Weighing Falling Bodies.
Galileo's Thought Experiment in the Development of his Dynamical Thinking.

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CONTENTS

"Galileo's mechanics returned to the dynamics of free fall at every critical point, and attempted to illuminate the whole of mechanics with its light."[1]

1. Introduction: The intelligibility of dynamics – the dynamics of intelligibility

In the present paper I will analyze Galileo's continuous attempts to come to grips with the dynamics of falling bodies. The central claim of the paper is that Galileo's famous thought experiment on the speed of falling bodies played different but equally crucial roles in these attempts at different stages of his thinking. Before entering in the detailed analyses intended to substantiate this claim, I will first have to make clear what I take to be involved in the expressions "coming to grips with" and "the dynamics of falling bodies."

In a paper from 1993, Alan Gabbey urged us not to use the term "dynamics" too lightly when considering the endeavours of sixteenth and seventeenth century mathematicians and natural philosophers. Not only was this term not in use before its (idiosyncratic) introduction by Leibniz, the disciplines of (mixed) mathematics and natural philosophy were significantly differently structured so as to make an uncritical adoption of the term with its modern connotations potentially distorting. Put more positively, paying attention to the inappropriateness of much of our modern disciplinary taxonomies might pay off in providing us with the possibility of understanding much better which were the "mechanical problems that contemporary authors saw as legitimate challenges."[2] That I nevertheless choose to speak of Galileo's dynamics and his dynamical thinking need not be opposed to this. I don't want to assume that this was a precisely delineated and pre-existing category – on the contrary, one of the main strands running through this paper is the story of the gradual development by which this category (not yet the term) could take shape through Galileo's work. As will become clear, it is as a result of his continuous involvement with his thought experiment on falling bodies that Galileo came to realize that if he wanted to understand its peculiar character he should distinguish between the causal analysis of moving bodies and of bodies constrained to remain at rest. Yet he came to realize this precisely because he was considered from the beginning with such causal analyses of motion. It is in the deliberately loose sense of investigation into the causes of the characteristics of motion that I use the term dynamics throughout this paper.

There is however more to be said about Galileo's involvement with dynamics as understood in this loose sense. This has to do with the nature of this investigation, and hence the nature of Galileo's realization that he should make such a distinction between moving bodies and bodies at rest if he wanted to *understand* certain facts about falling bodies: we need to ask ourselves what precisely is the nature of the understanding that Galileo tried to achieve. This is of course a contentious issue which

[1] Westfall 1971, p. 7.

[2] Gabbey 1993, p. 145.

can only be settled through a detailed analysis of both his pronouncements at different places in his writings and the historical context in which he produced these writings. At this place I can do no more than give a rough sketch of what I take to be the most convincing view of the matter. The analyses in the present paper will then provide some further substantiations of this preliminary sketch. As a starting point, let me first bring in some philosophical background.

Philosophers of science in the second halve of the twentieth century have been mainly interested in an analysis of the structure of scientific explanations, and tended to be rather critical about the notion of understanding which was often deemed to be too subjective to be of any real interest.[3] This is not the time and place to enter into a critical re-evaluation of these views, but let it suffice to point out that any view on explanation has to account for the status of certain basic brute facts which are apparently not in need of further explanation and hence can serve as explanatory bedrock for other phenomena. What I want to propose here is that we take seriously the idea that for any broadly conceived explanatory framework (whether it be "scientific," philosophical, or what may you have) there is always something about the proffered explanations that is responsible for them "making sense." This feature comes especially to the foreground in periods where competing frameworks struggle for the right to speak about a class of phenomena; periods where the allegation of unintelligibility is often levelled in both directions. In these circumstances it is clear that the sense of intelligibility is not merely a subjective feeling accompanying explanations, but refers to a basic way of going about in offering and receiving them. A basic way which can be shared by a large group of people and which most importantly can have a clear normative force.

Galileo's *Dialogue concerning the two chief world systems* is of course a classic in the polemic genre of undermining competing explanatory frameworks. As such it also provides us with a revealing view of the basic sense of intelligibility underlying Galileo's alternative. The Galilean Salviati's Socratic mode of questioning the Aristotelian Simplicio is always aimed at making the latter accept a set of basic facts which then can be turned against his prior convictions. What is important about this Socratic method is that in the end it is always Simplicio *himself* who makes the crucial judgements; i.e. he is always led to facts which have an incontestable character, albeit he might have had to be taught to notice them. As Sagredo explains to him:

> SAGR. … I say to you that if one does not know the truth by himself, it is impossible for anyone to make him know it. I can indeed point out things to you, things being neither true nor false; but as for the true – that is, the necessary; that which cannot possibly be otherwise – every man of ordinary intelligence either knows by himself or it is impossible for him ever to know it.[4]

[3] See Hempel 1965, p. 413 for an exemplary and influential statement of this view.

[4] Cf. *Opere* VII, p. 183. (Transl. from Galilei 2001 [1953], p. 183.)

Of immediate relevance for my purposes here is the clear implication that Galileo wants his opponents to see *what* (he claims that) *the things in the world themselves show* – it is only then that one is in the right position to discuss natural phenomena. In this way he is installing a set of facts that need not be further explained and hence open up the possibility of explaining further phenomena. What is most important is not so much the existence of such a set, but the grounds on which it is selected. Galileo wants to reduce phenomena to shared experiences which are close at hand and incontestable for "every man of ordinary intelligence." The notion of intelligibility that drives his investigations is that of having explanations that are anchored in this kind of familiar experiences.

There is of course much more to be said about this issue, but let me move on to how this connects with Galileo's investigations in the causes of the characteristics of motion, i.e. his dynamical thinking. When I said in the beginning that I want to analyze Galileo's continuous attempts to come to grips with the dynamics of falling bodies, I am referring to his attempts to render some of the characteristics of free fall intelligible through the possibility of introducing shared experiences where everybody can incontestably see the motive power of natural bodies at work. Experiences with Galileo in a first instance sought in the behaviour of bodies on a balance. This brings with it some important consequences. Firstly, the far-reaching suggestion that the things in the world themselves show their essential characteristics most clearly in our way of interacting with them. Secondly, the fact that these experiences readily lend themselves to be integrated within mathematical arguments. Peter Machamer recently introduced the notion of a *model of intelligibility* to capture the multiple functions of the balance within Galileo's thinking: "Its physical concreteness, mathematical describability, and physical manipulability leading to experimental possibilities gave intelligibility and structure to the abstract concepts of the mechanical world picture."[5] This notion is of course closely related to the kind of analysis that I am proposing here, and the present paper can hence be seen as an attempt to further spell out how this model of intelligibility directed Galileo's investigations. There is however a further and novel element to my analysis, and that is the story of how this model at the same time also became crucially transformed during the process.

It is of course one thing to have a model of intelligibility which in principle would allow you to anchor your explanations in shared and incontestable experiences, and another thing to put it fruitfully to work. This supposes that these experiences can be seamlessly integrated within the explanatory scheme, but the latter also has its own exigencies that at times potentially drive it towards another road leaving a gap between scheme and basic experiences. This is the natural result of the fact that the scheme is always supposed to explain a different and richer set of phenomena, and this is also what happened in Galileo's first attempts to come to grips with the dynamics of free fall. The first part of this paper is essentially devoted to laying bare this kind of gap in Galileo's attempted explanatory

[5] Machamer 1998, p. 71.

scheme in his youthful *De motu*. As will be seen, it is precisely in an attempt to cover up this gap that Galileo introduces his thought experiment for the first time. It hence is primarily *intended to restore intelligibility to his explanatory scheme*, rather than to provide independent empirical confirmation thereof.[6] It is in this function that it will continue to play a crucial role in Galileo's dynamical thinking. As is clear from some of his later writings, which will be analyzed in the second part of the paper, Galileo remained deeply concerned with the connection between on the one hand mechanical instruments such as the balance, and on the other hand the phenomenon of free fall; and it turns out that it is exactly the thought experiment that allowed him to mediate between both sets of phenomena. It is through rethinking the thought experiment that he was able to uncover the crucial facts that were responsible for the gap that – with hindsight – had to exist within his first attempts at natural philosophy. When we find him re-presenting the thought experiment more than forty years after its first introduction, he is precisely stressing the ambiguity it had first covered up, and apparently enjoying the confusion it causes in unprepared minds (such as those of his opponents). As will emerge from my analysis, once Galileo has reached this point, the thought experiment can truly start to function as a new model of intelligibility, hence moving the balance to a different, more restricted position within his dynamical thinking.

It is probably no accident that it was precisely a thought experiment that lay behind much of the dynamics of Galileo's thinking. Its seemingly paradoxical character still has the power to fascinate many people and the act of rethinking the thought experiment was probably stimulated by exactly this paradoxical character – with as effect that in unravelling the paradox Galileo was able to forge profound changes in his conceptual framework.[7] But the effect of this rethinking must remain hidden as long as we ignore the subtle but profound differences that exist between the different presentations of his thought experiment, as has been done up to now. As Winifred Wisan once aptly stated, however, "Galileo … lived long enough and maintained sufficient mental prowess to become in effect his own best disciple".[8] The fascinating creative process that lay behind the development of Galileo's

[6] Galileo's thought experiment has been the topic of some recent philosophical debates, but these primarily focussed on its epistemological status, and not so much on its role within Galileo's dynamical thinking. Koyré 1968 and Westfall 1966, 1971 are among the few authors who explicitly consider it from this perspective, as was already done by Mach 1960 [1893], p. 251; however, these authors also remain silent on the crucial role played by Galileo's *rethinking* of this thought experiment during different stages of his career. That is, they assume that Galileo could draw some important lessons from the thought experiment, but they do not treat the question how Galileo came to see that it implied these lessons.

[7] Paolo Palmieri has recently stressed the important cognitive role that paradoxes played for Galileo, both within his own thinking and in the presentation of his ideas (Palmieri 2005a). The present paper can also be read as a further confirmation of this claim.

[8] Wisan 1984, p. 271.

dynamical thinking – a process that spans a period of more than fifty years – bears striking witness to this fact.

PART I – Understanding weight as a dynamic factor: Ambiguities

"…quanto tutti gli altri ingegni a quello di Archimedo siano inferiori, e quanta poca speranza possa restare a qualsisia di mai poter ritrovare cose a quelle di esso simiglianti."
[…how inferior all other minds are to Archimedes's and what small hope is left to anyone of ever discovering things similar to his discoveries.][9]

2. *La Bilancetta*: Understanding mixtures and transforming gravities

Archimedes jumping out his bathtub is one of these images that have captured popular imagination. Historians of science are of course quick to point out how this is part of a romanticized image of science. It seems to have been no different at the end of the sixteenth century. The story was well known throughout the renaissance, through the numerous editions of Vitrivius's books on architecture,[10] and the image of Archimedes exposing the deceit of the goldsmith must have appealed enormously to mathematicians trying to secure their social position. After all, it was only Archimedes, through his knowledge of the principles of hydrostatics, who had been able to protect the highest authorities from being swindled by a mere artisan. However, the ones who were most self-conscious about their status as having a privileged understanding of mechanical principles were prone to be dismissive of Vitrivius's account. The method attributed by him to Archimedes falls short of the certainty and exactness of which they were capable, and which they had learned from Archimedes himself.

And so we find Galileo at age 22 tackling the problem of Hiero's crown in *La Bilancetta*, a short tract devoted solely to this problem.[11] He prides himself on having reinvented the true method that must have been used by Archimedes, having all the exactness required by the true mathematician. His solution is based on a hydrostatic balance, a device that had been used earlier to tackle this problem.[12] It is often claimed that the main interest of Galileo's manuscript lies in the technical innovations proposed with respect to the balance used.[13] Nevertheless, the theoretical treatment offered of the balance provides us with an invaluable picture of the young man attempting to gain full mastery of Archimedean hydrostatics; a mastery that he soon will be trying to exploit in building a natural philosophical treatment of motion on its basis. Crucial in this respect is the behaviour of mixtures of pure metals that lies at the heart of the solution to the crown problem. Of particular

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[9] *Opere*, I, p. 216. (Transl. from Fermi and Bernardini 1961, p. 114.)

[10] Clagett 1978, pp. 1066-1068, n.2 sketches the diffusion of the work in the renaissance; *ibid.*, pp. 1066-1085 is a useful account of the occurrences of the crown problem during the renaissance.

[11] *Opere*, I, pp. 210-220.

[12] Cf. Napolitani 1988, pp.163-164.

[13] Cf. e.g. Drake 1978, p. 6; Wallace 1984, p. 221.

interest are Galileo's peculiar handling of weight and specific weight, and his analysis of the effect of a medium on a body's weight.[14]

a. Solving the crown problem

How can we detect whether a crown of a given weight is fully made up of gold or of a mixture of gold and silver; and if a mixture, in what proportion? If we sink a body in water, it will loose weight by an amount equal to the weight of an equal volume of water (by the 7th proposition of Archimedes' first book on floating bodies). Hence, the smaller the difference between the specific weight of a metal and that of water, the more the metal will suffer a loss of weight. It is this proportionally different behaviour that Galileo wishes to exploit in determining the proportion of two different metals in one mixture. Take a sample of gold and one of silver, weigh them both in air and subsequently in water. By recording the weight-loss, one can determine the respective proportions in which gold and silver are alleviated, and hence their specific weights. Now weigh the crown in air and water, and determine the proportion in which it is alleviated. This last proportion can be related to the earlier determined proportions for the pure metals, hence fixing the proportion of gold and silver in the crown. Such is the broad outline of Galileo's method, in which he seems to follow the lines of earlier attempted solutions to the crown problem. Here is Galileo's own description:

> Let us suspend a [piece of] metal on [one arm of] a balance of great precision, and on the other arm a counterpoise weighing as much as the piece of metal in air. If we now immerse the metal in water and leave the counterpoise in air, we must bring the said counterpoise closer to the point of suspension [of the balance beam] in order to balance the metal. Let, for instance, *ab* be the balance [beam] and *c* its point of suspension; let a piece of some metal be suspended at *b* and counterbalanced by the weight *d*. If we immerse the weight *b* in water the weight *d* at *a* will weigh more, and to make it the same we should bring it closer to the point of suspension *c*, for instance to *e*. As many times as the distance *ac* will be greater than the distance *ae*, that many times will the metal weigh more than water. Let us then assume that weight *b* is gold and that when this is weighed in water, the counterpoise *d* goes back to *e*; then we do the same with very pure silver and when we weigh it in water its counterpoise goes in *f*. This point will be closer to *c* [than is *e*], as experience shows us, because silver is less heavy than gold. The difference between the distance *af* and the distance *ae* will be the same as the difference between the gravity of gold and that of silver. But if we shall have a mixture of gold and silver

[14] Most discussion's of Galileo's early work contain passing references to *La bilancetta*, but a detailed analysis of Galileo's actual proof procedure has not yet been provided. All more or less detailed expositions of Galileo's method that I know of translate it into modern terms and e.g. use algebraic methods.

> it is clear that because this mixture is in part silver it will weigh less than pure gold, and because it is in part gold it will weigh more than pure silver. If therefore we weigh it in air first, and if then we want the same counterpoise to balance it when immersed in water, we shall have to shift said counterpoise closer to the point of suspension *c* than the point *e*, which is the mark for gold, and farther than *f*, which is the mark for pure silver, and therefore it will fall between the marks *e* and *f*. From the proportion in which the distance *ef* will be divided we shall accurately obtain the proportion of the two metals composing the mixture. So, for instance, let us assume that the mixture of gold and silver is at *b*, balanced in air by *d*, and that this counterweight goes to *g* when the mixture is immersed in water. I now say that the gold and silver that compose the mixture are in the same proportion as the distances *fg* and *ge*.[15]

To our modern eyes, the absence of any explicit reference to the concept of specific weight is conspicuous. At the same time, we easily interpret Galileo's references to "gravity" as pertaining to it. After all, this is exactly what a hydrostatic balance does: it measures differences in specific weight. And of course, if specific weights can be measured, Hiero's crown problem is solved. The absence of the concept might seem even stranger when we take into account that the term was used from the Middle Ages on. However, there are good reasons for this absence.[16] For one thing, Archimedes himself never uses the concept – so if Galileo really wanted to claim that he could provide the original method used by his paragon, he should be able to do without it. But more importantly, it is absent in Archimedes for good reasons. Within the confines of classical proportion theory, as expounded in book five of Euclid's *Elements*, it is impossible to define the concept as the ratio of weight to volume, since ratios are only defined between magnitudes of the same kind.[17] There is no doubt that Galileo always regarded the mathematical instrument of proportional theory as regulative for his theorizing. That he consciously tried to evade the concept of specific weight is further corroborated by the belated introduction of it in the 1612 controversy on floating bodies. By that time he has discovered a flaw in his earlier analysis of the relation between a body and the medium in which it is immersed. It is only at this point, when no other routes are open to him, that he explicitly defines *"gravità in ispecie"* (which immediately forces him to belabour an extension of Euclidean proportion theory, analogous with the way in which he defines uniform speed).[18] I will come back to this in Section 5. Let us first see in

[15] *Opere*, I, pp. 217-218. (Transl. from Fermi and Bernardini 1961, pp. 115-116.)

