

CO-HYPERINTENSIONALITY

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Abstract

Co-hyperintensionality, or hyperintensional equivalence, is a relation holding between two or more contents that can be substituted in a hyperintensional context *salva veritate*. I argue that two strategies used to provide criteria for co-hyperintensionality (appeal to some form of impossible worlds, or to structural or procedural equivalence of propositions) fail. I argue that there is no generalized notion of co-hyperintensionality that meets plausible desiderata, by showing that the opposite thesis leads to falsity. As a conclusion, I suggest to take co-hyperintensionality as a primitive and I provide a general criterion of co-hyperintensionality whose content depends on each hyperintensional notion we aim to formalize.

1. Co-hyperintensionality: the problem

Necessary equivalence (co-intensionality, or co-extensionality in all possible worlds) is too coarse-grained for the equivalence of hyperintensional contexts.¹ Consider the following sentences.

- (1) Mary believes that ABC is triangular.
- (2) Mary believes that ABC is trilateral.

(1) and (2) are not equivalent beliefs, even if their prejacentes are necessarily equivalent.

On the other hand, (3) and (4) report equivalent beliefs, because their prejacentes are identical, and the matter is now trivial.

- (3) Mary believes that ABC is trilateral.
- (4) Mary believes that ABC is trilateral.

Something finer-grained than necessary equivalence, but coarser-grained than identity is needed to account for

¹ I do not consider here those views — like Stalnaker's for one — that aim to dispense with or explain away hyperintensionality.

equivalence of hyperintensional contexts. This notion I call co-hyperintensionality, or hyperintensional equivalence. Such a criterion would be able to tell whether:

- (5) Mary believes that ABC is trilateral.
- (6) Mary believes that ABC is three-sided.

report the same belief, for instance whether the prejacent of (5) and (6) can be substituted in a belief context *salva veritate*, that is, they are co-hyperintensional. Without criteria for co-hyperintensionality, we should either suspend judgment on all inferences involving hyperintensional contexts, or be prepared to live with some nasty consequences, like the paradox of omniscience (if Ruth knows a necessary truth, then Ruth knows all necessary truths, provided that 'know' is dealt with as an intensional operator).

Even independently from hyperintensional propositional attitudes (or hyperintensional non-propositional phenomena such as non-causal explanation and grounding), there are convincing arguments for the thesis that meaning is tracked by hyperintensionality:² co-hyperintensionality criteria would then track synonymy. If this were the case, there would be strong *prima facie* evidence of non-trivial cases of co-hyperintensionality, for most cases of synonymy are non-trivial. There at least two obvious strategies to deal with hyperintensionality, and consequently to provide some criteria for co-hyperintensionality: finessing propositions, or augmenting the modal space. In the following, I expose their inadequacy to the task of providing general criteria for co-hyperintensionality. I argue they all fail to meet one or more of the following desiderata:

- (I) *granularity* +: criteria for co-hyperintensionality should be fine-grained enough to account for problems like omniscience;
- (II) *granularity* -: criteria for co-hyperintensionality should be coarse-grained enough not to draw useless distinctions;
- (III) *generality*: criteria for co-hyperintensionality should be general enough to apply to all hyperintensional contexts, from attitudes report to non-propositional hyperintensional phenomena such as grounding or (non-causal) explanation.

² For a linguist's perspective see (Pollard, 2008a; Pollard, 2008b; Pollard, 2015).

2. Finessing propositions

How to account for the difference (or the equivalence) of (5) and (6)? Keeping fixed contextual parameters, their difference (equivalence) depends in substantive ways on their prejacent. It is on the prejacent, then, that our analysis should turn.

Naively, one may think that the difference between the prejacent of (5) and (6) is accounted for by the difference in the sentences ‘ABC is trilateral’, ‘ABC is three-sided’. There are many venerable arguments, however, against the thesis that the object of (propositional!) attitudes such as belief are sentences. If the objects of beliefs were sentences, one could not explain the equivalence of

(7) Ruth believes that ABC is trilateral.

