*–consequence of Γ at stage s of the proof iff $Dab(\Delta)$ is derived at stage s on the condition $\emptyset$.

Marking Criterium: Normal Selections Strategy

- *Dab*–consequences

  Dab(Δ) is a *Dab*–consequence of Γ at stage s of the proof iff *Dab*(Δ) is derived at stage s on the condition $\emptyset$.

- Marking Definition

  Line i is marked at stage s of the proof iff, where Δ is its condition, *Dab*(Δ) is a *Dab*–consequence at stage s.

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$: Proof Theory (4)

Derivability

A is derived from Γ at stage s of a proof iff A is the second element of an unmarked line at stage s.

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Derivability

A is derived from Γ at stage *s* of a proof iff *A* is the second element of an unmarked line at stage *s*.

Remark: Derivability is stage–dependent

$\Rightarrow$ Problematic: markings may change at every stage!

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$: Proof Theory (4)

Derivability

A is derived from Γ at stage s of a proof iff A is the second element of an unmarked line at stage s.

Remark: Derivability is stage–dependent

$\Rightarrow$ Problematic: markings may change at every stage!

Final Derivability

- A is finally derived from Γ on a line i of a proof at stage s iff (i) A is the second element of line i, (ii) line i is not marked at stage s, and (iii) every extension of the proof in which line i is marked may be further extended in such a way that line i is unmarked.
- $\Gamma \vdash_{\mathbf{RIT^s}} A$ iff A is finally derived on a line of a proof from Γ.

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The Adaptive Logics Approach

The Adaptive Logic **RIT$^{\mathbf{s}}$**: Example 1

The Knowledge Base

$\Gamma = \emptyset$

Example

The Knowledge Base

$\Gamma = \emptyset$

Example

1	$\neg K(p)$	–;RC	$\{K(p)\}$
2	$\neg K(\neg p)$	–;RC	$\{K(\neg p)\}$
3	$KC(p \wedge \neg p)$	–;RC	$\{\neg K \neg (C(p \wedge \neg p) \supset C\bot)\}$

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The Adaptive Logics Approach

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Example

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2	$\neg K(\neg p)$	–;RC	$\{K(\neg p)\}$
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4	$K(g(p \vee \neg p))$	1,2,3;RU	$\Delta_1 \cup \Delta_2 \cup \Delta_3$
5	$\neg K \neg (C(p \wedge \neg p) \supset C\bot)$	–;RU	$\emptyset$

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6	$Dab(\Delta_1 \cup \Delta_2 \cup \Delta_3)$	5;RU	$\emptyset$

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The Adaptive Logics Approach

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2	$\neg K(\neg p)$	–;RC	$\{K(\neg p)\}$
$3\checkmark_4$	$KC(p \wedge \neg p)$	–;RC	$\{\neg K\neg(C(p \wedge \neg p) \supset C\bot)\}$
$4\checkmark_5$	$K(g(p \vee \neg p))$	1,2,3;RU	$\Delta_1 \cup \Delta_2 \cup \Delta_3$
5	$\neg K\neg(C(p \wedge \neg p) \supset C\bot)$	–;RU	$\emptyset$
6	$Dab(\Delta_1 \cup \Delta_2 \cup \Delta_3)$	5;RU	$\emptyset$

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The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$: Example 2

The Knowledge Base

$\Gamma = \{K(p \vee \neg(q \wedge r)), K(\neg q \vee \neg r)\}$

Example

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT}^{\mathbf{s}}$: Example 2

The Knowledge Base

$\Gamma = \{K(p \vee \neg(q \wedge r)), K(\neg q \vee \neg r)\}$

Example

1	$K(p \vee \neg(q \wedge r))$	–;PREM	$\emptyset$
2	$K(\neg q \vee \neg r)$	–;PREM	$\emptyset$