[16] See Napolitani 1988 for much more on this issue, although he pays surprisingly little attention to Galileo's procedure in *La Bilancetta*.

[17] Grattan-Guinness 1996 offers a short and useful overview of the status of ratios and proportions within Euclid's *Elements*. There are some further potential problems with introducing the concept of specific weight as a quantity that can have a ratio, as explained in Napolitani 1988, pp. 190-196.

[18] This analogy is spelled out in detail in Napolitani 1988.

more detail how he tries to analyze the hydrostatic balance within the framework set by classical proportion theory.

When hanging a sample of gold from the balance at point *b* (see fig. 1), and weighing it first in air by hanging a counterweight at point *a*, and then in water by readjusting the position of the counterweight until at point *e* it anew equilibrates the sample, the law of the balance gives us for the ratio of the weight of gold in air to its weight in water:[19]

(gold : gold in water) :: (*ac* : *ec*).

Since we know that the weight of gold in water is equal to the difference of the weight of gold in air and the weight of an equal volume of water, we can transform this proportion in the following:[20]

$(\text{gold} : \text{water}_{\text{gold}}) :: (ac : ae)$, (1)

where the subscript "gold" refers to the fact that we are dealing with the weight of a volume of water equal in volume to the sample of gold. Equivalently we have:

$(\text{silver} : \text{water}_{\text{silver}}) :: (ac : af)$, (2)

with *f* the position of the counterweight when the sample of silver is immersed in water; and (again with *g* the second position of the counterweight):

$(\text{mixture} : \text{water}_{\text{mixture}}) :: (ac : ag)$. (3)

Commenting on (1) and (2), Galileo claims that it follows that the difference between *af* and *ae* is the same as the difference between the "gravity" of gold and the one of silver. What can this mean, and how does it follow?

It is clear that by "gravity," Galileo can only be referring here to something like what we would call specific weight. Nevertheless, he did start by measuring absolute weights, and applying the law of the lever to these. The transformation from absolute to "specific" weight is made possible by the physics of the situation, which seems to demand that the volume of water is always equal to the volume of the metal. Notwithstanding the fact that we are dealing with absolute weights in the first ratios of proportions (1)-(3), these proportions are valid regardless of the volume of the weighed bodies. This implies that *physically speaking* Galileo can consider the volumes of water mentioned in proportions (1) and (2) to be equal to each other, and by then applying the rule *ex aequali*[21] derive that (gold : silver) :: (*af* : *ae*), or equivalently[22] that (gold - silver : silver) :: (*af* - *ae* : *ae*), where gravity now must be understood as the weight of an unit volume of the metal.

Physically speaking, but not mathematically! As Galileo does not see weight as the product of specific weight and volume, there are no volumes for him to cancel out in the mentioned proportions

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[19] I follow the common practice of representing the ratio of *a* to *b* as (*a* : *b*) and the proportionality of two ratios (*a* : *b*) and (*c* : *d*) as (*a* : *b*) :: (*c* : *d*)

[20] By the rule *convertendo* which states that from (*a* : *b*) :: (*c* : *d*) one can derive (*a* : *a* - *b*) :: (*c* : *c* - *d*).

[21] From (*a* : *b*) :: (*d* : *e*) and (*b* : *c*) :: (*e* : *f*) derive that (*a* : *c*) :: (*d* : *f*).

[22] By the rule *dividendo*, which states that from (*a* : *b*) :: (*c* : *d*) one can derive (*a* - *b* : *b*) :: (*c* - *d* : *d*).

(which cancelling out, moreover, only makes sense from an algebraic point of view – and proportion theory is not algebra). And surely, the samples being weighed are not presumed to be equal in volume – as Galileo is attempting to reconstruct Archimedes' reasoning in solving the problem of Hiero's crown, this would not have made any sense: if the volume of the crown had been known, no hydrostatics would have been needed to expose the treacherous artisan.

We find Galileo reaching his result by equivocating: from the fact that the metal will always be opposed by an equal volume of water, he goes on to reason as if this equal volume was a unit volume. His terminology proved flexible enough to cover up possible ambiguities. Most often "gravity" stands for what we would call specific gravity, but depending on the context it can also refer to an absolute weight. Weight is most often used in the active sense as a verb, but sometimes it indirectly refers to the measurement of specific weight, again depending on the context. As mentioned, physically speaking he is justified in making these shifts from equal to unit volumes – and undoubtedly he realized this. However, only a few years later we will find him equivocating on exactly the same point, yet this time without having the same means to justify it. But before we come to that episode, let us return to Galileo's understanding of mixtures. This will prove crucial in his attempt to cover up the problems caused by that equivocation, through the introduction of his thought experiment.

Starting from proportions (1)-(3) it is possible to derive the following two proportions:[23]

(gold : gold - mixture) :: (*ag* : *ag* - *ae*),

(mixture - silver : silver) :: (*af* - *ag* : *ag*),

which can be compounded:[24]

(gold : gold - mixture) • (mixture - silver : silver) :: (*af* - *ag* : *ag* - *ae*).

Since the gravity of the mixture "has part of the silver" and "part of the gold", the ratio (gold - mixture : gold) can be taken as a measure for the amount of silver contained in the mixture (assuming the mixture to be homogenous); and equivalently for the second ratio on the left. From which the desired conclusion follows. A mixture of two elements will always be "in between" these elements with respect to its "gravity."

b. Balancing mixtures and speeds

The hydrostatic balance and its schematic representation function as a powerful embodiment of Galileo's knowledge about the relation between the "gravities" of a mixture and its component

[23] By the rules *invertendo* which states that from (*a* : *b*) :: (*c* : *d*) one can derive (*b* : *a*) :: (*d* : *c*), together with *convertendo* and *dividendo*.

[24] The symbol '•' is used to denote a ratio compounded of two ratios, which is not to be confounded with multiplication, although in the present case the results are the same.

elements. At the same time, the balance also embodied a rich tradition of dynamical thinking about the relation between weight, velocity, and mechanical effects. When these two aspects are put together a very suggestive picture emerges.

Let us first have another look at figure 1. It follows from Galileo's analysis that the lengths *ae*, *af*, and *ag*, stand for respectively the distances at which one counterweight must be hung to keep in equilibrium a body with more gravity, with less gravity, and a mixture of these (distances which can be related in exact proportion to the gravities, which are the same whatever the volume of the bodies). Let us now have a look at figure 2, which illustrates the main tenets of one influential way of understanding mechanical problems, which stretches back to the pseudo-Aristotelian *Mechanical problems* (written probably around 3thC BC) and which Galileo will incorporate in his *Mecaniche* (written at the end of the 1590's).[25] Basic to this view was an understanding of the law of the lever which crucially used the speeds of the bodies on a balance, and which was based on the geometrical properties of the circle. A body hanging in *A* can be held in equilibrium by a lighter body hanging at the point *B*. For consider what would happen if the bodies would start to move: since they are constrained by the balance they will move at the circumference of a circle; now, since they will always have moved over distances *AD* and *BE* in an equal time, the lighter body, which will have moved a over longer distance, will have travelled faster. We can understand that bodies of different weight can give rise to the same mechanical effect (i.e. equilibrium), by seeing that they also differ with respect to another crucial factor: speed, which can offset the differences in weight. To put it in a nutshell: associated with all points on the arm of a balance comes a different speed.

Both figures show how multiple explanatory schemes are embodied in one instrument: the balance. If we now mentally conceive the superposition of these pictures, since both refer to the same instrument, a suggestion emerges that maybe was too hard to resist. To put it in a nutshell: there is a different speed associated with every different ("specific") gravity – and this speed is independent of the volume of the bodies.

That the encounter with the hydrostatic balance indeed proved to be very enlightening for Galileo is testified by a fragment from the aborted dialogue version of his treatise on natural philosophy known as *De motu*:

> Now at last I can no longer avoid demonstrating some theorems to you. From the comprehension of these theorems you will clearly understand not only the answer to your question, but also the ratio of the speed or slowness of the motion of bodies both heavy and light, as well as the ratio of the heaviness or lightness of one and the same body weighed in different media. All these theorems had to be demonstrated when I tried to find the true

[25] For a useful overview of this tradition, see Brown 1978.

reasoning by which, in a mixture of two metals, we could indicate the exact amount of each separate metal.[26]

3. *De motu*: Attempts at an Archimedean natural philosophy

When writing the *Discorsi* at the end of his life, Galileo compounded a folder containing his "older notes on motion." These were written immediately after the completion of *La Bilancetta* and are generally considered to be Galileo's first truly original ventures into natural philosophy.[27] Although he no longer accepted many of its conclusions at the time of writing the *Discorsi,* important and noteworthy traces can be found in the first day thereof, among which the thought experiment. Following Galileo's own indication in the quote given at the end of Section 2, we can describe these notes as the outcome of confronting the natural philosophical problem of motion equipped with newly acquired Archimedean utensils. Let us have a look at the first book of the only completed treatise among these notes, the 23 chapter essay which was translated by Israël E. Drabkin as *On motion*.[28]

a. The dynamics of De motu

The first chapter is a kind of a terminological preliminary to the natural philosophical treatise, but a highly significant one. In it, Galileo stipulates when we should call one body "heavier" *(gravius)* than another: if it is measured to be heavier than the other in well-circumscribed circumstances – i.e. when the volumes *(moles)* of both bodies are equal. This stipulation betrays two facts: Galileo is clearly conscious about the possible ambiguities surrounding the use of *grave* and its cognates; at the same time, he still tries to evade the explicit introduction of specific weight. Possible ambiguities are not resolved by defining specific weight, but by regimenting the use of "heavier." He will continue to reason with absolute – not specific – weights, albeit always taken in relation to the volume. For most purposes this reduces to the same, but, as will become clear, not for all.

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[26] *Opere,* I, p. 379. (Transl. from Drake and Drabkin 1969, p. 344.)

[27] There have been several discussions about the exact order of composition of the fragments contained in the folder. Giusti 1998 is a recent assessment of the available evidence, which finds agreement with the order that was earlier proposed by Fredette 1969 and Drabkin 1960. Depending on the order one follows, different dating for the individual fragments follows, but there is general agreement that they all must be dated between 1586 and 1592.

[28] Fredette 1969 has shown that this treatise consists of two separate books, of which the first is devoted completely to the role of gravity in natural motion. I will only take into account the revised version of this first book where these revisions do touch directly on the central point that I am interested in; chapter 4 of Fredette 1969 discusses the textual differences in detail.

In the subsequent chapters, it becomes clear why Galileo opts for this stipulation. He prides himself on the fact that he can provide a rationale for the Aristotelian cosmological scheme, whereas other authors could only posit it without further rational foundation. This rationale is based on the geometric properties of a sphere, and on an atomistic conception of matter (undoubtedly immediately diminishing the attractiveness of his explanation to many contemporary eyes). If one body is heavier than another, this means (according to Galileo's stipulation) that an equal volume of it weighs more than the other. Following his atomistic conception of matter this implies that the heavier body contains a greater amount of matter in the same space; or equivalently, that heavier bodies contain the same amount of matter in smaller spaces. Now consider one of the essential properties of a sphere: that the spaces become narrower as we approach the centre, and larger as we recede from the centre. Wouldn't it then be a rational constitution if the heavy elements should be placed near the centre of the cosmos, and the light ones farther away? This explanatory scheme was probably suggested to Galileo by his study of Archimedes' treatise on floating bodies, which always demonstrates its propositions concerning equilibrium – whereby the lighter must stay on top of the heavier – for fluids and solids placed on a sphere.

Not only does Galileo's stipulation allow him to make sense of the Aristotelian cosmological scheme, it also allows him to infer the right dynamics from it. Herein we see the Archimedean import become even more dominant, yet with the natural motions still predetermined by natural places. Since the natural places are the places of Archimedean equilibrium, natural motion will always be motion towards such equilibrium. Extending this idea to motion through a medium, and keeping in mind that we only have to consider equal volumes, Galileo then can prove that bodies lighter than the medium do not descend whereas the heavier do. These proofs are always of the following structure: (1) suppose that situation X were an equilibrium state; (2) this cannot be so, because of the natural disposition, which is Y; (3) hence we have motion towards state Y. We hence see how the Archimedean scheme immediately *shows* the causes of natural motion. Once we have learned to look at nature in this way, it is impossible not to notice the relevant structures. An important first step in rendering the phenomenon of free fall intelligible is thus already taken.

In adducing these proofs that heavy bodies move down and light ones move up (heavy and light being understood relatively), Galileo cleverly exploits the fluid character of media. The body that is immersed in the medium pushes downward *(deorsum permit)* against the part of the medium *besides it*! This becomes clear if we have a look at one of Galileo's own pictures (see figure (3)). When the body *ef* is immersed in the medium, the level of the medium is necessarily raised: hence the body *ef*, in pushing downward, raises the part of the medium *so* (equal in volume), which is the part besides the body. In addition to this, we also can see how he conceptualizes the situation in terms of two bodies which are trying to raise the other, and at the same time resist being raised themselves. Both facts are of course extremely suggestive of an analogy with a balance, which Galileo then brings to the fore.

That the introduction of the balance within Galileo's natural philosophical treatise, which up to that point was modelled purely on hydrostatics, is highly significant has of course already been stressed many times. Among other things, it allows Galileo to import common sense knowledge, embodied in everyday experience with balances, into the natural philosophy he is attempting to develop. That he also understands the importance of this introduction in these terms is witnessed by the words with which he introduces the analogy:

> But the points set forth in these last two chapters cannot very well be further elucidated mathematically; they require rather a physical explanation (*minus adhuc mathematice, et magis physice, declarari possunt*). For this reason I propose, to reduce the matter to a consideration of the balance, and to explain the analogy that holds between bodies that move naturally and the weights of the balance. My aim is richer comprehension of the matters under discussion, and a more precise understanding on the part of my readers.[29]

Let us see what such a *magis physice* explanation brings to light. In the case of bodies on a balance, no one would doubt that the lighter body is moving up *because* it is lighter but that it nevertheless still has weight. The relative definition of heavy/light is hence shown to be part of common sense knowledge. Moreover, in a balance everyone readily sees that motion up and down are completely symmetrical and have the same cause; thus, some further consequences of the relative definition are laid bare through the mediation of the balance. The incorporation of this insight caused Galileo considerable trouble: there are hesitations on exactly this point at several places in his treatise.[30] These troubles are further aggravated by the fact that we are always dealing with a "force – resistance" pair in the model Galileo is working with. This pushes one to considering even natural motion as a kind of forced motion, and this brings Galileo close to crossing the borders of what could be considered natural philosophy at the time.

As a result of these problems, Galileo did not succeed in developing a completely worked out picture in *De Motu*, and these internal tensions were possibly among the reasons for its abandonment (together with the growing realisation of the importance of considering natural motion to be accelerated and the impossibility of incorporating this insight in an Archimedean framework).[31] Nevertheless, this explicit thematization of the balance as a model to understand natural phenomena

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[29] *Opere*, I, p. 257. (Transl. from Galilei 1960, p. 20.)

[30] These hesitations are closely connected with a constraint on Galileo's science that remained operative throughout all his writings: weight as a force downward is always a privileged kind of "natural" force, which cannot be assimilated to forces in general.

[31] Some more extended discussions of the tensions in *De motu* can be found in Fredette 1969, *passim* and especially chapter 4 and the general conclusion; in Galluzzi 1979, pp. 175-197; and in Damerow *et al.* 2004, pp. 141-157.

introduces one of the most important instruments of Galileo's thinking, which he will continue to exploit till the very last moments when expanding the *Discorsi* shortly before his death. Yet, as we will see in the second part of the present paper, by that time its status as a model of intelligibility will have undergone a profound modification.