(8) Ruth crede che ABC abbia tre lati (Italian).

Ruth believe-IND-PRS3sg that ABC have-SBJV-PRS3sg
three side-PL

even if they were in fact equivalent, in the sense of expressing equivalent senses (thoughts, propositions, etc., just pick your favorite).

For this and for other reasons, it is very plausible that the objects of attitudes such as beliefs are propositions, or at least are propositional-like.

Co-hyperintensionality, then, would be reduced to the task of finding criteria for the equivalence of propositions, now conceived as something finer than possible-world intensions.³

There at least two ways of doing this: by appeal to syntactic structure, or by appeal to their logical form.

Both strategies have problems of their own. I’ll analyze them in turn.

2.1. Syntactic structure

According to the first strategy, when the “contents” of any two propositions are the same (for instance, they are co-intensional),

³ There are also many venerable arguments either against identity conditions for propositions, or the meaninglessness of such concepts as synonymy, most famously held by Quine. Kratzer (Kratzer, 2012, Ch. 6), among others, holds that the “content” of beliefs are not propositions, but rather (possible) partial situations. Albeit these criticisms apply to this section, they are orthogonal to the whole issue of co-hyperintensionality. The problem of co-hyperintensionality applies to any kind of hyperintensional contexts, be their content propositions or not.

their difference (if any) is to be accounted for by the syntactic structure of the sentences expressing them.⁴

How finely the syntactic distinctions should be drawn depends on one's favorite theory, but it does not matter for the purposes of my argument: in fact, difference in structure is not enough to track hyperintensionality.

Consider the following sentences.

(9) Willard fears all woodchucks are woodchucks.

(10) Willard fears all woodchucks are whistle-pigs.

The prejacent of (9) and (10) are co-intensional and the sentences have the same syntactic structure. But (9) and (10) are not equivalent, and their prejacentes are not co-hyperintensional.

How would a structuralist account for this difference? There are several proposals in the literature: none that I know of makes any appeal to structure anymore.⁵ Admittedly, this does not rule out that structuralism may have further resources, but it is strong *prima facie* evidence that structuralism is not enough to account for hyperintensional differences (and, *a fortiori*, it is not enough for hyperintensional equivalence).

This strategy fails to meet at least *granularity* + (desideratum I): is not fine-grained enough, as it stands, to account for co-hyperintensionality.

2.2. "Logical form"

According to the second strategy, neither possible-word content nor syntactic structure are sufficient for the meanings of propositions, and therefore for their equivalence. What matters is what the proponents of *Transparent Intensional Logic* call the "logical form" of two propositions, that is, the procedure through which we understand or process them.⁶

⁴ This strategy starts possibly from (Carnap, 1947) (intensional isomorphism), and runs through (Cresswell, 1975; Cresswell, 1985), who first used the term 'hyperintensionality', albeit with reference to logical, and not necessary, equivalence.

⁵ For a similar argument, cf. (Ripley, 2012). He points to three alternative non-structuralist strategies a structuralist can appeal to: the nature of our attitudes (Crimmins, 1992), (Richard, 1990), pragmatics (Salmon, 1986), (Soames, 1987), two-dimensional semantics (on which see Sect. 3.2.)

⁶ The thesis that meanings are to be captured by procedures is found in Tichý's constructions (Tichý, 1988), Moschovakis's idealized algorithms (Moschovakis, 1993; Moschovakis, 2006) and others. *Transparent Intensional Logic* has in fact been inspired by Tichý's ideas. A simplified version thereof is the view I have in mind in this paragraph. Cf. (Duží, Jespersen, and Materna, 2010; Duží and Jespersen, forthcoming) and (Duží, 2014).

The difference between

(11) Roderick believes $12 + 13 = 25$.