The Adaptive Logics Approach

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The Knowledge Base

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1	$K(p \vee \neg(q \wedge r))$	–;PREM	$\emptyset$
2	$K(\neg q \vee \neg r)$	–;PREM	$\emptyset$
3	$\neg K(p)$	–;RC	$\{K(p)\}$
4	$\neg K(\neg(q \wedge r))$	–;RC	$\{K(\neg(q \wedge r))\}$
5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
6	$\neg K(\neg r)$	–;RC	$\{K(\neg r)\}$

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1	$K(p \vee \neg(q \wedge r))$	–;PREM	$\emptyset$
2	$K(\neg q \vee \neg r)$	–;PREM	$\emptyset$
3	$\neg K(p)$	–;RC	$\{K(p)\}$
4	$\neg K(\neg(q \wedge r))$	–;RC	$\{K(\neg(q \wedge r))\}$
5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
6	$\neg K(\neg r)$	–;RC	$\{K(\neg r)\}$
7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg(C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
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3	$\neg K(p)$	–;RC	$\{K(p)\}$
4	$\neg K(\neg(q \wedge r))$	–;RC	$\{K(\neg(q \wedge r))\}$
5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
6	$\neg K(\neg r)$	–;RC	$\{K(\neg r)\}$
7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg(C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
8	$KC(\neg q \wedge \neg r)$	–;RC	$\{\neg K \neg(C(\neg q \wedge \neg r)) \supset C\bot)\}$
9	$K(g(p \vee \neg(q \wedge r)))$	1–8;RU	$\Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_6 \cup \Delta_7 \cup \Delta_8$
10	$K(g(\neg q \vee \neg r))$	2,5,6,8;RU	$\Delta_5 \cup \Delta_6 \cup \Delta_8$

The Adaptive Logics Approach

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5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
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7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg(C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
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11	$K(\neg(q \wedge r))$	2;RU	$\emptyset$

The Adaptive Logics Approach

The Adaptive Logic $\mathbf{RIT^s}$: Example 2

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2	$K(\neg q \vee \neg r)$	–;PREM	$\emptyset$
3	$\neg K(p)$	–;RC	$\{K(p)\}$
4 $\checkmark_{11}$	$\neg K(\neg(q \wedge r))$	–;RC	$\{K(\neg(q \wedge r))\}$
5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
6	$\neg K(\neg r)$	–;RC	$\{K(\neg r)\}$
7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg(C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
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7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg (C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
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9	$K(g(p \vee \neg(q \wedge r)))$	1–8;RU	$\Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_6 \cup \Delta_7 \cup \Delta_8$
10	$K(g(\neg q \vee \neg r))$	2,5,6,8;RU	$\Delta_5 \cup \Delta_6 \cup \Delta_8$
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The Adaptive Logics Approach

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1	$K(p \vee \neg(q \wedge r))$	–;PREM	$\emptyset$
2	$K(\neg q \vee \neg r)$	–;PREM	$\emptyset$
3	$\neg K(p)$	–;RC	$\{K(p)\}$
$4\checkmark_{11}$	$\neg K(\neg(q \wedge r))$	–;RC	$\{K(\neg(q \wedge r))\}$
5	$\neg K(\neg q)$	–;RC	$\{K(\neg q)\}$
6	$\neg K(\neg r)$	–;RC	$\{K(\neg r)\}$
7	$KC(p \wedge \neg(q \wedge r))$	–;RC	$\{\neg K \neg(C(p \wedge \neg(q \wedge r)) \supset C\bot)\}$
8	$KC(\neg q \wedge \neg r)$	–;RC	$\{\neg K \neg(C(\neg q \wedge \neg r)) \supset C\bot)\}$
$9\checkmark_{12}$	$K(g(p \vee \neg(q \wedge r)))$	1–8;RU	$\Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_6 \cup \Delta_7 \cup \Delta_8$
10	$K(g(\neg q \vee \neg r))$	2,5,6,8;RU	$\Delta_5 \cup \Delta_6 \cup \Delta_8$
11	$K(\neg(q \wedge r))$	2;RU	$\emptyset$
12	$Dab(\Delta_3 \cup \Delta_4 \cup \Delta_5 \cup \Delta_6 \cup \Delta_7 \cup \Delta_8)$	11;RU	$\emptyset$

Outline

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Appendix

The Logic **RAD** (Verhoeven, 2007)

Semantic Characterization of the Disjunction

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The Logic **RAD** (Verhoeven, 2007)

Semantic Characterization of the Disjunction

For S a set of **CL**–models:

- **CL**–Characterization:
  - $S \vDash_{\mathbf{CL}} A \vee B$ iff S has a partition (S_1, S_2), such that
    - $S_1 \vDash_{\mathbf{CL}} A$, and
    - $S_2 \vDash_{\mathbf{CL}} B$.