After having introduced the analogy with the balance, Galileo moves on to the central goal of the first book of *De motu*: to establish a *quantitative* dynamics which could replace Aristotle's.[32] This dynamics is obviously based on the Archimedean scheme that had already provided the key element to Galileo's qualitative dynamics in the first chapters of *De* motu, but important aspects of the Aristotelian tradition are also retained. Let me quote two unambiguous examples which illustrate both aspects of Galileo's way of proceeding:

> And first, in connection with upward motion, let us show that, when solids lighter than water are completely immersed in water, they are carried upward with a force [*tanta vi*] measured by the difference between the weight of a volume of water equal to the volume of the submerged body and the weight of the body itself [*quanto … gravior erit*].[33]

The Archimedean import is clear: the force upon a body is measured by the difference in "gravity". But when we have a look at the way in which speeds are related to these forces, an Aristotelian aspect becomes obvious as well:

> If, for example, a piece of wood whose weight [*gravitas*] is 4 moves upward in water, and the weight [*gravitas*] of a volume of water equal to that of wood is 6, the wood will move with a speed that we may represent as 2.[34]

We immediately see how the speeds function as a kind of measure for the differences in "gravity" between the body and the medium through which it is moving. Galileo thus retains the basic Aristotelian idea that speeds and forces are proportional at the basis of his own quantitative dynamics. This is of course important, because otherwise he would have had no way to put his model of intelligibility to work.

The resulting scheme allows Galileo to escape from some difficulties which he skilfully lays bare in the original Aristotelian scheme, in which speed was not only proportional with the weight of the body, but also inversely proportional with the "density" of the medium. This inverse

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[32] More correct would of course be: what he and many others at that time took to be Aristotle's quantitative dynamics.

[33] *Opere*, I, p. 269. (Transl. from Galilei 1960, p. 33.)

[34] *Opere*, I, p. 270. (Transl. from Galilei, 1960, p. 34.)

proportionality implies a number of paradoxes, such as the impossibility of equilibrium. Since Galileo, contrary to Aristotle, and following Archimedes in this respect, sets force proportional to an arithmetic ratio (a difference) instead of a geometric ratio (a quotient), he can easily avoid these paradoxes. Moreover, this revision implies the possibility of motion in a void (for which the density is zero), a fact which he stresses abundantly in the remainder of the first book of his treatise.

Both Raymond Fredette and Paolo Galluzzi have stressed that Galileo, upon revising the first book of his treatise, discarded the chapter in which he introduced the balance analogy.[35] Fredette ascribes this primarily to the tensions arising because of the asymmetry between upward (forced) and downward (natural) motions. Galluzzi, however, sees another reason why Galileo might have judged the balance analogy to be improper. He claims that Galileo's Archimedean explanation of the causes for downward and upward motion is based upon the specific weights of the bodies and the media, whereas the balance only measures absolute weights. *We* can indeed easily see that these quantities are dimensionally incommensurable, but it should be clear from Galileo' treatment of the hydrostatic balance in *La bilancetta* that for him this distinction was not at all clear-cut. Remember that his treatment of this balance also started from absolute weights and then implicitly transformed these in specific weights. That something similar could be going on in *De motu* is clear when we have another look at the two last quoted passages. Both do reveal a crucial fact about Galileo's dynamical thinking in *De motu*. He is undeniably reasoning with the *actual volumes* of the moving bodies, and measuring the force by the difference in (absolute) weight for *these* volumes.[36] But this means that also within this hydrostatic context, a balance does indeed measure a body's tendency to downward motion! This direct identification is clearly illustrated by yet another quotation:

> We are said to be weighed down [*gravari*] only when some weight [*pondus*] rests on us which by its heaviness [*gravitate*] tends downward, while we must by our force [*vi*] resist its further downward movement; and it is that resistance which we call being weighed down [*gravari*].[37]

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[35] Fredette 1969, p. 272; Galluzzi 1979, p. 190.

[36] Further confirmation for this identification can be found in the second book of *De motu.* When discussing the possible cause of acceleration, Galileo first claims that "we know definitely, from what was proved at the beginning of this book [*in primo libri*], that speed and slowness are a consequence of weight and lightness." (*Opere*, I, p. 318. Transl. from Galilei 1960, p. 88.) Since Galileo uses *"gravitatem"* this might still be taken as ambiguous between absolute and "specific" weight (lightness – *"levitatem"* – must obviously be read as relative, as taught by the first chapter of the first book). Galileo however immediately continues by asking what could cause the change in weight that is responsible for the acceleration, and he adds that "the natural and intrinsic weight of the body has surely not been diminished, since neither its volume nor its density has been diminished." (*Ibid*.) It is clear that here the "naturalis et intrinseca mobilis gravitas" refers to an absolute weight, as it could also be changed by a diminution of volume.

[37] *Opere,* I, p. 288. (Transl. from Galilei 1960, p. 54.)

A body's tendency to motion is thus responsible for its experienced weight, which is measured by the force that is necessary to resist that motion. Again, we see how fundamental the force-resistance pair is in Galileo's thinking. Moreover, both this quotation and the two earlier ones come from chapters which are completely retained in the revised version of the first book.[38] Galileo's experience with balances is hence still implicitly structuring his thinking, even after he has discarded the explicit analogy with a balance. This justifies our considering this model as really anchoring his dynamical thinking to shared experiences that must provide for the intelligibility of his explanatory scheme.

That Galileo is undeniably reasoning on actual volumes, and that he accordingly sets the force equal to a difference in absolute weights might come as a surprise to many, given that it is always stated (as is done by Galluzzi) that in *De motu* Galileo sets the speed of a falling body proportional to the difference of the specific weights of body and medium.[39] It is moreover undeniable that this is indeed how we would interpret the actual proportions that he assigns to the speeds of different bodies. The central question to a satisfactory understanding of Galileo's *De motu* hence becomes: how and why does he make this transition?

b. From equal volumes to unit volumes

Let me first quote a crucial passage in which Galileo makes exactly this transition:

> In the case of bodies differing both in size and weight, if we take from the larger a part equal [in size] to the smaller, we shall … have two bodies differing in weight, but not in size. And the part [of the larger body] will, with the smaller, keep the same ratio [in the speed] of their motions, as will the whole of the larger body. *For … in the case of bodies of the same material, the part and the whole move with the same speed*. It is therefore clear that, if we know the ratio of the speeds of those bodies that differ only in weight, but not in size, we also know the ratios of those that differ in every other way.[40]

[38] See Fredette 1969, chapter 4.

[39] Wisan 1978, p. 7, e.g. states that it follows from Galileo's natural philosophy that "'natural' motion is caused by relative heaviness and lightness" and immediately adds between parentheses: "Galileo intends relative density". I propose that we be more careful with ascribing intentions to Galileo and pay attention to the actual ambiguities with which his texts present us. Even Westfall, who is unusually careful in stating that Galileo claims that the force of a body in a medium equals the "amount by which its weight exceeds that of an *equal* volume of water", also states on the same page that "when Galileo said that speed in a void depends on the total weight of a body, *he meant* its specific weight", without explaining how such a transition would be effected. (Westfall 1971, p. 15; my emphases.)

[40] *Opere,* I, p. 267. (Transl. from Galilei 1960, p. 31. My emphasis added.)

Notwithstanding the fact that Galileo is reasoning on absolute weights of equal volumes, *he claims that he can always generalize his results by pretending that these equal volumes were unit volumes*. The clue to the transition from absolute to "specific" weights thus lies in the equality of the speeds of bodies of the same material. That Galileo truly starts from absolute weights is made further clear by the fact that he tries to justify this equality of speeds precisely on the basis of an argument starting from the absolute weights of the bodies. This further testifies to the fact that it is the latter property that is really basic in Galileo's thinking: a satisfactory explanation is one which reduces an explanandum to facts involving absolute weights – which facts are presumable intelligible in themselves through our shared experiences with them. Absolute weight is hence undeniably the central dynamical factor.

It is at the beginning of the 8th chapter, "in which it is shown that different bodies moving in the same medium maintain a ratio [of their speeds] different from that attributed to them by Aristotle," that Galileo tries to establish the equal speeds for bodies of the same material. He begins by asking whether it wouldn't be ridiculous to imagine a direct proportionality between weight and speed, but immediately goes on "to employ reasoning … rather than examples (for what we seek are the causes of effects, and these causes are not given to us by experience)".[41] And this reasoning goes as follows:

> Thus, if we imagine that the water on which a large piece of wood and a small piece of the same wood are afloat, is gradually made successively lighter, so that finally the water becomes lighter than the wood, and both pieces slowly begin to sink, who could ever say that the large piece would sink first or more swiftly than the small piece? For, though the large piece of wood is heavier than the small one, we must nevertheless consider the large piece in connection with the large amount of water that tends to be raised by it, and the small piece of wood in connection with the correspondingly small amount of water. And since the volume of water to be raised by the large piece of wood is equal to that of the wood itself, and similarly with the small piece, those two quantities of water, which are raised by the respective pieces of wood, have the same ratio to each other in their weights as do their volumes … – i.e., the same ratio as that of the volumes of the large and the small piece of wood. Therefore the ratio of the weight of the large piece of wood to the weight of the water that it tends to raise is equal to the ratio of the weight of the small piece of wood to the weight of the water that it tends to raise. And the resistance of the large amount of water will be overcome by the large piece of wood with the same ease as the resistance of the small amount of water will be overcome by the small piece of wood.[42]

[41] *Opere,* I, p. 263. (Transl. from Galilei 1960, p. 27.)

[42] *Opere,* I, p. 264. (Transl. from Galilei 1960, pp. 27-28.)

It seems that Galileo is claiming that the equal speeds follow from Archimedean considerations. But this does not really make sense. What he actually proves is that W_{body}/W_{medium} is invariant for bodies of the same material, but to conclude from this that hence the speeds are equal implies that he would be employing an Aristotelian geometric ratio (with the resistance of the medium measured by its weight) instead of the Archimedean arithmetic ratio which he explicitly favours as the central dynamical formula. Given Galileo's dynamical scheme which was sketched above, and the fact that $W_{body}/(W_{body} - W_{medium})$ will also be invariant, all that we can conclude is that for any two bodies of the same material, there is a constant ratio between the speeds of these bodies in void and in a medium. This only implies that the speeds of these bodies are diminished in the same proportion by a medium, not that they are the same.[43] Only upon the supposition that the speeds of all bodies of the same material would be the same in the void would the equality of their speeds in a medium follow. But why would these speeds in the void be the same? This in no way follows from his Archimedean-Aristotelian dynamical scheme – it is even in explicit opposition to it. It seems that he is hence left without a way of rendering this fact intelligible.

It is clear that the proportional alleviation effect of a medium cannot account for the equal speeds of bodies of the same material. Strictly speaking, Galileo cannot make the transition from absolute to specific weights. This raises the further question: why does he nevertheless want to make it? After all, he could as well have developed a theory which is strictly based on his Archimedean-Aristotelian scheme, and set $v \sim W_{body} - W_{medium}$.

A first clue to a possible answer is given by Galileo himself, when he raises empirical objections against a direct proportionality between speed and weight which he dubs “ridiculous.” Moreover, when he recounts his own development in the 1630’s, he will again stress these considerations as the first to have raised his suspicion against Aristotle’s explanations.[44] In doing so, he (implicitly) also dismisses the proportionality with an alleviated weight, which however would be less ridiculous (since the differences would be smaller). It is nevertheless quite possible that he was convinced that also these differences in speed would be too large to be empirically credible. But it is also true that in the same *De motu*, he is quite willing to invoke *ad hoc* explanations to account for the striking differences between the accelerated character of the motion of all actually falling bodies and the uniform character of the motion of his theoretical models. In this case he did let his theoretical model overrule the empirical observations. It seems that there must be a hidden motivation behind his choice which cannot be traced back solely to its empirical plausibility.

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[43] This is easily seen when we translate the situation in modern terms: that speed v is proportional with $W_{body} - W_{medium}$, implies that $v \sim (density_{body} - density_{medium}) \times volume$; this implies that for bodies of the same material but of different absolute weight, the speeds in a medium will be proportional with their respective volumes.

[44] See Section 6.

I submit that Galileo's experience with the hydrostatic balance provides the most important clue for understanding the tension in his dynamical thinking in *De motu.* It is the hydrostatic balance which had taught him that bodies of the same material are equally affected by a medium, regardless of their volume. Moreover, as was already pointed at the end of Section 2, the properties of a balance were closely linked with the "speeds" of the bodies balanced by it. I believe that Galileo's inconsistent argument in *De motu* should be seen as a failed attempt to mimic the cogent reasoning behind the irrelevance of volume for a hydrostatic balance, with the results now translated to speeds.

4. *De motu*: Introducing the thought experiment

"But we may reach this same conclusion by another argument."[45] Such is Galileo's own introduction to his famous thought experiment in *De motu*. This other argument for the equality of the natural speeds of bodies of the same material has received much more attention than the confused attempt based on the proportional alleviation effects of the medium. This is undoubtedly due to a fascination for the cleverness of the argument, but it may also result from the simple fact that this argument does seem to reach its goal cogently. I agree that the argument is indeed unassailable, but it remains to be pointed out that the premises are not as innocent as they might look. We will see how Galileo's presentation of the thought experiment provides further indications of the far-reaching repercussions of his earlier encounter with the hydrostatic balance.

a. A hidden assumption

Let us first consider Galileo's own presentation of his thought experiment.

> Let us first make this assumption: if there are two bodies of which one moves [in natural motion] more swiftly than the other, a combination of the two bodies will move more slowly than that part which by itself moved more swiftly, but the combination will move more swiftly than the part which by itself moved more slowly. …
> On the basis of this assumption, I argue as follows in proving that bodies of the same material but of unequal volume move [in natural motion] with the same speed. Suppose there are two bodies of the same material, the larger *a* and the smaller *b*, and suppose, if it is possible, as asserted by our opponent, that *a* moves [in natural motion] more swiftly than *b*. We have, then, two bodies of which one moves more swiftly. Therefore according to our assumption, the combination of the two bodies will move more slowly than that part which by itself moved more swiftly than the other. If, then, *a* and *b* are combined, the combination will move more

[45] *Opere,* I, p. 264. (Transl. from Galilei 1960, p. 28.)

> slowly than *a* alone. But the combination of *a* and *b* is larger than *a* is alone. Therefore, contrary to the assertion of our opponents, the larger body will move more slowly than the smaller. But this would be self-contradictory.[46]

The argument inevitably leads to its conclusion: bodies of the same material have the same speeds in free fall. Following Gendler's neat reconstruction we can summarize the argumentative structure as follows:[47] (1) natural speed is mediative (the natural speed of a combined body will fall between the natural speeds of the component bodies); (2) weight is additive (the weight of a combined body will be the sum of the weights of the component bodies); hence (3) natural speed is not directly proportional to weight; and, moreover the only way to hold on to (1) – (3) simultaneously is by asserting that (4) natural speed is independent of weight.

The crux of the argument seems to lie in premise (1). One could wonder how Galileo can claim to know that this is a valid assumption. A first possible answer is provided by the following note which he wrote in a margin in the original manuscript: "Aristotle makes this same assumption in the solution of the 24th Mechanical Problem." Now, this is a little bit of a stretch on Galileo's part. The 24th Mechanical Problem deals with the famous paradox of Aristotle's wheel, not at all with the natural speeds of falling bodies. The importation of that assumption, in the context of the thought experiment would require a much more substantial argument. It is not at all obvious that rolling wheels and falling bodies partake in the same principles. Moreover, if this assumption were accepted only on Aristotle's authority, then it might well function in a reduction of the Aristotelian theory, but not in an argument which seeks to establish an alternative theory. For the conclusion (4) to hold generally, independent grounds for accepting premise (1) must be present. Such grounds are provided by Galileo, however:

> Thus, if we consider two bodies, e.g., a piece of wax and an inflated bladder, both moving upward from deep water, but the wax more slowly than the bladder, our assumption is that if both are combined, the combination will rise more slowly than the bladder alone, and more swiftly than the wax alone. Indeed it is quite obvious. For who can doubt that the slowness of the was will be diminished by the speed of the bladder, and, on the other hand, that the speed of the bladder will be retarded by the slowness of the wax, and that some motion will result intermediate between the slowness of the wax and the speed of the bladder?[48]

[46] *Opere*, I, pp. 264-265. (Transl. from Galilei 1960, 28-29.)

[47] Gendler 1998, p. 404.

[48] *Opere*, I, p. 265. (Transl. from Galilei 1960, 28-29.)