(12) Roderick believes $13 + 12 = 25$.

is explained by an appeal to the different procedures we use to calculate the prejacent, and therefore the logical form strategy can account for the difference of (11) and (12) even though they are co-intensional and share the same syntactic structure.

But now take another hyperintensional notion, namely (non-causal) explanation (which is often taken to be hyperintensional), and consider the following sentences.

(13) Now it's 2 pm: In two hours' time, it will be 4pm because $2 + 2 = 4$.

(14) Now it's 2 pm: In two hours' time, it will be 4pm because $4 - 2 = 2$.

(13) and (14) have the same syntactic structure and are co-intensional, but the procedures differ. Yet, they are the same explanation, whereas according to the logical form strategy the two explanations should differ. (An argument for the thesis that (13) and (14) are the same explanation will be given in Sect. 3.2, when discussing (19) and (20).)

This second strategy overkills, drawing (at least in this case) useless differences. This strategy fails to meet *granularity* – (desideratum II): is not coarse-grained enough, as it stands, to account for co-hyperintensionality.

Both the structuralist and the procedural strategies focus on propositions, but cannot account for co-hyperintensionality in a convincing manner: since hyperintensionality is not only a matter of propositions, just finessing propositions is clearly insufficient, or at least off-target.

3. Augmenting the modal space

Many of the troubles caused by applying possible-world intensions to hyperintensional phenomena are due to the coarse-grained structure of the standard modal space. A strategy to account for hyperintensionality is to augment the modal space, either by adding impossible or non-normal worlds to the model, or by adding another kind of worlds to the model in order to account for

distinctions that ordinary metaphysically possible worlds cannot draw.

I argue both strategies fail, either because they are either too coarse-grained or too fine-grained, or because of their lack of generality.

Take your favourite account of metaphysically possible worlds: necessary (or impossible) propositions have the same extension in all possible worlds, and therefore cannot be distinguished in hyperintensional contexts. Add impossible worlds: now, two necessary (or impossible) propositions, while keeping their co-extensionality in all *possible* worlds, may differ in their extension in one (or more) *impossible* world(s).

It is now easier to account for the difference of (2) and (1): there is (an impossible) situation where ABC is trilateral without being triangular: their prejacentes have the same extension at all possible worlds, but a different extension in one (or more) impossible worlds.

Two contents are now co-hyperintensional when they have the same extension in all possible *and* impossible worlds, provided that hyperintensional operators are defined over possible and impossible worlds (non-hyperintensional operators behave standardly, being defined over possible worlds).⁷

3.1. *An argument against unrestricted worlds*

But now consider the following argument. Assume an *Unrestricted Comprehension Principle*, according to which ‘for any way the world could not be, there is an impossible world which is like that (Berto, 2013).’

Although seemingly implausible, this principle makes impossible worlds a powerful tool, and has advantages of its own.⁸

Now, let’s stipulate that the following two sentences:

⁷ This recipe is flexible enough to adjust for non-congruential intensional phenomena (think of objective chance), congruential non-intensional phenomena (think of subjective credence), besides fully hyperintensional (ie non-congruential, non-intensional) ones. A context is congruential iff it is closed under logical equivalence; intensional iff it is closed under necessary equivalence. Congruentiality and intensionality may come apart if logical truths are defined with regard to real-world validity, rather than general validity, in the parlance of (Crossley and Humberstone, 1977).

⁸ According to (Nolan, 1997), for instance, by adopting unrestricted worlds one could even keep the underlying logic classical (but note the use of unrestricted worlds in (Rantala, 1982a; Rantala, 1982b)); moreover, one is not forced to make unprincipled distinctions between different kinds of impossibility. Versions of this view have been adopted in different fashions by (Vander Laan, 1997), (Zalta, 1997) and more recently by (Berto, 2014).