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Semantic Characterization of the Disjunction

For S a set of **CL**–models:

- **RAD**–characterization:
  - $S \vDash_{\mathbf{RAD}} A \vee B$ iff
    - S has a partition (S_1, S_2), such that
      - $S_1 \vDash_{\mathbf{CL}} A$, and
      - $S_2 \vDash_{\mathbf{CL}} B$.
    - For all partitions (S_1, S_2) of S for which $S_1 \vDash_{\mathbf{CL}} A$ and $S_2 \vDash_{\mathbf{CL}} B$,
      - $S_1 \vDash_{\mathbf{RAD}} A$, and
      - $S_2 \vDash_{\mathbf{RAD}} B$.
    - $S \nvDash_{\mathbf{CL}} A$, and
    - $S \nvDash_{\mathbf{CL}} B$.

Comparison with the **RIT**^{s}–approach

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Hypothesis: Both approaches are equivalent, in case

- Ω is restricted to Ω^K, and
- the functions g and $g^\star$ are defined in a slightly different way.

Comparison with the **RIT**^{s}–approach

Hypothesis: Both approaches are equivalent, in case

- Ω is restricted to Ω^K, and
- the functions g and $g^\star$ are defined in a slightly different way.

$\Rightarrow$ It is possible to provide a standard adaptive logic characterization of the logic **RAD**.

Problem for the **RAD**–approach

Some formulas for which the informational content is empty are **RAD**–derivable.

EXAMPLE: $\vdash_{\textbf{RAD}} p \vee \neg p$

Appendix

The Logic **RAD** (Verhoeven, 2007)

Problem for the **RAD**–approach

Some formulas for which the informational content is empty are **RAD**–derivable.

EXAMPLE: $\vdash_{\mathbf{RAD}} p \vee \neg p$

JUSTIFICATION: "*This is completely in accordance with Grice's theory of conversation, which interprets an assertion of $p \vee \neg p$ in standard contexts as containing the conversational meaning that the speaker does not know whether p or $\neg p$ is the case and therefore considers $p \vee \neg p$ worth asserting (in the appropriate context).*" (Verhoeven, 2007, p. 360)

Appendix

The Logic **RAD** (Verhoeven, 2007)

Problem for the **RAD**–approach

Some formulas for which the informational content is empty are **RAD**–derivable.

EXAMPLE: $\vdash_{\mathbf{RAD}} p \vee \neg p$

HOWEVER: This justification refers to some of the reasoning processes of the hearer, while **RAD** was developed to capture some of the reasoning processes of the speaker.

$\Rightarrow$ The **RAD**–approach confuses the perspective of the speaker with the perspective of the hearer.

Problem for the **RAD**–approach

Some formulas for which the informational content is empty are **RAD**–derivable.

EXAMPLE: $\vdash_{\mathbf{RAD}} p \vee \neg p$

MOREOVER: All Grice's maxims may be ignored by the speaker in *an appropriate context*!

$\Rightarrow$ If this is taken into account, you may as well stick to **CL**!

Conclusion

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The relevance conditions for asserting disjunctions can be captured formally by relying on the adaptive logics approach.

= by means of the adaptive logic $\mathbf{RIT}^{s}$

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The relevance conditions for asserting disjunctions can be captured formally by relying on the adaptive logics approach.

= by means of the adaptive logic $\mathbf{RIT^s}$

Further Research

- To extend the approach to relevance conditions related to other connectives.
- To extend the approach to other Gricean maxims.

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