The same argument is then repeated for a piece of wood and an inflated bladder falling downward in air. These are of course very revealing examples. The first thing to notice is that they involve bodies of *different material*. Now, since Galileo wants to conclude that for bodies of the same material the speed of fall is equal, it would have been clearly self-defeating if he could have adduced empirical examples of this kind to illustrate his assumption. But this also points toward the fact that *Galileo considered his assumption to be an empirical fact of the matter*, possibly following a theoretical principle, but surely recognizable without such a principle at hand. Secondly, the provenance of this empirical fact of the matter is easily recognizable. Take two bodies of different material and compare their behaviour with the behaviour of a mixture of these materials…

Once again we find Galileo translating the situation of *La bilancetta* by having natural speeds mirror the positions of the counterweight on the hydrostatic balance. These positions on the balance arm had indeed undeniably shown that "specific weight" is mediative. But this implies that the proportionality of speed with "specific weight" is a hidden assumption of his thought experiment. The thought experiment thus accomplishes the transformation from absolute to "specific" weights by presupposing the latter.

b. The dynamical conundrum

Once that the conclusion of the thought experiment is reached, it becomes impossible to hold on to a proportionality between speed and absolute weight. However, this leaves Galileo without any intelligible dynamics, as the balance is his paradigm case of a situation in which the force of weight can be immediately understood. In *La bilancetta,* he had been able to take these forces, as measured by absolute weights, as the starting point for analysing specific weights, by exploiting the fact that any body is always opposed by an equal volume of water in a hydrostatic balance. At this point he thus did also not consider specific weights as giving rise to forces directly. That he still holds on to this indirect relation in *De motu* is clear if we remember that at several places (after already having presented the thought experiment), Galileo does set speeds proportional to forces which are measured by differences in absolute weights – differences which then can be transformed into differences of "specific" weights by pretending (on the basis of the thought experiment) that the results hold independently of the volumes. But if we are not mistaken in imputing to Galileo a dynamics which still refers back to experiences with absolute weights, then the conclusion of the thought experiment must have presented a potential conundrum for him.

The absence of an explicit concept of specific weight undoubtedly helped to mask the dynamical problem. By not explicitly thematizing the dimensional differences within the undifferentiated concept of "grave", the conundrum might have seemed less pressing (and indeed seems to have been largely ignored by most Galileo scholars). There was of course also the attempt at explaining the equality of speeds by considering the alleviation effect of a medium, which might have eased Galileo's mind at this point – provided he did not realize himself that he was being incoherent. But it must anyway have been clear to him that this was insufficient. This can be judged from the fact that after that he has established the possibility of motion in a void, he proclaims that the thought experiment must also be valid in this situation.[49] Given that the argument is supposed to remain precisely the same, it is clear that the effect of the medium can not be operative in reaching the desired conclusion. This helps us to pinpoint more precisely the gap that remains in Galileo's dynamical conceptualization of motion. As the transformation procedure which he used to such great effect in *La bilancetta* completely breaks down in the void, he is left without any way to connect his mathematical scheme with the shared experiences that had to secure its intelligibility. What he offers instead is his thought experiment, which supposedly can provide for an equally incontestable experience that could possibly anchor his explanatory scheme (albeit it does this, as we saw, by actually presupposing further experiences which go back to phenomena involving dense media). That it is indeed supposed to render the dynamics of free fall immediately intelligible is further proved by the following passage, which follows almost directly after the presentation of the thought experiment:

> And who, I ask, will not recognize the truth at once, if he looks at the matter simply and naturally? For if we suppose that bodies *a* and *b* are equal and are very close to each other, all will agree that they will move with equal speed. And if we imagine that they are joined together while moving, why, I ask, will they double the speed of their motion, as Aristotle held, or increase their speed at all?[50]

The question is to the point, and it will be the starting point for a successful solution of the conundrum in the *postils to Rocco*, but at this point it must remain a rhetorical question. If a balance does indeed measure a body's tendency for downward motion, as repeatedly implied by Galileo in *De motu*, then the only natural response to the question would be: why not? This is not to deny that Galileo was convinced that they do not: he clearly believed that specific gravity provided a much better measure for the speed of fall. But it is the argumentative structure of *De motu* itself that leaves a gap at exactly this point.

One might wonder whether it is really justified to call this gap a "conundrum", as there are no clear signs that Galileo was puzzled by it in any significant respect.[51] As far as *De motu* goes, this might be true, but as will become clear in the second part of the present paper, at a later time Galileo indeed began to wonder about how to connect the behaviour of the bodies in his thought experiment with their behaviour on a balance. At this point he has clearly become aware of the gap that exists

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[49] *Opere*, I, pp. 283-4.

[50] *Opere,* I, p. 266. (Transl. from Galilei 1960, 30.)

[51] I have to thank Paolo Palmieri for pushing me on this point.

between his explanatory scheme and the basic experiences that were first thought to render it intelligible. If we would not be allowed to think of this gap as a conundrum, we might hence loose the means to understand the dynamics behind Galileo's thinking, as it seems that it really did trigger Galileo's rethinking of the thought experiment in a fundamental new way. As was already noticed, once the gap is perceived as a conundrum, the crucial question becomes why bodies of the same material would have to move with the same speed in the void. Indeed, in this situation the empirical examples which were adduced by Galileo to justify the first premise of his thought experiment loose their intuitive plausibility, which was based on the experience with the behaviour of mixtures in dense media. This shows that, although he does not need to change the argument itself, he would need some other kind of justification for the mediative character of natural speeds. In the later presentations of the thought experiment (to be discussed in Sections 6 and 7) in the *postils to Rocco* and in the *Discorsi, exactly such a justification will be provided,* which will be clearly dynamical in character. As we will see, once that he has provided the dynamical justification for the first premise, Galileo will also be in a better position to solve the conundrum raised by the conclusion.

Recapitulating our long analysis of *De motu*, Galileo's thought experiment plays a crucial role therein in at least two respects. It enables him to make the transition from absolute to "specific" weight as the relevant factor for the natural motion of bodies, without having to define the latter explicitly. At the same time, it covers up the fact that Galileo by his own standards misses a fully intelligible dynamics for free fall. It is indeed clear that this transition from absolute to "specific" weight cannot be based on the effect of a medium on the weight of bodies, while Galileo nowhere gives a hint of how to understand "specific" weight as a primordial and immediately intelligible dynamical factor: the only model which he possesses for understanding forces is the balance which measures absolute weights; and all his dynamical thinking is based on the idea that speeds are caused by such forces.

5. *Discorso*: The impotence of specific gravity as a dynamic factor

Galileo never published or even circulated the manuscript of *De motu*. We can hence safely conclude that he was not convinced of the resulting natural philosophy, whatever the precise reasons for his own dissatisfaction. However, throughout his career he kept returning to topics and concepts which were already introduced within *De motu*. We will have a brief look at another context in which he further developed and articulated some aspects of his dynamical thinking. The main reason for doing so is that it will further corroborate my analysis of the argumentative gap that is left in *De motu*.

In 1610 Galileo moved to Florence to become court mathematician and philosopher of the grand duke of Tuscany, where he almost immediately became invested in a controversy on the reason why bodies stay atop on water. In the course of these discussions he realized the need to define specific gravity explicitly, an event which will further clarify the status of this concept within his thinking. But before entering into this episode, it is necessary to briefly recapitulate some well-known basic facts about Galileo's conceptualization of mechanical effects.

a. Moment and absolute weight

In the most extended version of his mechanical treatise, *Le mecaniche*, written sometime during the 1590's, Galileo after having explained the subject matter of the treatise introduces a set of definitions for his basic concepts. The first is immediately very interesting:

> We call *heaviness* [*gravità*], then, that tendency to move naturally downward which, in solid bodies, is found to be caused by the greater or lesser abundance of matter [*materia*] of which they are constituted.[52]

We see a clear return to weight taken absolutely, still indissolubly connected to a tendency for downward motion. What is added here is the specification that the more matter a body contains, the more heaviness and thus tendency for motion downward. But the real innovation of the mechanical treatise is the next concept to be introduced:

> *Moment* is the tendency to move downward caused not so much by the heaviness of the movable body as by the arrangement which different heavy bodies have among themselves. … Thus *moment* is that impetus to go downward composed of heaviness, position, and of anything else by which this tendency may be caused.[53]

This would prove to be a very fruitful concept, which allows Galileo to give his mechanical treatise a clear and powerful structure.[54] All machines work by having some force overcome a greater resistance, and the concept of moment would provide the clue to analyzing this kind of situation. A body's heaviness combined with other factors gives rise to its moment, and its moment gives rise to the dynamical effects the body might have. An analysis of the simple machines thus boils down to finding out these other factors making up the body's moment. The ones singled out as relevant in *Le mecaniche* are the relative positions of the bodies, and the velocity and distance of their motions (distance and velocity are interchangeably on a balance, where the weights always move in the same time). Once the moments are found out, Galileo can consider the machines as closed systems, obeying

[52] *Opere*, II, p. 159. (Transl. from Galilei 1960, p. 151.)

[53] *Opere*, II, p. 159. (Transl. from Galilei 1960, p. 151.)

[54] Galluzzi 1979 is the seminal study on the concept of moment, and its pivotal role in the development of Galileo's thinking.

a conservation law for moment. The machines then produce their useful effects by transforming the moment of the working force into the one of the resisting force.

To our purposes, one aspect of Galileo's treatment of the moment of a body is crucial: its measurement. As witnessed by the expression "moment is that impetus to go downward," moment is intimately related with dynamical effects, yet it is always measured by a resisting counterweight (which is completely unproblematic given Galileo's definition of heaviness as also a tendency to go downward). If we e.g. consider Galileo's analysis of motion on an inclined plane, we see that each body's impetus to go downward on such a plane is measured by the weight of a body keeping it in equilibrium, attached to it by a balance with bent arms, suspended above the plane.[55]

We can immediately learn two crucial facts about Galileo's dynamical thinking at this stage. Firstly, dynamical effects are measured by static weights. The balance remains the one and only instrument to understand force. The transition from the static measure to the dynamical effect is then made by the principle that the addition of "an insensible weight"[56] is sufficient to set in motion a weight that is held in equilibrium on a balance or an inclined plane. Secondly, moment as the cause of these dynamical effects arises from the modification of absolute weight. Although there is a clear broadening of Galileo's dynamical framework through the introduction of "moment", it is still indissolubly tied to absolute weight. Specific weight appears impotent to cause any effects.

Paolo Galluzzi has stressed that Galileo is cautious to remain silent on any link between moment and the resulting speeds in *Le mechaniche*.[57] As the treatise is devoted to mechanics, and as an investigation into precise measures of speed hence falls outside its scope, it is hard to decide what to make of such silence. Anyway, for my present purposes it is enough to notice that absolute weights remain the paradigm cases of forces; and if Galileo possibly did no longer hold on unequivocally to a proportionality between forces and speed (although, as we will see, there are passages in the later *Discourse on bodies…* which suggest that he had not yet let go this idea), he certainly has not found a way to make sense of any other possible connection.

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[55] Galileo's discussion of the inclined plane in *Le mecaniche* is an expansion of an earlier discussion in the second book of *De motu*. As was already remarked by Damerow *et al.* 2004, p. 147, n. 39, the presence of this discussion in the latter work gives rise to another incoherence, as the speed of the motion is measured by its "moment" (a term not yet introduced in *De motu*) and hence is proportional with the body's absolute weight (modified by the inclination of the plane).

[56] *Opere*, II, p. 163.

[57] Galluzzi 1978, p. 219. Galluzzi ascribes this caution to Galileo's realization that any straightforward relation between moment and speed would be unable to account for the acceleration along an inclined plane. I would add the dynamical conundrum of *De motu* as a further problem that might have exercised Galileo's mind at the time of writing *Le mechaniche.*

b. Moment and specific weight

That specific gravity cannot unproblematically function as a force emerges most clearly from Galileo's *Discourse on bodies that stay atop of water, or move in it* from 1612. The *Discourse* was an outcome of Galileo's involvement in a public dispute concerning the reason why ice floats on water.[58] The opening sections of the work are of particular interest to us, since Galileo starts by reconsidering the foundations of Archimedean hydrostatics. As was pointed out by William Shea, Galileo started a first draft of the work by repeating the analyses of floating, sinking, and rising of bodies in a medium as they were already presented in *De motu.* Subsequently he discovered that these were insufficient because they are not generally applicable, a discovery that forced him to belabour an original new approach to hydrostatics.[59]

The complication that arose for Galileo's former treatment of hydrostatics is that he realized that a body immersed in water is not always opposed by an equal volume of water. (Just imagine the case of a large body immersed in a very narrow vessel.) That this must have had profound implications for Galileo's understanding of hydrostatic phenomena should be clear from our discussions in Sections 2 and 3. This vitiated his strategy of transforming differences in absolute weights to differences in (unconceptualized) "specific weights". How could he furthermore understand cases of equilibrium in such situations – when the absolute weights of an immersed body and a much smaller amount of water could differ greatly, although both being equal in "specific weight"?

The first and foremost thing to notice is that Galileo presents this as an "admirable and almost incredible event"[60] which stands in need of an ingenious explanation. Although he will go on to give, for the first time, an explicit definition of specific weight (by stating that "the absolute weights of solids have the compounded ratios of their specific weights and their volumes"[61]), *he clearly does not see it as immediately explanatory to claim that the body and the medium have equal specific weight*. Once again, we find further corroboration for the fact that Galileo did not consider specific weight as a primordial explanatory factor. He nevertheless had to introduce it explicitly in the *Discourse*, for reasons that we will now briefly discuss.

Galileo's explanation, which is ingenious indeed, for this admirable event is based on his concept of mechanical moment. The general cause of equilibrium is equality of moments, not equality of absolute weights (which is only a special case of the former). The truly central model for understanding natural phenomena is the balance with unequal arms, where we can *see* equilibrium

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[58] For an account of the circumstances surrounding the publication of the *Discourse*, see Biagioli 1993, chapter 3, which also contains interesting discussions on some other aspects of its contents.

[59] Shea 1972, pp. 18-20. Besides Shea 1972 and Biagioli 1993, other extended analyses of this approach, and Galileo's path leading up to it, are Galluzzi 1979 (pp. 227-246), and Palmieri 2005a.

[60] *Opere*, IV, p. 67. (Transl. from Drake 1981, p. 26.)

[61] *Opere*, IV, p. 74. (Transl. from Drake 1981, p. 44.)

obtaining between bodies of different absolute weight. We already saw that one of the possible factors making up a body's moment is the speed of its motion. Galileo will now also introduce this factor in his discussion of hydrostatic phenomena by taking into account the reciprocal motions of a body and the medium in which it is immersed. To this end he proves some geometrical theorems relating the volumes of the body and the medium with the path over which they respectively ascend and descend when the body is raised by hydrostatic pressure. One can intuitively see that when a body that is immersed in a very narrow vessel is expelled from the medium, the medium will descend over a proportionally much larger distance than the body will ascend, since the level of the medium will be lowered considerably by the expulsion of the body. If the proportion between the lengths over which body and medium move are known, the proportion between the speeds is known as well, since both motions take place in the same time. This theorem, together with the explicit definition of specific weight allows Galileo to analyse all cases of immersion, emersion, and floatation.

If the ratios of the specific weights of a body and a medium are given, the ratios of their absolute weights can be compared with the ratios of their volumes due to the definition of specific weight. The ratios of the volumes then can be transformed into a ratio of speeds due to the geometric theorem. As a result, the ratios of absolute weights can be compared with the ratios of the speeds, and hence the respective moments can be evaluated (resulting in equilibrium or disequilibrium). As an extra pay-off, Galileo now can also give a quantitative determination of the exact conditions of equilibrium, i.e. how much of a floating body will be immersed in the medium before it comes to a rest.

Once again, we see that absolute weights remain the primordial dynamical factor through their participation in a body's moment.[62] A body's specific weight, on the other hand, merely expresses some proportionality between this absolute weight and the body's volume. This proportion then controls the specific proportion between the moments of the body and the medium in which it is immersed. Hence, specific weight can function as a kind of measure for the behaviour of a body in a medium, but it cannot be said to cause this behaviour in any unproblematic way. And if we consider the situation in a void, specific weight again loses all relevance. It is only when analyzing the interaction between a body and a medium that it functions as a relevant concept, as witnessed by close attention to Galileo's explanatory scheme.