- (15) Mary believes that ABC is trilateral.
 (16) Mary believes that ABC is three-sided.

report the same belief, and therefore their prejacentes are co-hyperintensional, that is, they have the same extension in all possible and impossible worlds. Yet, there must be an impossible world, w^* , where ‘ABC is trilateral’ and ‘ABC is three-sided’ differ in their extension, *via* the unrestricted comprehension principle. Sure, this is impossible. But impossibility is the whole point of impossible worlds, after all, especially under the unrestricted account.

It follows that (15) and (16) are not co-hyperintensional, because the extensions of their prejacentes differ in at least one impossible world, w^* , by construction via the *Unrestricted Comprehension Principle*. Suppose there is another prejacent, φ , such that

- (17) Mary believes that ABC is trilateral.
 (18) Mary believes that φ .

(17) and (18) are in fact co-hyperintensional. Yet, there must be an impossible world where ‘ABC is trilateral’ and ‘ φ ’ differ in their extension, *via* the unrestricted comprehension principle. Repeat *ad libitum* up to identity. Unrestricted impossible worlds, while being a powerful and general tool, cannot account for co-hyperintensionality in a useful way, on pain of *reductio*.

Two possible replies on the unrestricted impossibilist’s behalf: *first*, one may introduce *degrees* of co-hyperintensionality, based on the distribution of impossible worlds where two contents have the same extension. This intuition is, however, hard to pin down precisely, given the arguably uncountable number of impossible worlds, provided the impossible worlds domain is even set-sized. *Second*, one may introduce subsets of impossible worlds, individuated by relevant (to each hyperintensional notions) accessibility relations. Then, provided that in all accessible impossible worlds as selected by the relevant doxastic relation two contents have the same extension, they are (belief-)co-hyperintensional. This, if not *ad hoc*, just shifts the task to the accessibility relation. This strategy seems, however, independently unacceptable because it just inverts the order of explanation: for instance by making meaning (if one thinks that hyperintensionality tracks meaning, as many theorists believe) dependent on a theory of belief (or on a theory of the hyperintensional notion one is modeling), captured by the constraints imposed on the accessibility relation(s).

3.2. *Restricted worlds*

One may instead take an “Australasian” stance on impossible worlds, where impossible worlds are closed under some notion of (non-classical) logical consequence, therefore relaxing the Unrestricted Comprehension Principle.⁹ The problem is how to provide the correct notion of logical consequence for each hyperintensional context, because the right resolution needed for belief is arguably different from the one needed for (non-causal) explanation:

(19) Now it's 2 pm: In two hours' time, it will be 4pm because $2(\text{pm}) + 2 = 4(\text{pm})$.

(20) Now it's 2 pm: In two hours' time, it will be 4pm because $4(\text{pm}) - 2 = 2(\text{pm})$.

(19) and (20) are the same explanation, but believing that $2 + 2 = 4$ is arguably different than believing that $4 - 2 = 2$.

Why? Any answer would depend on a specific understanding of explanation, but in general suppose (19) and (20) were not the same explanation. Then, assuming they are an explanation for something, they either (i) would explain different phenomena, or (ii) would explain the same phenomenon in a significantly different manner. But it is easily verified that, keeping certain mathematical, astrophysical, and conventional facts fixed, neither (i) nor (ii) hold. And this is the case because, contrary to belief, explanation is less sensitive to the agent's capacities or sensibility to modes of presentation.

Impossible worlds are now just standard models of the “right” logic, rather than a direct means to approach (co-)hyperintensionality. My argument does not directly apply to anyone endorsing the Australasian stance; on the other hand, this approach to impossible worlds is much less powerful and has independent problems in explaining (co)hyperintensionality, above all, a lack of generality.