In the concluding section of the *Discourse*, we find Galileo writing that the "heaviness [*gravità*] of the medium must be compared with the heaviness of the moveable" and "that is the single, true, proper, and absolute cause of swimming above or going to the bottom.".[63] We are confronted

[62] Another way to state this would be that Galileo's conceptualization still starts from the balance as its model of intelligibility, but that he now has generalized this model to include the case of an unequal arm balance. (A similar move had already been made with his treatment of the inclined plane.)

[63] *Opere*, IV, pp. 139-140. (Transl. from Drake 1981, p. 194.)

with an apparent return to the original Archimedean scheme where the concept of moment does not occur. The extension of Galileo's explanatory scheme with that concept is indeed only needed in those situations where the hydrostatic paradox can arise. However, the preceding pages of the treatise give the impression that Galileo might really have had specific weights in mind when writing this sentence – and many readers have understood him exactly that way.[64] He claims there that "it is not the greater absolute heaviness, but greater specific heaviness, that is the cause of greater speed, nor does a ball of wood weighing ten pounds descend more swiftly than one of the same material that weighs ten ounces."[65] The presence of this old *De motu* theory in his *Discourse* testifies that Galileo had not yet found a way to fill in the gap introduced into his natural philosophy by the absence of any fully intelligibly dynamics for natural motion. Although the latter treatise is not focussed on the problem of explaining natural motion, the dynamical ideas which are introduced in it cannot help to make sense of the equal speeds of bodies of the same material. Indeed, when we consider the motion of bodies in a medium that is not enclosed in a vessel, as is the case for natural motion, the speeds of the body and the medium will always be equal, and the moments hence again reduce to the absolute weights.

c. The extrapolation argument

The publication of Galileo's *Discourse* was followed by several published replies by Aristotelian philosophers. Together with Benedetto Castelli, a former pupil, Galileo prepared a set of answers to some of these, which were published in 1615. They contain the typical scathing remarks and repetitions of earlier arguments, but hidden in the train of one line of argument is presented a remarkable new argument.[66] This argument would have momentous consequences, but these are not stressed at all in 1615. It is as if Galileo was not yet sure himself about what to do with the new insight.

Drop a ball of ebony and one of lead into water. One will observe that their speeds differ considerably. Now let the same balls fall through air. One will observe that their speeds differ only to a very small degree. Hence we can conclude that it is very likely that if we would further rarefy the medium until we would reach a void, the speeds would be equal.

Galileo stresses that the conclusion is valid for bodies of *different* specific gravity. Now, this insight was of course destined to change his conceptualization of natural motion. I have argued that his

[64] Stillman Drake, e.g., adds in his translation the following note to the passage just quoted: "Galileo considered his three kinds of floating to have been reduced to a single cause, the lesser specific weight of the floating object in comparison with water." (Drake 1981, p. 231). Cf. also Wallace 1983, p. 619: "he feels that he has successfully determined the true, natural, and primary cause of a body's floating or sinking, namely, its specific gravity relative to that of the medium in which it is immersed."

[65] *Opere*, IV, p. 133. (Transl. from Drake 1981, p.180.)

[66] *Opere*, IV, p. 659.

earliest attempts at developing a natural philosophy were modelled to a large extent on hydrostatic phenomena, even if he did not always succeed in explicitly drawing out the analogy, since the role of specific gravity had to remain ambiguous. This ambiguity was seen most clearly for the situation of fall through a void, for which hydrostatics could impossibly offer a fruitful model.

This new empirical argument might have helped Galileo to further realize that specific gravity indeed could also not provide the central key to understanding natural motion. Nevertheless, it would be almost a full twenty years before we find records of the fact that he had found a possible clue to a new and fruitful understanding. These will be dealt with in Part II.

PART II – Understanding weight as a dynamic factor: Towards a resolution

"Surely I won't loose my head to such an extent that, while falling, I wouldn't study the laws of free fall."[67]

6. *Postille a Rocco*: Rethinking the thought experiment

Galileo worked on his *Dialogue concerning the two chief world systems* mainly during the 1620's, and finally saw them to press in 1633. Dispersed throughout the work are allusions to the new science of motion discovered by the "Academician." For many seventeenth century philosophers, this was the only first hand knowledge they had of Galileo's work on natural motion. In one of these digressions, Galileo has Salviati state that Aristotle was mistaken in claiming that speed of fall is proportional to the weight of the falling body. He does not adduce any arguments for his statement, except for the empirical implausibility of such proportionality, but he does limit his remarks to bodies of the same material.[68]

It is of course an understatement to claim that the *Dialogues* spurred some debate. One of the philosophers who took up Galileo's challenge and tried to stand up in Aristotle's defence was Antonio Rocco, who in 1634 published his *Esercitationi filosofiche* in response.[69] Among the many things for which he took Galileo to task was his ignorance of the true reasons behind the phenomenon of free fall. As Galileo was not the man to let criticism that he considered misdirected easily pass, he prepared some notes (never published during his lifetime) in which he had his usual sarcastic fun with Rocco, and in which he gave the arguments which he had omitted from his *Dialogues.* It is at this point that he finally faces the gap that he was left with in *De motu.* How can he understand weight as a dynamic factor without thereby having to claim that speed of fall must be proportional with it?[70]

a. Re-presenting the thought experiment

One of the remarkable things about Galileo's *postils* is their unusually direct style. Galileo seems not so much to be trying to convince Rocco, as that he is rehearsing his arguments for himself.

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[67] The dadaist Hugo Ball, quoted in Safranski 1998 [1994], p. 115.

[68] *Opere*, VII, pp. 249-250.

[69] Rocco's *Esercitationi* were also reprinted by Favaro in his edition of Galileo's works (*Opere*, VII, pp. 567-712).

[70] I owe the suggestion that I should have a look at Galileo's *postils to Rocco* to Paolo Palmieri. These *postils* have up to now not received much attention; Drake translates some passages in his *Galileo at work* (Drake 1978, pp. 361-367); and Shea 1972 and Galluzzi 1978 pay passing attention to some passages (see the respective indexes), as does McMullin 1978, p. 226. Palmieri 2005b provides a first more detailed analysis of these *postils* (which are strictly speaking much more than mere postils).

He moreover introduces the central and most interesting part of his arguments by claiming that he will now be presenting the reasons by which he convinced himself of the falsity of Aristotle's teachings. We always have to be careful with such autobiographical reconstructions, but they undeniably give an invaluable insight in Galileo's thinking at this stage – if not necessarily in his earlier thoughts. Such an exercise in reconstruction forces him to think through the problem again, consciously trying to unravel the most central aspect of it, which could then lead to a natural and gradual dawning of insight. It is as if in this place he is practicing his favourite Socratic questioning on himself.

First, Galileo claims, he "immediately felt repugnance" in his intellect upon reading Aristotle's texts, for "how could it be that a body ten times or twenty times heavier than the other should fall downwards with ten times or twenty times the speed"?[71] Taking this as his starting point, he then "formed an axiom that could not be doubted by anyone," i.e.:

> that any heavy body [*corpo grave*] that is descending has in its motion degrees of speed, limited by nature and so predetermined, that to alter them, by increasing the speed or diminishing it, could not be done without using violence against it in order to retard it or to prevent its abovementioned limited natural course.[72]

This axiom will serve as a justification for the crucial premise of his thought experiment. It will be remembered that in his initial presentation of the thought experiment in *De motu,* this premise was justified on grounds of the empirical plausibility of the mediative character of natural speeds. The fact that the new justification introduces explicitly dynamical considerations already testifies to the fact that Galileo has gained confidence in his understanding of the dynamics behind the thought experiment.

Next, Galileo introduces not the full blown thought experiment, but the limited version for two equal bodies that are falling with the same speed. In *De motu* this version came after the general thought experiment, and it served there to hide the absence of a fully intelligible dynamics behind the thought experiment. Having now started by laying out a dynamical principle, Galileo will use the same limited situation to show how this principle plays out in the case of these falling bodies being tied together. The interesting fact about this situation is that no one would doubt that two equal bodies do fall with the same speed. But if the body that results from their tying together would have a different speed, Galileo now asks "which one of them [original bodies] will be the one which, adding impetus to the other, will double its speed"? Whereas in *De motu,* he rested content with claiming that such a doubling of the speed would be unintelligible, he is now trying to come to grips with this

[71] *Opere,* VII, p. 731.

[72] *Opere,* VII, p. 731.

unintelligibility. Given his dynamical principle, it is clear that at least in this situation *none of the bodies will exercise a force on the other*.

After this preparatory stage, Galileo presents the thought experiment. Again conspicuous is the explicitly dynamical formulation with which he describes the set-up:

> Assume now, mister Rocco, that these assumptions are true, which I don't think you are able to doubt. Thus, every descending weight [*grave*] has degrees of speed determined by nature, and that those degrees cannot be increased if not by violating its abovementioned natural constitution. Consider the two moving bodies *A*, the major, and *B*, the minor, of which, if it is possible, *A* is naturally faster and *B* less fast. Since, given the above, the natural speed of *B* can only be increased by violence, if we would want to increase it by attaching the faster *A* to it, it will be agreed that the speed of that body *A*, in violating *B*, would diminish partially, since there is no more reason that the bigger speed of *A* operates in the minor speed of *B*, than that the slowness of *B* reoperates in the velocity of *A*.[73]

The reduction argument then follows as before.

Not only is the formulation of the thought experimental set-up explicitly dynamical, it also betrays the origin of these dynamical ideas. I already stressed how the balance model shaped Galileo's understanding of forces, and that one of the central facts about this model was the presence of force-resistance pairs. This clearly surfaces in the passage just quoted, but even more importantly, it is now transformed into a true action-reaction pair (which from our vantage point is not strictly speaking the same as the equilibrating forces on a balance, which both exert their force – actually their moment – on a third body, the balance). If the faster body exerts a force on the slower, the slower will also have to exert an opposite force on the faster. This explicit recognition of the presence of a reaction for every action, at least in this kind of situation, will prove to be of the utmost importance in shaping Galileo's further dynamical thinking.

True, in the *De motu* presentation of the thought experiment Galileo had already stated: "who can doubt that the slowness of the wax will be diminished by the speed of the bladder, and, on the other hand, that the speed of the bladder will be retarded by the slowness of the wax"[74]. Nonetheless, the explicit insight that this mutual retardation and acceleration is the effect of interacting forces is conspicuously missing.[75] Most importantly, he does not think through its possible consequences for

[73] *Opere*, VII, p. 732.

[74] *Opere,* I, p. 265. (Transl. from Galilei 1960, p. 29.)

[75] It is perfectly possible (and I tend to believe: true) that at the time of *De motu*, Galileo understood the effect of combining the wax with the bladder (and vice versa) purely in terms of the effect on their "specific" gravity (in perfect analogy with what happens with the alloys of the king's crown), which is then only indirectly reflected in the speeds.

what happens in the thought experiment – as is testified by the different treatment of the case of the two equal bricks. Considerations of empirical and intuitive plausibility seem to do most of the work in this early version. The true innovation of the *postils* lies in the attempt to uncover the grounds behind these judgements.

b. Resolving the dynamical conundrum

Immediately after the formal presentation of the thought experiment follows the most interesting passage of the *postils* – and, I would add, one of the most fascinating pieces of writing ever produced by Galileo. I will hence quote in full:

> These are mathematical advances, mister Rocco. They are consequences that, as far as I can ascertain, were not expected by you. And since I am certain that you persist in believing that once the gravity in *A* is increased by the addition of *B*, its velocity should also increase, if not proportionally to the weight [*peso*] as you required up to now with Aristotle, then at least in some way; how much would it not surprise you if I would show you that the addition of *B* does not increase the gravity of *A* with one hair, nor would the addition of a thousand *B's* increase it, and that given that is doesn't grow in weight [*peso*], by consequence its speed doesn't grow either, thus making you touch with your own hand how you are totally misled in this matter! So you will say: how could it be true that, *A* and *B* being two pieces of lead, the one put on top of the other, it will not increase its gravity? And I would add that even if *B* was made of cork the weight [*peso*] will increase, and I agree with you in admitting that *A*, placed on a scale, will weigh [*peserà*] more with the addition of *B*, even if it was not of cork, but a flake of cotton wool or one leaf of flax; and if *A* would weigh [*pesasse*] a hundred pounds, and *B* an ounce of plumes, on the scale their compound will weigh [*peserà*] a hundred pounds and one ounce. Yet to take advantage of this experience in reference to what we are concerned with is a useless and irrelevant matter. But at any rate, mister Rocco, if you put the palm of one hand under a cannonball weighing a hundred pounds [*100 libbre di peso*], which is suspended and supported by a rope, and you would only touch it, tell me whether you would feel weighed down [*aggravarvi*]? I know that you will answer no, for its weight [*peso*] is supported by a rope, and its descending is entirely prevented. When the rope is cut, and you would interdict this effect by the strength of your arm, you would indeed feel a burden [*gravarvi*] on your hand, which [hand] should do the job of the rope by prohibiting to the ball its natural descent. But when you would not oppose the ball which has been let free, but you would give in to its impetus by lowering the hand with the same speed at which the ball would descend, tell me anew if you, apart from touching it, would feel yourself weighed down by its weight [*dal suo peso gravarvi*]? It is absolutely necessary to reply that this is not the case,

> because you don't offer any resistance to the pressing [*premura*] of that weight [*peso*]. Conclude now from this clear and brief reasoning, since it is not possible to define being weighed down [*aggravato*] if not as that opposition to a weighing body that is descending, that by the addition and superimposition of the abovementioned bricks the one to the other, which even you will allow to be descending with equal velocity because they are the same, the gravity of the one is not increased by the other. Hence, also the velocity is not increased.[76]

"Yet to take advantage of this experience in reference to what we are concerned with is a useless and irrelevant matter." In this one sentence is contained the resolution of the conundrum. In one master stroke Galileo restructures the whole of his natural philosophy. To put it in a nutshell: *by asking Rocco to imagine using a falling balance, he shows its inapplicability as a model for a very central class of natural phenomena.* The balance hence loses the centrality which it had always had within his philosophy. He now urges us that if we want to understand the dynamics of falling bodies, we should not be misled by what happens on a balance!

The way Galileo establishes this limitation of his model of intelligibility merits closer attention. The most important step in his attempts to convince Rocco (and himself, I would suggest) occurs when he substitutes the hand and arm for the previously assumed balance. This substitution enables him to physically grasp the absence of action-reaction pairs in the case of the falling body and the hand moving down with the same speeds. Indeed, everybody can feel this for himself – even the illustrious signor Rocco could do so. We are hence witnessing the introduction of a new set of shared experiences, possibly allowing for a better understanding of the phenomenon of free fall. Experience, which as Galileo himself adds, need not be confined to the sense of sight, but which the senses of hearing and touch can also perfectly have.[77] Tellingly, we still find Galileo exploiting the physical concreteness and manipulability of his old model of intelligibility (features singled out by Machamer as essential to its functioning), even in overcoming its own limitations. The hand and arm are moreover easily assimilated to a second body falling along with the first body. And in the absence of any interaction, it then makes no sense to speak about the falling bodies weighing more or less. This latter conclusion is of course justified through the claim that "it is not possible to define being weighed down if not as that opposition to a weighing body that is descending." At first sight it might seem that Galileo is reversing to some kind of subjective notion of weight by placing this stipulation at the centre of his explanations.[78] Yet on this interpretation we would lose sight of the essentially interactive aspect of the action of the force of weight which he is laying bare here. His terminology makes clear that he is interested in the two sides of this interaction: there is no "pesare" of the body without the

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[76] *Opere*, VII, pp. 732-733.

[77] *Opere*, VII, p. 724.

[78] Palmieri 2005b, p. 232, n. 26, speaks of a "'psychological' definition of weight".

experience of being burdened ("aggravato"), which in turn finds its origin in the counter-force we have to keep on exerting on the body. Galileo is hence able to extract something fundamental about the property of weight from our way of experiencing it: *a body's gravity gives rise to "peso" only if it is opposed by a continually (re)acting resisting force*.

Already in *De motu*, Galileo had given the following definition:

> We are said to be weighed down [*gravari*] only when some weight [*pondus*] rests on us which by its heaviness [*gravitate*] tends downward, while we must by our force [*vi*] resist its further downward movement; and it is that resistance which we call being weighed down [*gravari*].[79]

It is important to ask why this definition had not already in this early work led up to the conclusions which are now shown to follow from it. In the first place it is important to note that Galileo had introduced this definition of being weighed down in *De motu* to back up his claim that elements have no weight in their own place. Now, since such elements simply do not tend downward anymore when they are in their natural place, this situation is considerably more straightforward than when one is dealing with falling bodies. These do have a tendency for downward motion, and the balance hence seemed eminently applicable. The balance itself moreover serves to hide the necessary action-reaction pairs in the measurement of weight. After all, the seemingly crucial elements for such measurements are the weight and counterweight and their respective distances from the fulcrum. The physical role of the fulcrum itself is often passed over in silence, although it is precisely the fixed nature of the latter which enables the measurement. The counterweight can only resist the downward motion of the weight because the fulcrum introduces a reaction force on the combined action of both weights into the system. (If the bodies weren't continually weighing down on the fixed point this reaction force would not arise, and the system would simply fall down.) Yet, the confusion easily arises that it is the counterweight which plays the resistive role of the given definition, which would make the non-sense of using a falling balance less obvious. In this respect it is suggestive to note, as has been done by Paolo Palmieri, that in *De motu* Galileo had presented the two equal bricks as falling adjacent to each other, while in the *postils* he is considering bricks which are put upon each other.[80] This seems to be exactly what is needed to bring the interactive character of weighing down to the fore, whereas the former presentation was still very much tied to the image of a balance.[81]

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[79] *Opere*, I, p. 288. (Transl. from Galilei 1960, p. 54.)

[80] Palmieri 2005b, p. 232.

[81] Guided by this image it might even have appeared as if the two falling bodies were keeping each other in equilibrium, hence mutually weighing down on each other – this is the kind of image which Galileo will repudiate in the fragment on the law of the lever, referred to in the next paragraph. This is moreover precisely the image which clearly guided Bennedetti in presenting his version of the thought experiment, since he suggests that four equal bodies fall down with the same speed as the body that is composed by their conjunction because

That Galileo brought precisely these features to the focus of his attention after having rethought his thought experiment is testified by a dialogue fragment which was probably intended for inclusion in either the first or the second edition of the *Discorsi*, but which remained in manuscript form.[82] In this fragment, Galileo expresses doubts about the conclusiveness of the pseudo-Aristotelian proof method for the law of the lever (he had always significantly refrained from granting it the status of a demonstration in his writings). Instead he offers a more satisfactory proof; a proof which however is not the Archimedean proof that was given by Galileo both in his *Mecaniche* and in the second day of the *Discorsi* (presumably because Galileo sought a more physically appealing proof). If one puts two weights on a balance, and then let it go freely, it will fall perpendicularly along the line connecting the common centre of gravity of the two weights with the centre of heavy things. But if we fix the balance in this common centre of gravity, there will be no motion and the balance will be in equilibrium (and if this fulcrum does not coincide with the centre of gravity, the arms of the balance will respectively move up and down). Now, this proof was not original with Galileo, as it faithfully recapitulates the teachings of Guidobaldo del Monte (without mentioning the latter).[83] However, the fact that we find Galileo reversing to exactly this kind of explanation is significant. In the *Dialogue concerning the two chief world systems* of 1633, he had still presented the pseudo-Aristotelian proof method without any sign of dissatisfaction (but with the usual caution in not calling it a demonstration, but referring to its confirmation by many experiments "*con molte esperienze*").[84] But now, after having rethought his thought experiment, he apparently comes to prefer an explanation which explicitly singles out the necessity of a fixed fulcrum. To put it a little bit more suggestive: his new method of proof is designed to show that a falling balance is no longer an instrument for the measurement of weight.

That it is furthermore precisely the interactive aspect which is still missing at the time of *De motu* is proved by a passage in which Galileo seems to come close to the insights which he reached only here in the *postils*. In offering an explanation of the accidental acceleration of free fall, he had already stressed the fact that when a "stone is at rest in someone's hand, we must not say that in that case the holder of the stone is impressing no force upon it. Indeed, since the stone presses downward with its own weight, it must be impelled upward by the hand with a force exactly equal, neither larger nor smaller."[85] Yet when the stone is let go, the force of the hand remains for some time with it,

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the separated bodies will together be able *to equilibrate* the body composed of them *during their fall*; cf. his *Resolutio…* from 1553 (translated in Drake and Drabkin 1969, see especially pp. 150-151).

[82] *Opere*, VIII, pp. 438-440.

[83] Cf. Van Dyck 200+ for Guidobaldo's mechanics; Micheli 1995, p. 150-151, points out the similarity between Guidobaldo's and Galileo's treatments, yet in a slightly different context. Drake and Drabkin 1969 offer translations of large parts of Guidobaldo's *Liber mechanicorum.*

[84] *Opere*, VII, pp. 241-242

[85] *Opere*, I, 320. (Transl. from Galilei 1960, p. 91.)

although continually diminishing in strength. A few pages earlier, Galileo had moreover already explained how we should conceptualize such impressed force. The body in which it is impressed retains its natural and intrinsic weight, but it assumes a preternatural lightness "in the same way as when it is placed in media heavier than itself."[86] Now, the first book already made abundantly clear how we should model this effect of a medium. The idea of impressed lightness thus actually becomes an attempt to have the balance model transferred *into* the body. To put the situation graphically: Galileo imagines the body during its fall as if it is continually in a balance with as counterweight the impressed force which is gradually diminishing, causing the body to become heavier in fall and speeding up. He had not yet freed himself from the falling balance; and the resisting force was indeed assimilated to a counterweight.

As a result of his deconstruction of the balance model (as applicable to falling bodies), Galileo can now uphold seemingly conflicting theses. Weight is indeed a force, and if a body has more matter (and hence more gravity), it exerts a greater force that can be measured using a balance. Exerting more force does moreover result in an increase of speed. And yet, speed of free fall can be independent of gravity, the reason being that falling bodies do not necessarily weigh down more by the addition of more matter – or to say the same thing, that this extra added matter exerts an extra force on the body which would cause it to speed up.

Galileo surely also had felt the uneasiness that anyone must feel who is first confronted with this insight. After all, as was already repeatedly claimed in his earlier writings, weight as measured by a balance is caused by the body's gravity, which is a tendency to move naturally downward. In his *postils to Rocco* Galileo stresses that this still holds true,[87] but he also warns Rocco that it does not necessarily follow that this greater tendency causes a greater speed, only that the body "has to tend more downwards."[88] It is true that Galileo does not give an explicit explanation of how we *should* understand the precise link between this tendency and the resulting speed, but we will see in Sections 7 and 8 that there are some clear hints in his latest thoughts on natural motion – starting from the insights contained in these *postils,* and reflecting the lasting influence of the thought experiment in his thinking.

One other thing of importance that is presented in the *postils* is the extrapolation argument establishing that in a void all bodies, regardless of their material, will have the same speeds of fall. We saw that Galileo had already experimentally established this fact in the period 1612-1615, but it is only at this point that it becomes integrated in a broader natural philosophical scheme. This incorporation is of course highly significant. It is only at this point that the seemingly paradoxical fact can start to

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[86] *Opere,* I, pp. 311-312. (Transl. from Galilei 1960, p. 81.)

[87] *Opere,* VII, pp. 722, 725.

[88] *Opere,* VII, p. 722.

become intelligible. As we saw, the instrument which allowed Galileo to make this situation intelligible was the thought experiment.

7. *Discorsi*: Presenting the thought experiment

Almost immediately after his fateful encounter with the Roman inquisition that followed upon the publication of the *Dialogue*, Galileo began preparing a work in which he would finally expound his theory of motion. The work, which would go to press in 1638 as the *Discourses and mathematical demonstrations concerning two new sciences pertaining to mechanics and local motions*, was essentially a continuation of many earlier researches on both natural motion and the strength of materials.

As we saw, at the time of composing the *Discorsi*, Galileo also wrote down his *postils to Rocco*, so it is not surprising to find much of its contents reappearing in the book.[89] However, there are also some minor, but relevant changes in the presentation. In the first part of this section, I will be mainly interested in laying bare the argumentative strategy used by Galileo in the part of the first day of the *Discorsi* where he presents the thought experiment. In the next parts, I will take up in turn Galileo's treatment of the effect of a medium, and of the accelerated character of free fall.

a. From the model to the world

The first of the four days of the *Discorsi* is a collection of discussions on miscellaneous topics. These discussions take place between three men: *Simplicio*, standing in for the Aristotelian philosopher, *Sagredo*, the intelligent and open minded lay-man (often suggesting theses that Galileo had defended earlier, but had by now discarded), and *Salviati*, the spokesman of Galileo. Their discussions introduce many of the themes to be treated in the next days and, most importantly, lay some of the necessary groundwork – with respect to the dynamics of free fall, the paradoxes involving infinities, and the nature of matter.[90] Among these are the passages that introduce the thought experiment and the extrapolation argument.

In arguing that in a void the speed of fall is equal for all bodies, Galileo prepares the ground for his science of local motion, which he will present in the third and the fourth days of the book. This independence from the kind of matter guarantees the universality of that science, and delimits its true

[89] It is interesting to note that also many of the passages on infinity in the first day of the *Discorsi* were already contained in these *postils*, which hence provide a fairly extensive sketch of the discussions in this first day.

[90] That the first day provides some of the necessary groundwork with respect to the treatment of free fall in the third and fourth day has been forcefully stressed by Clavelin 1968; for the paradoxes involving infinities, see e.g. Galluzzi 1978; and for the link with the second day through its treatment of matter, see Biener 2004.

domain of application: the void. Galileo also gives an intriguing argument to show that this science will still be valid for actually occurring instances of motion, i.e. motion in a medium.[91]

The context within the first day in which these discussions arise is worth mentioning. In discussing the constitution of all matter, Salviati at one point invokes the possible existence of voids. From here on the discussion is diverted to a discussion on Aristotle's proof of the impossibility of void, which turns around the presumed impossibility of motion in a void. This allows Galileo to have Salviati expose the errors in Aristotle's philosophy of local motion. As an alternative, Galileo's own philosophical scheme will be proposed, which not only allows for motion in a void, but which even makes this the central case to be considered. I think it is not unimportant not to forget this occasion on which Galileo introduces his discussions of Aristotle's and his own schemes. This restricts Galileo in that he cannot start his arguments by considering the case of fall in a void, since he first has to make plausible that such a case is not unthinkable.

Galileo's refutation of Aristotle's teachings on free fall follows almost exactly the more than forty years old lead of *De motu*. He first attacks the idea that the speed of fall is proportional to the weight of the bodies by stating that such proportionality is simply ridiculous, since empirically wildly implausible. Thereupon follows the thought experiment, explicitly restricted to bodies of the same specific gravity. After a few preliminary remarks on the effect of impediments (which are much more extensively discussed further on in the discussions in the first day – see section 7.b of the present paper), responsible for the observed differences in speeds, Galileo then introduces his arguments against the second Aristotelian claim: that speed of fall is inversely proportional with the resistance of the medium, and hence that the absence of all resistance would imply an instantaneous, and thus unthinkable motion. These arguments are modelled again on some *De motu* arguments, but this time Galileo does not explicitly state and defend the Archimedean scheme which lay behind them. As a result his argument for the possibility of motion in a void is less perspicuous as it was in the earlier unpublished treatise. It seems that he was much more interested in immediately introducing the following step in his line of arguments: the extrapolation argument for the equality of speeds of fall for *all* kinds of bodies in a void. Once this is established, Galileo then goes on to show how to understand the proportionally different behaviour of bodies of different material when falling in a medium.

The presentation of the thought experiment itself is clearly modelled on the earlier recapitulation in the *postils to Rocco.* It is however no longer preceded by the limited argument for two equal bodies. Apparently, Galileo now had become so confident in his understanding that he no longer thought that he needed this preliminary situation, which had served him so well to unravel the conundrum. The argument itself is also presented in a tighter form, apparently the result of a conscious rewriting, but the crucial premise on the mediativity of natural speeds is again introduced on the basis of exactly the same explicitly dynamical considerations.

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[91] I have analysed some aspects of this argumentative strategy and its implications in Van Dyck 2005.

After the presentation of the reductio argument follows a discussion between Simplicio and Salviati, which Galileo uses to convey the same crucial message as in his earlier reprimand against Rocco. The presentation is again much more streamlined, thereby loosing some of its earlier forcefulness, but there is an interesting novel feature, which I have emphasized in the text:

> SIMP. I find myself in a tangle, because it still appears to me that the smaller stone added to the larger adds weight [*peso*] to it; and by adding weight, I don't see why it should not add speed to it, or at least not diminish its speed in it.
> SALV. Here you commit another error, Simplicio, because it is not true that the smaller stone adds weight [*peso*] to the larger.
> SIMP. Well, that indeed is quite beyond my comprehension.
> SALV. It will not be beyond it a bit, when I have made you see the equivocation in which you are floundering. *Note that one must distinguish heavy bodies [*gravi*] put in motion from the same bodies in a state of rest.* A large stone placed in a balance acquires weight [*peso*] with the placement on it of another stone, and not only that, but even the addition of a coil of hemp will make it weigh [*pesar*] more by the six or seven ounces that the hemp weighs [*peserà*]. But if you let the stone fall freely from a height with the hemp tied to it, do you believe that in this motion the hemp would weigh on [*graviti sopra*] the stone, and thus necessarily speed up its motion? Or do you believe it would retard this by partly sustaining the stone?
> We feel weight [*sentiamo gravitarci*] on our shoulders when we try to oppose the motion that the burdening weight [*peso*] would make; but if we descended with the same speed with which such a heavy body would naturally fall, how would you have it press and weigh on us [*graviti sopra*]? Do you not see that this would be like trying to lance someone who was running ahead with as much speed as that of his pursuer, or more? Infer, then, that in free and natural fall the smaller stone does not weigh upon [*non gravita sopra*] the larger, and hence does not increase the weight [*peso*] as it does at rest.[92]

First notice the complete reversal with respect to the earlier presentation of the thought experiment in *De motu*. There the reductio argument was immediately followed by the (rhetorical) question: why would the bodies change speed on being tied together? Here we are confronted with the opposite question: why wouldn't they? But most importantly, the question is now followed up with an answer. It seems that it is only now, when he is in the position to dismantle the paradox, that Galileo dares to bring it into the open. Now he can play his favourite argumentative game of first completely destabilizing his opponents' prior convictions by making them admit what they seemingly have to

[92] *Opere,* VIII, p. 108. (Transl. from Galilei 1974, pp. 67-68.)

deny, followed upon by the presentation of his own alternative view which enables him to restore coherence in at least the reader's mind (if not the opponent's).

The innovation with respect to the treatment in the *postils to Rocco* is subtle but of the utmost importance.[93] Whereas in the earlier exposition, Galileo merely claimed that the balance could not be used to measure the weight of falling bodies, he now sees a distinction *within* these bodies themselves. That is, he explicitly moves from a limitation in the model to an essential difference in the target system. *We* would say: either a body's weight is used in accelerating it, or in pressing down on the balance which resists its motion, but it cannot do both things simultaneously. We will see in the next part of this section how we can impute to Galileo something rather similar on the basis of his treatment of fall in a dense medium.

At this point, we thus witness how a peculiar feature of a model of intelligibility (its inapplicability) is transferred to the world. This feature can now become one of the immediate characteristics that the things in the world "show themselves;" i.e. it provides new bedrock for judgements of intelligibility. Of course, one first has to be taught to see (or feel) this fact – through thinking through the thought experiment – but once one has learned to notice it, it becomes one of these incontestable experiences that can back up explanations of more complicated phenomena. This is of course not to deny that learning how to exploit this fact in explaining further phenomena takes a lot of hard work, which it finally would take someone of the stature of Newton to fully accomplish. Yet, we will see in section 8 how Galileo himself already made some preliminary but promising attempts in this direction.

b. Understanding the effect of a dense medium

After having presented the thought experiment and his claim that in a void *all* bodies would fall with the same speeds, Galileo goes on to explain why we do not observe this equality in dense media. The explanatory scheme is immediately recognizable: the primary effect of a medium is to subtract from the weight of an immersed body, following Archimedean hydrostatics.[94] The re-

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[93] Another innovation introduced in the *Discorsi* is the example of the lance, which seems to open up Galileo's insight in action-reaction to a more general treatment of impact. Interestingly enough, Galileo indeed takes up the very same example later in the fourth day when he discusses the differences in impact of projectiles depending on the state and characteristics (elastic vs. inelastic) of the thing struck. At the same time, this treatment clearly shows the limitations of Galileo's understanding of the generality of action-reaction, as in this context he remains almost completely (but only almost!) silent on the effect that he impact has on the motion of the projectile itself. (*Opere*, VIII, p. 291.)