For a concrete example, consider Jago's account (Jago, 2014), (Jago, 2015), which is possibly the most refined applied approach employing impossible worlds on the market. Jago suggests to consider hyperintensions to be functions from epistemically possible worlds to extensions: ‘hyperintensions have a finer grain than

⁹ This view is usually endorsed by relevance and paraconsistent logicians. Cf. (Mares, 1997), (Restall, 1997), and, in general, (Priest, 2005; Priest, 2008).

intensions because they are defined over the larger class of all epistemically possible worlds, which includes some (but not all) logically impossible worlds (Jago, 2014, p. 254).’ Since Jago takes epistemic accessibility relations to be vague (and ‘epistemic contents are indeterminate in membership (p. 265)’), he is able to respect desideratum (I) and (II), at least with regard to knowledge and belief.

But Jago’s theory, which nominally employs (logically) impossible worlds, is best assimilated to a version of 2-D semantics, to which my attention turns in the next section. We will see that this strategy fails to meet desideratum (III): even if it fares well with regard to epistemic notions, it cannot structurally account for other hyperintensional phenomena.

Without severe limitations or *ad hocness*, which would diminish the advantages gained by adding impossible worlds, the (restricted or unrestricted) impossible worlds strategy cannot account for co-hyperintensionality as finer-grained than necessary equivalence, and coarser-grained than identity, eventually collapsing onto it. This strategy fails to meet either desideratum (I) or desideratum (II): is not fine-grained enough, as it stands, to account for co-hyperintensionality, or it is too fine-grained, eventually collapsing onto identity.

3.3. 2D semantics

Two-dimensional semantics proposes to assign to expressions (in a given context) two intensions:¹⁰ primary intensions (functions from epistemic scenarios to extensions) and secondary intensions (standard functions from possible worlds to extensions). For the purposes of this paper, I abstract away from Chalmers’s refined discussion of two-dimensional semantics for attitude-ascriptions, and just focus on one particular trait.

On Chalmers’s two-dimensional account, two contents are necessary equivalent iff they have the same secondary intension. Two contents are epistemically equivalent iff they have the same primary intension.

If one thinks that hyperintensionality just tracks cognitively relevant phenomena (those that can be explained by epistemic scenarios), then one may be able to give an account of co-

¹⁰ I consider here Chalmers’s approach, in particular his application to propositional attitudes in (Chalmers, 2011).

hyperintensionality: two contents are co-hyperintensional iff they have the same primary (and secondary) intension.

But hyperintensionality is not just sensible to cognitively relevant phenomena: think of explanation, or of grounding: there, two contents may well be *non* co-hyperintensional even if they have the same cognitive significance, that is, they have the same primary intension. Just consider the following sentences.

- (21) Now it's 2 pm: In two hours' time, it will be 4pm because $2 + 2 = 4$.
- (22) Now it's 2 pm: In two hours' time, it will be 4pm because 4 is the successor of the successor of 2.

' $2 + 2 = 4$ ' and '4 is the successor of the successor of 2' have arguably the same cognitive significance because understanding the concept of natural numbers (greater than zero) is understanding the concept of "the successor of" operation (under at least one such explanation), even if, in the context of explanation, ' $2 + 2 = 4$ ' and '4 is the successor of the successor of 2' do not explain in the same way why, given now it's 2pm, it will be 4pm in two hours' time.

As it stands, this strategy fails to meet desideratum (III): is not general enough to account for co-hyperintensionality.

At this point it is highly plausible to extend this paradigm either by adding other intensional levels, tracking explanatory scenarios (or the relevant hyperintensional notion), or by identifying primary intensions with explanatory scenarios (or the relevant hyperintensional notion).

But there are three general worries to be raised: *first*, not all hyperintensional contexts are propositional or quasi-propositional: think of grounding. One would then need to add translation principles from the hyperintensional relata (say, facts), to relevant propositions. Not impossible to do, but cumbersome nonetheless. *Second*, adding other intensional levels has been considered equivalent to adding impossible worlds,¹¹ therefore inheriting not only the standard problems of impossible worlds, but also the additional problem I raised in Sect. 3.1. *Third*, adding one intensional level *per* hyperintensional notion seems helplessly *ad hoc*. This is not perforce a shortcoming, as I will argue in the next section.

¹¹ Cf. (Yagisawa, 2010, Ch. 8) and (Ripley, 2012).

4. No general hyperintensionality

I give a *reductio* argument for the thesis there isn't a notion of co-hyperintensionality adequate for all hyperintensional contexts. More precisely, there is no notion of co-hyperintensionality meeting the following conditions

- (I) *granularity* +: it is finer-grained than logical or necessary equivalence, in order to account for problems like omniscience;
- (II) *granularity* -: it is coarse-grained enough in order to not collapse on to identity;
- (III) *generality*: it is general enough to apply to all hyperintensional contexts, from attitudes report to non-propositional hyperintensional phenomena such as grounding or (non-causal) explanation.

These three desiderata are jointly impossible.

Assume general co-hyperintensionality is true, that is, for every hyperintensional notion H , the same co-hyperintensionality relation picks out exactly all the H -equivalent relata. Take any two hyperintensional notions H' , H'' , such that A and B are H' -equivalent. By assumption, they are H'' -equivalent. Now simply show that given two hyperintensional notions H' , H'' , while A and B are H' -equivalent, they are not H'' -equivalent. Contradiction. For such a counterexample, consider again

- (23) Now it's 2 pm: In two hours' time, it will be 4pm because $2(\text{pm}) + 2 = 4(\text{pm})$.
- (24) Now it's 2 pm: In two hours' time, it will be 4pm because $4(\text{pm}) - 2 = 2(\text{pm})$.

(23) and (24) are the same explanation, but believing that $2 + 2 = 4$ is not believing that $4 - 2 = 2$, contrary to hypothesis.

There is no notion of co-hyperintensionality which is general, because there is at least a counterexample to the general statement.

This does not show there is no notion of co-hyperintensionality whatsoever, but only that the three desiderata, which are reasonable and intuitive, are jointly impossible.

(Bader, ms.) for example, endorses a neat distinction between "worldly" and "non-worldly" matters: the former refers to properties, facts, states of affairs (among others); the latter to propositions and other "conceptual" matters. In Bader's approach the

notion of hyperintensional equivalence is different, albeit structurally analogous, in the worldly and nonworldly domains.

Let's assume that there is in fact a substantial distinction between worldly and conceptual matters. This would require (i) just a small modification in the desiderata, splitting generality:

- (III-a) *worldly-generality*: there is a notion of co-hyperintensionality that is general enough to apply to all worldly hyperintensional contexts;
- (III-b) *nonworldly-generality*: there is a notion of co-hyperintensionality that is general enough to apply to all non-worldly hyperintensional contexts;

and (ii) running two parallel arguments to find counterexamples, one for each domain.

For a concrete example, take Bader's theory. Two properties are hyperintensionally equivalent iff they share the same grounders and are grounded in the same way, ie the grounding relation is of the same flavor (metaphysical, nomological, normative). This is at least an implicit admission that there is no unique notion of hyperintensional equivalence even in the worldly domain, because the grounding relation does substantive qualitative work. For a (rough) comparison, consider necessary equivalence and the accessibility relation in possible-world semantics. Obviously, $\Box(\alpha \equiv \beta)$ "means" different things depending on the interpretation of the necessity operator; (possible-world) semantically, depending on the accessibility relation. One may then argue that there are multiple notions of necessary equivalence, structurally analogous but different nonetheless. However, the role played by the accessibility relation is not qualitative at all, and does not substantive work, rather, it just enlarges or shrinks the domain: that's why necessary equivalence is just one single notion.

One could try to read Bader's account in this way. Bader's grounding relation, instead, makes qualitative distinctions when the domains are already identical: that's why his account does not provide a unique notion of hyperintensional equivalence even in the worldly domain.

However, this argument could be taken to weaken the plausibility of the desiderata, rather than the plausibility of co-hyperintensionality understood in such a way as to respect all three of them. Since the strongest desideratum is *generality*, it is *generality* itself that might be thought to be false.