[94] Galileo nowhere explicitly differentiates between buoyancy and a medium's frictional effect. Nevertheless, Clavelin 1968, pp. 331-353 (especially pp. 342-343), has claimed that it is possible to discern a coherent distinction between these effects in Galileo's treatment of them. In the present paper, I will not try to decide the

emergence of this framework within the *Discorsi* raises some problems for Galileo, which he nowhere explicitly tackles, but which he tries to circumvent in his presentation.

The guiding idea behind Galileo's explanation is simple. Assuming the empirically suggested equality of speeds in a void, the alleviation effect of a medium serves as a measure for the way in which this speed is affected by the medium. The only innovation with respect to *De motu* thus seems to be the assumption of equal speeds in a void – it can moreover be noticed that already in *De motu* Galileo had stressed that only in a void we are dealing with the truly natural motion.[95] But why would the ratio between a body's weight and an equal volume of the medium's weight serve as a measure for the way the body's speed is affected, if this speed is not caused by the body's weight in the first place? How can Galileo justify this reappearance of weight as a dynamic factor after having discarded its role? The *De motu* explanation of the effect of a medium sits uncomfortable within the *Discorsi*.[96]

It is very improbable that Galileo would not have noticed the tension within his discussions in the first day. That he nevertheless extensively discusses this analysis of the medium's effect testifies to the fact that he must have been satisfied with its empirical plausibility. In introducing this analysis he moreover briefly touches on this problematic issue:

> If we then assume the principle that in a medium no resistance exists at all to speed of motion, whether because it is a void or for any other reason, so that the speeds of all moveables would be equal, we can very consistently assign the ratios of speeds of like and unlike moveables, in the same and in different filled (and therefore resistant) mediums. This we shall do by considering the extent to which the heaviness [*gravità*] of the medium detracts from the heaviness [*gravità*] of the moveable, which *heaviness is the instrument by which the moveable makes its way*, driving aside the parts of the medium. *No such action occurs* in the void, and therefore no difference in speed is derived from different heaviness.[97]

The description is suggestive, but a little too cryptic to impute to Galileo a definite solution to the tension. Yet, if we remember how he made intelligible the non-operativeness of absolute weight in

hard question whether this is truly possible and justified. I will rather take a necessary first step towards a satisfactory answer: to ascertain the different status (with respect to the earlier *De motu* treatment) that attaches to the buoyancy effect after Galileo reached his new understanding of weight as a dynamic factor through rethinking the thought experiment.

[95] "[W]e may assert beyond any doubt that only in the void can the true and natural ratios of velocities occur." *Opere*, I, p. 296. (Transl. from Galilei 1960, p. 6.)

[96] The tension was already eloquently summarized by Dijksterhuis 1922, p. 233 (my translation): "by subtracting the upward pressure, the effect of the medium has been taken into account; it is now as if we were again in a void, but with a lighter body. But this is supposed to fall as fast as the heavier…"

[97] *Opere*, VIII, p. 119. (Transl. from Galilei 1974, p. 78. My emphasis added.)

free fall, we can see how this is already constraining his attempts at such a solution. It is entirely coherent to assume that weight again becomes operative at the moment that a body encounters a medium in its fall, since the parts of the medium are at rest and hence truly resistive – *there is something for the body at which it can weigh down*.[98]

The remaining puzzle resides in the fact that this should have an effect at the body's speed. This does suggest that whatever it is that is operative in giving a body its downward motion, it is somehow intimately related to weight without being identical with it. Either it is giving a body its downward motion (and in such a way that all bodies receive the same speeds), or it is giving it weight by which it can push aside the parts of the medium. In the latter case, the fact that it gives the body weight also implies that it gives the body less of its downward motion.

> The medium … opposes that transverse motion now with less, and now with greater resistance, according as it must be slowly or swiftly opened to give passage to the moveable … . This means some retardation and diminuation in the acquisition of new degrees of speed… .[99]

Apparently, a body can also be at rest and in motion at the same time. The distinction between moving bodies and bodies at rest is only an absolute distinction when we neglect the presence of a resisting medium. But this latter conclusion remains unsaid in the *Discorsi*.

c. Understanding the accelerated character of free fall

One striking fact about Galileo's presentation in the first day remains to be mentioned: the almost casual treatment of the accelerated character of free fall. It is true that when treating the frictional effect of a medium he cleverly exploits this acceleration, but the overall impression is undeniably that Galileo seems more concerned about the fact that in a void all bodies have the same speeds, i.e. that there is no direct correlation between (specific) weight and natural motion, than he is about the accelerated character of that motion.

It is important to remind ourselves of the fact that from the beginning Galileo was presented with different challenges in his attempts at developing a new science of motion. In discussing *De motu* in Section 3, we limited our attention to the first book in which Galileo tried to conceptualize the role

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[98] Damerow *et al.* 2004, pp. 269-70, claim that Galileo simply takes over the older *De motu* theory with the addition of the proposition that in a vacuum all bodies fall with the same speed – implying that his dynamical thinking has remained basically unchanged in between the two treatises. It is clear that I cannot accept such a conclusion.

[99] *Opere,* VIII, p. 119. (Transl. from Galilei 1974, p. 78.)

of gravity in free fall. The most important problem to which the second book was devoted, was explaining the apparently accelerated character of free fall. The fact of acceleration clearly creates important tensions within Galileo's hydrostatic model, which seems to have only room for uniform motions. The specifics of his ingenious solution in *De motu* need not concern us here. It is only important to realize that the tension is created by the dictum that causes and effects must be proportional. That a body's speed changes during natural motion whereas its weight remains constant hence further complicated Galileo's attempt at understanding weight as a dynamic factor; i.e. it is another fact that sits very uneasily with his original model of intelligibility.

Of course, by the time of the *Discorsi,* Galileo had abandoned his hydrostatic model for free fall, and he was strongly convinced of the fact that acceleration was an essential characteristic of natural motion. This conviction seems to have been mainly the result of the discovery that he could give an exact mathematical description of this acceleration, coupled with the internal problems which anyway had already threatened his original natural philosophical scheme. Winifred Wisan and Paolo Galluzzi have shown how Galileo at first tried to come to grips with this acceleration through the exploitation of his understanding of motion on an inclined plane.[100] Such an attempt had appeared destined to fail however, because it seemed that it could not accommodate the fact that this acceleration should be independent of weight. By the time of the *Discorsi* he had not come up with a satisfactory understanding of acceleration, and it seems to be accepted there without further ado as a basis fact of nature:

> A heavy body has from nature an intrinsic principle of moving toward the common center of heavy objects (that is, of our terrestrial globe) with a continually accelerated movement, and always equally accelerated, so that in equal times there are added equal new momenta and degrees of speed.[101]

In the introductory discussions of the third day, Galileo moreover has Salviati famously declare that "for the present, it suffices our Author that we understand him to want us to investigate and demonstrate some attributes of a motion so accelerated (whatever be the cause of its acceleration)…"[102] That Galileo truly saw this only valid "for the present," and always remained concerned about providing causal analyses of natural phenomena – albeit possibly changing the criteria about what counts as a successful analysis – has by now been sufficiently argued by many writers. I will hence not repeat these arguments here.[103] This is further corroborated by the fragments

[100] Wisan 1974, pp. 222-229; Galluzzi 1978, especially chapter 4 of the second part.

[101] *Opere*, VIII, p. 118. (Transl. from Galilei 1974, p. 77.)

[102] *Opere*, VIII, p. 202. (Transl. from Galilei 1974, p. 159.)

[103] Cf. e.g. Machamer 1978 and Wallace 1983.

to be discussed in the next section that postdate the *Discorsi* and where Galileo explicitly engages in such causal analysis. And it is also proven by the fact that Galileo tried to indicate that his constant acceleration need not be in contradiction with the proportionality between cause and effect.

> When I consider that a stone, which falls from some height starting from rest, constantly acquires new increments of velocity, why should I not believe that these additions are made in the simplest and easiest manner of all? The moveable remains the same, as does the principle of motion. Why should the other factors not remain equally constant? You will say: the velocity then remains the same. Not at all! The facts establish that the velocity is not constant, and that the motion is not uniform. *It is necessary then to place the identity,* or if you prefer the uniformity and simplicity, not in the velocity but *in the increments of the velocity*, that is, in the acceleration.[104]

The constant effect shows in the acceleration, not in the velocity of natural motion. The constant cause somehow lies in the falling body and is connected with the effect as a "principle of motion". That the cause lies in the falling body irrevocably brings to mind the body's matter – certainly if we take into account Galileo's scorn for explanations through "occult" properties. And at the beginning of *Le mecaniche* Galileo had already defined a body's weight to be caused by its matter.[105] Again, there seems to be "something" about the body that is both responsible for its natural motion downward and for its weight – but as the thought experiment has by now taught, without being simply identifiable with the latter.

Both the accelerated character of free fall and the interaction with a dense medium inevitably bring a question to the fore that was left unanswered by the thought experiment: granted that it is intelligible that natural motion is not determined by weight, it is only natural to further inquire into what it is that *does* determine its character. At first sight the thought experiment could not offer any further help on this score. It had anyway always been presented without taking into account acceleration. Yet in the next section we will see how it played a role in Galileo's final efforts, in which he came close to a satisfactory understanding of the phenomenon of free fall. At this point, the thought experiment has hence taken over the role of the balance as a model of intelligibility directing Galileo's dynamical thinking, by drawing particular facts about the relation between weight, free fall, and equilibrium to his attention.

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[104] *Opere,* II, p. 262. (Transl. from Westfall 1971, p. 5. My emphasis.) This is a passage from a first draft of the third day of the *Discorsi*, which is commonly dated around 1609.

[105] *Opere*, II, p. 159. (Also cited in section 5a of the present paper.)

8. *Intorno a due nuove scienze*: Unifying "statics" and "dynamics"

At the closing sections of the *Discorsi,* Galileo repeats a promise which he had already expressed earlier in the fourth day: that he will also discuss the phenomenon of percussion, by which a moving weight exerts a much greater power on any resistance than does a body which is merely weighing down.[106] On this topic, Galileo admits, through the intermediary voice of Salviati, to have "long remained in … shadows", and only "after he had spent thousands of hours during his life in theorizing and philosophizing about this, he had arrived at some ideas very distant from our first conceptions".[107] Galileo however never was able to complete the projected fifth day of the *Discorsi* in which he would live up to that promise, but among his manuscripts are contained a dialogue which was intended to that end as well as some further notes on the topic.[108]

That these attempts to come to grips with the phenomenon of percussion in part postdate the publication of the *Discorsi* implies that Galileo could tackle the problem starting from the dynamical insights which he had already reached within the *postils to Rocco* and the first day of the *Discorsi*. As we will see, by bringing together the problems treated in these works with the problem of percussion, he was able to come very close to a more or less satisfactory solution to the remaining puzzles within his understanding of free fall. These leads would afterwards be further taken up by Evangelista Torricelli, who had assisted Galileo in the final months of his life. In the first two subsections, I will first offer a summary of an important conclusion that Galileo reaches in his notes on percussion, and then provide a new interpretation of how this conclusion became integrated in Galileo's attempts at developing a satisfactory dynamics for free fall. In the third subsection, I will finally show how we can see the thought experiment still driving these investigations.

a. Measuring the force of percussion

The first traces of Galileo's involvement with the problem of percussion date from the time of *Le mecaniche*.[109] When he comes back to the problem at the end of his life, he still tries to subsume it under his analysis of the mechanical machines (but as was announced in the fourth day the results reached thus are considerably distant from these first attempts). This implies that he tries to understand the mechanism by which the force of the weight of the body is multiplied so that it can give rise to

[106] *Opere*, VIII, pp. 292-293, 312-313.

[107] *Opere*, VIII, p. 293. (Transl. from Galilei 1974, p. 242.)

[108] See Drake 1978 (chapters 20, 21) for the historical circumstances surrounding both the announcement and non-delivery of the fifth day.

[109] Galileo's theory of percussion has not received much attention in the literature. Yet both Westfall 1971 (chapter 1) and Galluzzi 1979 (chapter 7 of the second part) contain very useful and insightful discussions, as do Moscovici 1967 and de Gandt 1987 who both also discuss Torricelli's exposition of this theory.

potentially useful effects by means of the concept of (mechanical) *moment*. It can be remembered from Section 5 that a body's moment expresses its tendency for downward motion, and that it arises from its heaviness combined with either its relative position (with respect to the fulcrum of a lever), or with its velocity. It is clear that in the case of percussion velocity will be the relevant parameter.

The main part of the dialogue on percussion consists in the exposition of several possible ways of measuring the moment of percussion of a falling body. The recurring theme during these discussions is the *infinity* of this moment. One proposal is to take as measure the static weight that drives a pole as far in the ground as does the blow of a percussant body. Galileo explicitly uses the term "dead weight" for this measuring body which operates through its heaviness alone. The problem with this proposal is that the measure is dependent on the resistance of the pole – the more resistant it is, the proportionally heavier the dead weight must be to have the same effect as the falling body. Yet, although this procedure is not appropriate as a uniform measure of the moment of percussion, it already teaches Galileo something important. If a body has fallen on the pole and driven it a certain distance in the ground, and if we then let it fall again on the pole, its second blow will drive it still further in the ground (although a smaller distance). The same is obviously not true of the dead weight: it operates by pressing, which effect can not be accumulated once the pole has been driven a certain distance. No matter how long it will lie on top of the pole, its effect is already completely exhausted. This implies that the effects of percussion and of a dead weight are truly incomparable. Any resistance which is not infinite will always give way to a blow of a percussant body, which thus can be said to have an infinite moment.

Another proposal to measure the moment of percussion is to use a system consisting of two weights connected by a rope over a pulley, one weight lying on an inclined plane, the other hanging freely along the vertical side of the plane. By letting the free body fall over a certain distance until it pulls the other body through the rope, the moment of its percussion can be measured by determining the distance over which the resisting weight is moved on the inclined plane. The necessary conclusion is again that any weight will be lifted by a falling body, since the counterweight is initially at rest, and thus has a moment which is zero compared to that of the moving body.

Both instances make clear that the infinity of the moment of percussion is actually the result of the incommensurability of the effect of a falling body with the effect of a dead weight. This incommensurability can be understood by considering the role played by time. As was already clear from the case with the dead weight pressing on the pole, the effect of its moment is exhausted in a single instant. The same is obviously not true of the falling body, which can accumulate its moments of gravity before actually hitting the pole.[110] In one of the fragments attached to the dialogue, we find

[110] The possibility of such accumulation is a belated consequence of Galileo's initial choice to conceptualize moment as the *combination* of the effects of weight and speed; i.e., moment is not merely a restriction that is placed on the effect of a constrained weight but something that adds to its effect. The former possibility could

the following summary of the situation by Galileo, where he discusses the differences between a body that presses against another and a body that strikes it:

> …the one that moves [a thing] by pressing without striking, and the other that acts by striking. The mover that operates without impact moves only a resistance which is less, though [it may be] only insensibly [less], than the power [*virtù*] of the pressing heaviness; but that will move it through an infinite distance, accompanying it always with its same force. That which moves by striking moves any resistance, thought [this may be] immense; but [moves it only] through a limited distance.
> Hence I consider these two propositions true: that the percussent moves an infinite resistance through a finite and limited interval, while the pressing [force] moves a finite and limited resistance through an infinite interval; hence to the percussent, the interval is proportionable, and not the resistance, while to the pressing [force] the resistance, and not the interval [is proportionable]. These things make me doubt whether Sagredo's question has an answer, as one that seeks to equate things that are incommensurable; for such I believe are the actions of percussion and of pressing.[111] (EN VIII, 343)

Hence, time is a potential measure for the moment of percussion, but (static) weight is not, whereas weight is a measure for the moment exercised by gravity alone, but time is not.

b. Moment of gravity and acceleration

The intimate relationship between moment of percussion and time is a conclusion of potentially great moment.[112] Galileo in his definition of naturally accelerated motion had already proclaimed that since "the closest affinity holds between time and motion," the uniformity of acceleration had to be understood as the fact that "in any equal times, equal additions of swiftness are added on."[113] Obviously, Galileo also reflected on the relationship between his analysis of the moment of percussion and his earlier work on naturally accelerated motion. In another note appended to the dialogue on percussion, we find the following passage:

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also have sufficed to make sense of the pseudo-Aristotelian "proof" of the law of the lever as given in *Le mechaniche*, but it would have excluded the possibility of including percussion as a mechanical effect.