In the next section I show instead how *generality* can be retained, although the price to pay is a higher degree of abstractness.

5. Primitivism and a general formal method for co-hyperintensionality

No criterion for co-hyperintensionality considered so far has met all the desiderata. There are at least two reasons: *first*, none of the theories considered is originally conceived as a theory of hyperintensionality (and *a fortiori* of co-hyperintensionality) *per se*. These are theories devised to account only for (some) hyperintensional phenomena. It is then not a surprise they lack in generality. *Second*, hyperintensionality, as currently defined and understood, is no more than a *negative* notion, that is, simply everything-not-intensional. This lack of a positive definition is a mixed blessing: on one hand, it's a shortcoming. On the other, it's a feature because such a negative definition can group many diverse phenomena.

There are at least two features common to current accounts: (i) *flatness* and (ii) *fixed domains*. Both produce bad consequences. As for (i), flatness is an essential feature of set-theoretic environments: $\{0, 1\}$ is the same set as $\{1, 0\}$, because they have the same elements, if further ordering conditions are not imposed. More generally, set-theoretic approaches are hostage to extensionality-like axioms, which result in coarse-grainedness. The approaches considered in Sect. 2 tried to remedy this by imposing some structure. As for (ii), domains (of propositions, beliefs, worlds) are required to be fixed. This is a problem because, especially in hyperintensional contexts, we might consider some things the same and yet different at the same time. The approaches considered in Sect. 3 tried to remedy this by introducing impossibilities in the first place.

I will now sketch a general *method*, rather than a single *criterion*, to account for co-hyperintensionality, that combines the insights gained from both (i) and (ii) in a coherent manner and hopefully satisfies all three desiderata.

As for (i), I employ lattice-theoretic structures, in order to eschew the flatness typical of set-theoretic environments (like standard possible-world semantics).¹² These may be lattices of

¹² PW semantics are flat in the sense that (a) individual worlds are themselves unstructured; (b) the structure one might think to recover from the accessibility relation is not suitable to work as an ordering, in general.

states, situations, propositions, etc.; the details can be abstracted away as it is the algebraic construction that matters. This also makes my approach immediately compatible with both propositional and non-propositional phenomena.¹³

As for (ii), I employ not just *one* structure (or one per propositions, one per state of affairs, etc.), but rather an indexed family of lattices A_i , $i \in G$, where G , the indexing set, is a poset, where the \leq_G represents the degree of coarseness of the state description, such that for $g \leq_G g'$, A_g is at least as fine-grained as $A_{g'}$.

This is easily seen as formally regimenting contextual variations and interactions, or simply by changing the hyperintensional notion in question. The A_i represent the same “stuff” but with different resolutions, or zooms.

This is formally achieved by defining for all $g, g' \in G$ such that $g \leq_G g'$ a homomorphism $\eta_{gg'}: A_{g'} \rightarrow A_g$ such that:

1. for all $g \in G$, η_{gg} is the identity function on A_g
2. for all $g, h, i \in G$, such that $g \leq_G h \leq_G i$, $\eta_{gi} = \eta_{gh} \circ \eta_{hi}$

It is clear that, given the homomorphisms, a state that is atomic on a certain (coarse-grained) lattice (description) can in fact be the sum of more complex states on another (finer-grained) lattice (description). Whether absolutely atomic states exist is a problem we do not tackle here; it is enough to show that the formal framework can capture both options, we are content to say that most of the time this problem is a matter of our aims of enquiry, level of description, etc.

As for the co-hyperintensionality problem, the desiderata can be formally captured in our family of structures by considering homomorphisms defined either on the same lattice, or on different lattices, such that there is a difference between identical states (an “intralattice” notion, which is obtained when the homomorphism is the identity function) and homomorphic states (an “interlattice” notion), when homomorphism is not the identity function, ie defined on the same lattice.