[111] *Opere*, VIII, p. 343. (Transl. from Galilei 1974, pp. 303-304.)

[112] Despite my different conclusion, for the following I am much indebted to the discussions in Galluzzi 1979.

[113] *Opere*, VIII, pp. 197-198. (Transl. from Galilei 1974, p. 154.)

> The moment of a body in the act of percussion is nothing but a composite and aggregate of infinitely many momenta, each of them equal only to a single moment, either internal and natural *per se*, as is that moment of its own absolute weight [*gravità assoluta*] which it eternally exercises when placed on any resistant body, or else extrinsic and violent, as is that moment of the moving power. Such momenta go accumulating during the time of [naturally accelerated] motion of the heavy body from instant to instant with equal increments, and are stored therein, in exactly the way that the speed of a falling body goes increasing; for as in the infinitely many instants of a time, however short, a heavy body goes ever passing through new and equal degrees of speed, always retaining those acquired in the previously elapsed time, so also in the moveable those momenta (either natural or violent, conferred on it by nature or by art) go conserving themselves and compounding from instant to instant, etc.[114]

As has been stressed by Paolo Galluzzi, Galileo refrains here from explicitly stating that we are dealing with a direct causal relationship between the accumulation of the momenta (which must be here understood as momenta of gravity, as indicated by Galileo himself) and the acceleration of the motion.[115] He "merely" points out a striking analogy between both phenomena. According to Galluzzi this must be attributed to the independence of acceleration from weight – how could this fact have possible been squared with such a causal relationship?

Since the notes we are discussing here are among the last of Galileo's life, it is possible that he had no time left to think this problem through, and was forced to end with the cautionary tone that is discerned by Galluzzi. Given his earlier analyses of the thought experiment, he nevertheless had all the elements at his disposition to come up with a solution. It was already concluded there that adding extra matter does not press on a falling body, and that therefore no extra speeds are added – although such a body with greater gravity will have to "tend more downwards". Seeing the thought experimental situation through the mechanical conceptual apparatus which Galileo has been exploiting in his analysis of percussion, it is clear that this extra matter *does* add moment of gravity. This extra moment will then also be accumulated during the time of fall. And it is indeed undeniable that a heavier body will have a greater moment of percussion at the time it meets a resistance. What remains is the question why the greater moment of gravity has its effect in a greater percussion, but not in a greater increment of speed. That Galileo knew how to understand this, is evidenced by the following fragment, again from the notes appended to the dialogue on percussion. I will quote a long part, to give a taste of Galileo's knack of extracting physical insight from everyday phenomena.

[114] *Opere*, VIII, p. 344. (Transl. from Galilei 1974, p. 304.)

[115] Galluzzi 1979, p. 403.

> He who shuts the bronze door of San Giovanni will try in vain to close them with one single push; but with a continual impulse he goes impressing on that very heavy movable body such a force [*forza*] that when it comes to strike and knock against the jamb, it makes the whole church tremble. From this one sees how there is impressed in moveables – and the more, the heavier [*più gravi*] these are – and how there is multiplied and conserved in them the force [*forza*] that has been communicated to them over some time.
> A similar effect is seen in a great bell, which is not set in strong and impetuous motion with a single pull of its rope, nor with four, or six [pulls], but [is] with a great many. These being long repeated, the final [pulls] add force [*forza*] to that acquired from the preceding pulls; and the thicker and the heavier [*grave*] the bell shall be, the more force [*forza*] and impetus it acquires, this being communicated to it in a longer time and by a larger number of pulls than are required for a small bell, into which impetus is readily put, but from which it is also readily taken away, this [small bell] not drinking in, so to speak, as much force [*forza*] as the larger one.[116]

If we are allowed to translate this insight to the case of falling bodies, we finally reach a completely coherent understanding of the phenomenon of free fall. The body's gravity is continually pulling the body down, adding increments of speeds, yet the heavier the body, the stronger the pulls shall have to be. More matter adds more moment, but not more speed, since now there is also more matter that must be put in motion.[117] Are we allowed to translate this insight? I would urge that Galileo was moving towards a position in which this made perfectly sense. Have another look at the previously cited fragment in which the analogy between the accumulation of momenta of gravity and increments of speeds was expounded, and notice how Galileo is clear on the fact that it is indifferent whether these momenta are natural or violent. This reading is further confirmed by another fragment which was dictated by the by then blind Galileo's to Viviani in which he compares the action of gravity in natural motion with the wind which moves a boat.[118]

Further indirect proof is provided by the fact that Galileo was not as reluctant as suggested by Galluzzi to consider the continuous action of the momenta of gravity as the cause of the acceleration

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[116] *Opere*, VIII, p. 345-346. (Transl. from Galilei 1974, pp. 305-306.) A comparable passage, also dealing with the sounding of a great bell is found in the first day of the *Discorsi* (*Opere,* VIII, p. 141).

[117] Westfall 1966, pp. 77-78, n. 13, also cites a fragment from the first day of the *Discorsi* where Galileo in discussing the frequency of vibrations of strings notes the following: "Here note that the heaviness of the moveable is more resistant to speed than is its thickness" (*"E qui notisi come alla velocità del moto più resiste la gravità del mobile che la grosezza"). Opere,* VIII, p. 146. (Transl. from Galilei 1974, p. 103.)

[118] *Opere*, VIII, pp. 441-442. Galluzzi also cites this fragment as evidence for the fact that Galileo in the end came close to the kind of view just expounded in this paragraph. He also cites from writings of Torricelli and Baliani a similar view is expressed. (Galluzzi 1979, pp. 323-326.)

in free fall. On introducing the system with the two connected bodies on an inclined plane as a way to measure the moment of percussion, Galileo also considers a special case: what happens if the bodies have the same weight? The body moving along the vertical is in free fall until it snaps the cord. At this point the weights of both bodies cancel out, and the combined system has a speed conferred to it by the moment of percussion of the first body. Given that there now is equilibrium of forces, this speed will be equably conserved. Significantly, Galileo himself explicitly likens this situation to what happens on a perfectly horizontal plane.[119] Even more suggestive, he then adds the following explanation for this situation, linking it with the acceleration which gave the percussion its moment:

> Now it is evident that this degree of speed will not go on increasing when *its cause of increase* is taken away, this *being the weight [*gravità*] of the descending body itself*; for its weight [*gravità*] no longer acts when its propensity to descend is taken away by the repugnance to rising of its companion of equal weight [*peso*].[120]

A similar view is also contained in the fragment where Galileo compared the action of gravity with the wind blowing in the sail of a boat: in both cases the motive force acts to add extra speed on a body which is already in motion due to the earlier action of the force – the accelerated character is thus explained as the joint effect of a constant force and the conservation of motion, both linked with a uniform flow of time.

If we take all this together, the following picture emerges: at every instant of time the body's gravity gives rise to a moment of gravity, which in its turn gives the body a degree of speed – which will be independent of the particular strength of this moment. Both these momenta and degrees of speed are conserved during the next instants of time, respectively explaining the percussive effect and the natural acceleration. This also provides an alternative explanation for Galileo's reluctance about claiming a direct causal relationship between the accumulation of momenta of gravity and the degrees of speed, which merely were said to increase in the same way. To claim such a direct causal relationship would indeed be too hasty, since this would not take into account the independent conservation of momenta and speeds – or to put it differently, *this would ignore the crucial role played by time*. Yet, this does not preclude that each individual moment is the cause of each individual degree of speed.

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[119] A discussion of Galileo's proto-inertial principle lies outside the scope of the present paper, but a detailed analysis thereof would again have to stress Galileo's emphasis on shared and familiar experiences.

[120] *Opere*, VIII, p. 337. (Transl. from Galilei 1974, p. 297.)

c. A new model of intelligibility

It is clear that the foregoing attribution of these ideas to Galileo is in part a reconstruction on the basis of what may seem rather scant information. The main reason for doing so lies in the intimate link of these ideas with the lessons learned from the thought experiment. Without taking the latter into account as a natural source for the further development of Galileo's dynamical thinking, these latest ideas might indeed appear as a loose set of fragmentary insights.[121]

There are a few places where we can most clearly detect the influence of the way Galileo rethought his thought experiment in his attempts to ascertain the moment of percussion. In at least two passages in his dialogue on percussion, there is a direct return to the analysis of weight that he had attempted in his *postils to Rocco*. At one point he describes an attempt to measure percussion involving a balance with at one end a counterweight and at the other end a bucket filled with water, under which was hung another empty bucket. The upper bucket was then pierced with a hole, and the idea was that the percussive effect of the water could then be ascertained through the extra counterweight that had to be added. Yet a complication arises because the water, while it is in the air in between both buckets,

> does not weigh [*non gravita*] at all against either upper or lower bucket. Not against the upper, for the parts of water are not attached together, so they cannot exert force [*far forza*] and draw down on those above, as would some viscous liquid, such as pitch or lime, for example. Nor [does it weigh] against the lower [bucket], because the falling water goes with continually accelerated motion, so its upper parts cannot weigh down [*gravitare*] or press against its lower ones. Hence it follows that all the water contained in the jet is as if it were not in the balance.[122]

It is noteworthy that by now Galileo explicitly stresses that it is the relative acceleration that is of importance rather than the speeds, a fact which was not mentioned in the *postils* (where acceleration remained completely out of the picture – although Galileo consciously seems to have left ample room for its introduction by always using "degrees of speed"). In a second passage Galileo repeats the example of the ball and hand moving down with the same speed.[123]

The effect of the thought experiment is much more pervasive, however: it does not just provide a few striking examples, *it offers a new way of thinking about weight itself*. The use of active language is conspicuous throughout the notes on percussion. Galileo continually speaks about a body

[121] As implied by Westfall 1971, p. 39; Wisan 1984, p. 286.

[122] *Opere*, VIII, 324-325. (Transl. from Galilei 1974, p. 285.)

[123] *Opere*, VIII, 331. (Transl. from Galilei 1974, p. 292.)

exerting its gravity ("essercitasse sua gravita"[124]) and about the *operation* of its gravity ("operando colla gravità"[125]). This is obviously linked with his central goal, i.e. measuring the *effect* of percussion. But is seems that he had now found a way of moving ahead towards this goal, precisely because he had realized that he had to conceptualize a body's dead weight ("peso") as an effect as well.[126] This moreover immediately paved the way for a reintegration of this weight in Galileo's still developing dynamical scheme which he is exploring in these notes on percussion. It was only a small conceptual step from the realization that the measurement of weight is only possible if there is a continually re-acting force to the point where we find Galileo explicitly speaking of a body's "moment of its own absolute weight [*gravità assoluta*] which it eternally exercises when placed on any resistant body".[127] The thought experiment thus had already provided Galileo with the necessary basis to conceive of a body's weight peculiar non-relation with time. Every body has its gravity, which at every moment of time generates a moment of gravity. Either this moment of gravity is opposed by a resisting force which arises because the body presses which its moment on another body, or a degree of speed is generated.[128] If the resisting body remained in place, because it is somehow fixed, all the continuously arising momenta of gravity will in their turn be continuously annihilated. If the body is not opposed at all, the continuously arising momenta will cause a universal uniformly accelerated motion as explained in the previous section.[129]

By consciously separating the behaviour of heavy bodies constrained to remain at rest and bodies in free motion, Galileo hence effectively separated what we would call the domains of statics and dynamics.[130] His treatment of these domains moreover shows some structural similarities with our

[124] *Opere*, VIII, 325. (Transl. from Galilei 1974, p. 285.)

[125] *Opere*, VIII, 325. (Transl. from Galilei 1974, p. 286.)

[126] As already indicated there is short section in *Le mecaniche* which deals with percussion, but in it Galileo did not reach any interesting results. Torricelli also describes some early experiments of Galileo, which he did in Padua, but which again were unable to lead to any unambiguous conclusions (cf. Moscovici 1967, pp. 433-435).

[127] *Opere*, VIII, p. 344. (Transl. from Galilei 1974, p. 304.)

[128] The latter generation is hence an action without a reaction. This is another example of the limited nature of Galileo's action-reaction principle (see also supra, note 93). This is of course again connected with Galileo's conceptualization of gravity as internal to a body, whereas Newton's gravitational force will have an unproblematic reaction in the attraction of the earth by the falling body. (It is somewhat imprecise to refer to the moment of gravity as a force internal to the body, since in its pressing and percussion it has an external action on other bodies – however when its effect is the addition of a degree of speed its action remains internal and devoid of reaction.)

[129] As is often the case with Galileo, he is not entirely consistent in his terminology, but one can see a fairly general attempt to use "peso" exclusively for what we would call static weight, and "gravita" for the underlying dynamical cause.

[130] It must always be kept in mind that for Galileo any body is always under the influence of the force of its own internal gravity.

classical understanding of them. It is probably not accidental that the element which allowed him to effect this separation has a classical counterpart in Newton's third law. Yet we should of course not forget the essential differences between Galileo's understanding and a modern one. He might have separated what we can recognize as statics and dynamics, but he had "dynamicized" *all* motion. Even "inertial motion" is essentially an effect of a special kind of dynamical situations.[131] Paradoxically, Galileo who is often hauled as the father of modern kinematics, couldn't conceive of kinematics strictly speaking. Motion remained unthinkable for him in the absence of all forces. Moreover, even if he approached some kind of inertial ideas in his final conceptualization of free fall, the only resistance against acquiring new degrees of speed that he could think of was a body's weight (again due to the identification of gravity as something internal to bodies).

On the other side of the historiographical spectrum, one could also recognize some traces of the medieval impetus theories in Galileo's independent conservation of the accumulated momenta responsible for the force of percussion. Yet more important than what remains of the older views, is what has changed in the meantime. In his *De motu* explanation of the accidental acceleration of bodies in free fall, Galileo had already explicitly conceptualized the force which is impressed on a body by someone or something preventing its motion as an artificial lightness. We have also seen how in his notes on percussion Galileo still conceptualized artificially impressed momenta as commensurable to the internal and natural momenta of gravity. But by now he concept of moment has replaced the concept of heaviness/lightness. This has enabled Galileo to see static weight as an effect of something more fundamental. "Statics" is hence no longer the basis of all his thinking; it is only the special situation in which the natural momenta are opposed by a resisting force. Natural motion can be understood "dynamically" within its own right, with time appropriately being the determining factor that sets apart dynamics from statics. That it can be understood within its own right testifies to the fact that Galileo has by now found a way of offering new incontestable experiences which can anchor his explanatory scheme. Once the thought experiment has taught us to look at the world in the right way, the things themselves indeed show us that we should distinguish between bodies constrained to remain at rest and bodies in free motion. At this point we can start conceiving of Galileo's treatment of free fall as providing a model of intelligibility in its own right, exactly as Newton will do.

[131] This is due to the fact that "inertial states" are only thinkable for Galileo in the presence of forces – in complete opposition to the classical viewpoint. A body lying on a perfectly horizontal plane is indifferent to motion according to Galileo, not because there are no forces, but precisely because its moment of gravity is offset by the force eternally exerted on it by the plane. This is of course due to Galileo's conviction that gravity is something internal to matter, responsible for its tendency toward downward motion. The example of the two connected bodies (see section 8.b) also shows that within Galileo's thinking inertial motion must not necessarily be restricted to horizontal motion, as long as there are the right kind of opposing forces.

Figures

Figure 1 (*Opere*, I, p. 217).

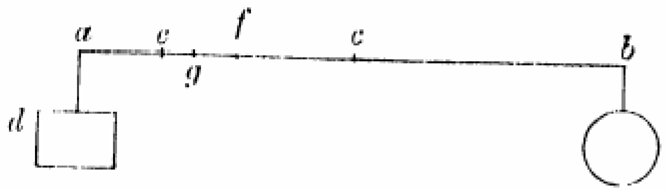


Figure 2 (*Opere*, II, p. 163).

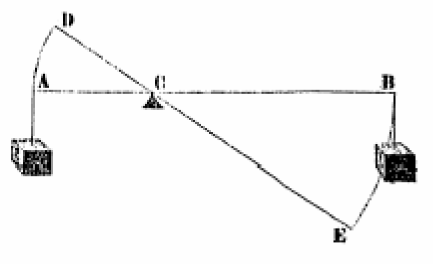
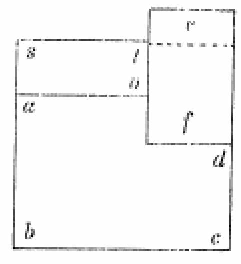


Figure 3 (*Opere*, I, p. 255).



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