Note that, for each homomorphism $\eta_g: A \rightarrow B$, we could quite naturally consider its kernel $\ker(\eta_g)$ as the equivalence relation \sim on A such that $a \sim b$ iff $\eta_g(a) = \eta_g(b)$. \sim is in fact a congruence

¹³ Modal features can be recovered in a lattice-theoretic framework, for instance in the way Fine (Fine, 2014), (Fine, 2016) suggests.

Table 1 Desiderata for co-hyperintensionality

<i>Theory</i>	granularity +	granularity -	generality
necessary equivalence	-	-	-
structure	+	-	-
logical form	-	+	-
impossible worlds	-	-	+
2D semantics	-	-	-
identity	-	+	+
primitivism	+	+	-/+

relation, and we can consider the quotient $A \setminus \sim$ which is isomorphic to the image of η_g by the fundamental homomorphism theorem.

(The kernel is trivial if the homomorphism is injective, i.e. no one element of the domain is mapped to the same element of the codomain.) We can therefore consider the kernel as a measure of “how much” a homomorphism fails to be injective, that is, preserve the distinctions among elements, and a way to capture the granularity of the second structure in the first.

This result is highly generalizable: we can consider just the finest-grained structure with no loss of generality. Provided we have all the homomorphisms, we can reconstruct different level of co-hyperintensionality by considering the different equivalence classes generated by the kernel operation in the finest-grained lattice.

‘ \sim ’ is now our co-hyperintensionality notion, whose exact content depends on each hyperintensional notion, formally captured by each particular homomorphism.

Are the desiderata satisfied? As for the first (granularity +) and the second (granularity -) desiderata, they are satisfied by construction. As for the third (generality) desideratum, it might be both satisfied and not satisfied (this is the reason why, in table 1, primitivism gets both a plus and a minus sign). It is not satisfied in a strong sense because I do not provide a general, unique and *substantial* criterion, valid for each and every hyperintensional notion. It is satisfied, however, in a weaker sense, because I provided a sketch of a general *formal* method, whose precise contents

are to be fixed for each and every hyperintensional notion when specifying the homomorphisms.¹⁴

As a modest conclusion I offer some considerations in support of the thesis that a strong substantial, rather than formal, criterion for hyperintensionality is not desirable, even if it is possible. *First*, hyperintensional phenomena are diverse. Unless we have a unified substantial theory of hyperintensionality, every criterion for co-hyperintensionality aiming to be general enough (in keeping with desideratum III) will turn out to be either too coarse-grained (against desideratum I) or too fine-grained (against desideratum II). Every criterion for co-hyperintensionality aiming at the right fineness of grain will turn out to be not general enough to account for all hyperintensional phenomena, because they require different resolutions. *Second*, intensionality is understandable precisely only against a formal framework. There are no reasons why hyperintensionality should differ, although at present there is no such formal framework employed in theorizing about it.

All these considerations, although not definitive, point towards a primitivist approach, i.e. one that take the relevant notion of co-hyperintensionality as a primitive, when theorizing and modeling a given hyperintensional notion. One result of such an approach has been to solve a good number of puzzles and paradoxes plaguing standard accounts of deontic modals and deontic reasoning, for instance in (Faroldi, ms.).¹⁵

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¹⁴ There are some type-theoretic accounts, in the literature, that diverge from standard intensional, Montagovian approaches by allowing some degree of hyperintensionality, such as (Fox and Lappin, 2005), (Pollard, 2008b; Pollard, 2015), (Musken, 2007). All are decidedly primitivist, introducing directly hyperintensional equivalence, either just for propositions or for all higher-order types. The approach developed in this section is superior because, while being compatible with type-theoretic theories when those are suitably partialized, i.e. structured, it carries out more explanatory work, since co-hyperintensionality is explained with the help of other, more basic, notions, rather than assumed abruptly.

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